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Longitudinal static stability requirements for wing in ground effect vehicle

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ABSTRACT: *The issue of the longitudinal stability of a WIG vehicle has been a very critical design factor since the first experimental WIG vehicle has been built. A series of studies had been performed and focused on the longitudinal stability analysis. However, most studies focused on the longitudinal stability of WIG vehicle in cruise phase, and less is available on the longitudinal static stability requirement of WIG vehicle when hydrodynamics are considered: WIG vehicle usually take off from water. The present work focuses on stability requirement for longitudinal motion from taking off to landing. The model of dynamics for a WIG vehicle was developed taking into account the aerodynamic, hydrostatic and hydrodynamic forces, and then was analyzed. Following with the longitudinal static stability analysis, effect of hydrofoil was discussed. Locations of CG, aerodynamic center in pitch, aerodynamic center in height and hydrodynamic center in heave were illustrated for a stabilized WIG vehicle. The present work will further improve the longitudinal static stability theory for WIG vehicle.*

KEY WORDS: Ground effect; Longitudinal stability; Aerodynamics; Hydrodynamics; Hydrofoil; Vehicle dynamics.

NOMENCLATURE

$M_{a,h}, M_{a,z}, M_{a,\dot{q}}, M_{a,\dot{w}}, M_{a,\theta}, Z_{a,h}, Z_{a,z}, Z_{a,\dot{q}}, Z_{a,\dot{w}}$ Differentiation of aerodynamic forces with respect to corresponding variable

$M_{h,h}, M_{h,z}, M_{h,\dot{q}}, M_{h,\dot{w}}, M_{h,\theta}, Z_{h,h}, Z_{h,z}, Z_{h,\dot{q}}, Z_{h,\dot{w}}$ Differentiation of hydrodynamic forces with respect to corresponding variable

$M_{hb,h}, M_{hb,\theta}$ Differentiation of hydrodynamic forces of fuselage with respect to h and θ

$M_{hf,h}, M_{hf,\theta}$ Differentiation of hydrodynamic forces of hydrofoil with respect to h and θ

INTRODUCTION

The Wing-in-Ground (WIG) effect vehicle is considered to be a promising form of transport, due to its high speed and low fuel consumption (Rozhdestvensky, 2000; Yun et al., 2010; Yang and Yang, 2009). WIG craft lies between a sea-going ship and an aircraft in terms of its characteristics. It is generally slower than airplane but has much lower fuel consumption than ship. The WIG craft would have application wherever there are: (a) significant spans of overwater operations; (b) inadequate aircraft operational bases to support airline operations; (c) beaches or simple port unloading facilities for roll on-roll off operations.

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WIG craft characteristics exceed those of ship and aircraft because of it can carry greater than aircraft payloads over significant distances at general aviation aircraft speeds (Yang and Czysz, 2011). Unlike conventional air vehicle, the characteristics of force and moment of a WIG vehicle vary due to pitch and height, when the vehicle approaches the ground. Only appropriately designed, can WIG vehicle escape from the potential danger of kissing the ground (or water surface). The most important issue for the operation of WIG vehicle is the longitudinal static stability, for the longitudinal dynamic stability can be usually fulfilled when the system parameters, which provide the static stability, are within a certain range.

The issue of longitudinal stability of Wing-in-Ground effect vehicle has existed for decades as a very critical design factor since the first experimental WIG vehicle has been built. A series of studies were performed and focused on the longitudinal stability analysis. Kumar (1968) derived equations of longitudinal motion of a WIG vehicle by using quasi-steady aerodynamic derivatives. Irodov (1970) carried out his work on longitudinal stability of ekranoplans and formulated the criterion of longitudinal static stability as the requirement. Rozhdestvensky (1997) applied mathematics of Extreme Ground Effect (EGE) to study the cross-section of wing in ground effect. Taylor (1995) has carried out an elegant experimental verification of the stability of a schematized Lippisch configuration. The longitudinal stability analysis on a 20-passenger WIG was conducted by Chun and Chang (2002) based on wind tunnel test data. The effect of configurations on longitudinal stability was studied by Yang et al. (2010) and Lee et al. (2010). Considering hydrodynamics, Shi et al. (2007) studied the course stability and longitudinal stability of WIG vehicle in takeoff and landing phases by compling hydrodynamic and aerodynamic forces. Yang et al. (2011) investigated the influence of hydrodynamic forces on longitudinal motion of WIG vehicle. Benedict et al. (2001) proposed a complex mathematical model of WIG motion including the takeoff mode to provide a predictive tool for the design of WIG vehicle. At the same time, WIG vehicles with different configurations considering longitudinal stability have been built (Halloran and O'Meara, 1999; Rozhdestvensky, 2006; Matveev, 2008; The WIG Page, 2009).

In regard to the longitudinal stability of WIG vehicle, the equations of equilibrium in the longitudinal plane has been addressed in the current study, taking into account the aerodynamic, hydrostatic and hydrodynamic forces acting on WIG vehicle. Static stability criterion has been discussed, and effects of hydrodynamics and hydrofoil were also considered. The present work aims to provide reference for design and control of WIG vehicle, and shows a picture of longitudinal stability requirements of a WIG.

LONGITUDINAL DYNAMIC MODEL

Axis system

To describe the motion of a WIG vehicle and the forces acting on it, a stability-axis (XOZ) system was used. The relationship between body-axis (X_BOZ_B) and stability-axis are presented in Fig. 1. They are all right-handed and orthogonal. The origin of axis system is located at the Center of Gravity (CG). In the following is illustrated the mathematical model developed for this work, based on the model proposed by Collu et al. (2010).

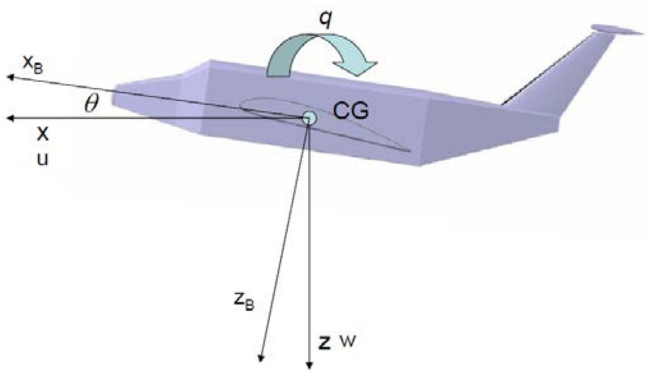


Fig. 1 Stability-axis and Body-axis.

Main forces

The forces acting on WIG vehicle are relatively complex, but it is possible to define and classify them with the approach presented. If the forces due to a control system are not taken into account, in general the forces and moments acting on WIG vehicle can be divided into five groups: gravitational force, thrust force, aerodynamic forces, hydrodynamic forces, disturbance force. So, the total force *F* acting on vehicle can be expressed as,

$$F = F_g + F_a + F_h + F_p + F_d \tag{1}$$

where F_g is gravitational force, F_a is aerodynamic force, F_h is hydrodynamic force, F_p is thrust foce and F_d is environmental disturbance force. Each component consists in:

$$F^i = \begin{bmatrix} X^i & Z^i & M^i \end{bmatrix}^T \tag{2}$$

where X is the force is x direction, Z in z direction and M is pitch moment. Forces and moments are assumed to depend on the values of the state variables and their derivatives with respect to time. Each force and moment is the sum of it value during the equilibrium state plus its expansion to take into account the variation after a small disturbance, which is,

$$\left.\begin{aligned} F &= F_0 + F^{'} \\ F_0 &= \begin{bmatrix} X_0 & Z_0 & M_0 \end{bmatrix}^T \\ F^{'} &= \begin{bmatrix} X^{'} & Z^{'} & M^{'} \end{bmatrix}^T \end{aligned}\right\} \tag{3}$$

where the subscript ($_0$) denotes equilibrium state and superscript (`) denotes perturbation. Depending on the steady state of WIG vehicle, it is possible to make some assumptions on the forces which are negligible.

In this study, it is assumed that the thrust does not vary with small perturbation:. $F_p = F_{p0}$ Environmental disturbances, like waves are beyond the scope of this work; so a stable undisturbed environment is assumed:. $F_d = F_{d0}$

The disturbance from gravity comes from its component in flying direction:

$$\left.\begin{aligned} F_g &= F_{g0} + F_g^{'} \\ F_{g0} &= \begin{bmatrix} 0 & mg & 0 \end{bmatrix}^T \\ F_g^{'} &= \begin{bmatrix} -mg\theta & 0 & 0 \end{bmatrix}^T \end{aligned}\right\} \tag{4}$$

For a wing in ground effect vehicle, the aerodynamics is affected not only by angle of attack, but flight height, which is different to conventional airplane. The aerodynamics and its disturbance can be written as:

$$\left.\begin{aligned} F_a &= F_{a0} + F_a^{'} \\ F_{a0} &= \begin{bmatrix} X_{a0} & Z_{a0} & M_{a0} \end{bmatrix}^T \\ F_a^{'} &= \begin{bmatrix} X_{a,h} \\ Z_{a,h} \\ M_{a,h} \end{bmatrix} h + \begin{bmatrix} X_{a,u} & X_{a,w} & X_{a,q} \\ Z_{a,u} & Z_{a,w} & Z_{a,q} \\ M_{a,u} & M_{a,w} & M_{a,q} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \end{bmatrix} + \begin{bmatrix} X_{a,\dot{u}} & X_{a,\dot{w}} & X_{a,\dot{q}} \\ Z_{a,\dot{u}} & Z_{a,\dot{w}} & Z_{a,\dot{q}} \\ M_{a,\dot{u}} & M_{a,\dot{w}} & M_{a,\dot{q}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} \end{aligned}\right\} \tag{5}$$

In the takeoff and landing regime, the vehicle experiences hydrodynamic forces. There are two main components: hydrodynamic force from fuselage and hydrodynamic force from hydrofoil: $F_h = F_{hb} + F_{hf}$, where, F_{hb} is hydrodynamic force from fuselage and F_{hf} is hydrodynamic force from hydrofoil. Considering hydrodynamic force, the dynamics of wing in ground vehicle are highly non-linear. So, the first step is to linearize the non-linear system. Martin (1978) and Troesch and Falzarano (1993) showed that the added mass and damping coefficients are nonlinear functions of the motion but also that their nonlinearities are very small compared to the restoring forces nonlinearities. Martin (1978) presented an approach to estimate added mass damping and restoring coefficients. Faltinsen (2005) presented an alternative method to compute the added mass damping and restoring coefficients. Then the hydrodynamic force from fuselage can be written as:

$$\left.\begin{aligned} &F_{hb} = F_{hb0} + F_{hb}^{'} \\ &F_{hb0} = \begin{bmatrix} X_{hb0} & Z_{hb0} & M_{hb0} \end{bmatrix}^T \\ &F_{hb}^{'} = \begin{bmatrix} 0 & X_{hb,z} & X_{hb,\theta} \\ 0 & Z_{hb,z} & Z_{hb,\theta} \\ 0 & M_{hb,z} & M_{hb,\theta} \end{bmatrix} \begin{bmatrix} x \\ z \\ \theta \end{bmatrix} + \begin{bmatrix} X_{hb,u} & X_{hb,w} & X_{hb,q} \\ Z_{hb,u} & Z_{hb,w} & Z_{hb,q} \\ M_{hb,u} & M_{hb,w} & M_{hb,q} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \end{bmatrix} + \begin{bmatrix} X_{hb,\dot{u}} & X_{hb,\dot{w}} & X_{hb,\dot{q}} \\ Z_{hb,\dot{u}} & Z_{hb,\dot{w}} & Z_{hb,\dot{q}} \\ M_{hb,\dot{u}} & M_{hb,\dot{w}} & M_{hb,\dot{q}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} \end{aligned}\right\} \tag{6}$$

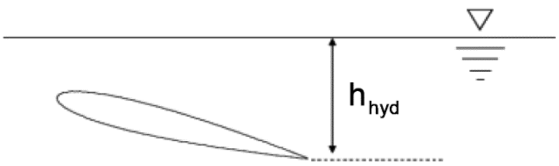


Fig. 2 2D hydrofoil in water.

At the same time, we have $\dot{h}_{hyd} = -\dot{h}$ (Fig. 2). Then the hydrodynamic force from hydrofoil can be written as:

$$\left.\begin{aligned} &F_{hf} = F_{hf0} + F_{hf}^{'} \\ &F_{hf0} = \begin{bmatrix} X_{hf0} & Z_{hf0} & M_{hf0} \end{bmatrix}^T \\ &F_{hf}^{'} = \begin{bmatrix} X_{hf,h} \\ Z_{hf,h} \\ M_{hf,h} \end{bmatrix} h + \begin{bmatrix} X_{hf,u} & X_{hf,w} & X_{hf,q} \\ Z_{hf,u} & Z_{hf,w} & Z_{hf,q} \\ M_{hf,u} & M_{hf,w} & M_{hf,q} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \end{bmatrix} + \begin{bmatrix} 0 & X_{hf,\dot{w}} & 0 \\ 0 & Z_{hf,\dot{w}} & 0 \\ 0 & M_{hf,\dot{w}} & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} \end{aligned}\right\} \tag{7}$$

Equations of motion

Ignoring the coupling of lateral motion and longitudinal motion and eliminating the negligible terms, the linearized equations of longitudinal motion can be expressed as

$$\left.\begin{aligned} &m\dot{u} = X \\ &m(\dot{w} - qU) = Z \\ &I_{yy}\dot{q} = M \end{aligned}\right\} \tag{8}$$

The generalized equations of motion are linearized in the frame of small-disturbance stability theory. In this work a given equilibrium state is assumed, that is the rectilinear uniform level motion, with a nearly steady forward velocity. The system can be expressed as,

$$\left.\begin{aligned} &X_{a0} + X_{h0} + X_{p0} = 0 \\ &mg + Z_{a0} + Z_{h0} + Z_{p0} = 0 \\ &M_{a0} + M_{h0} + M_{p0} = 0 \end{aligned}\right\} \tag{9}$$

By definition for small disturbances, all the linear and the angular disturbance velocities are small quantities. The linearized equations of motion can be expressed as,

$$\left.\begin{aligned} m\dot{u} &= X_u u + X_w w - mg\theta + X_q q + X_{\dot{w}}\dot{w} + X_z z + X_h h \\ m(\dot{w} - qU) &= Z_u u + Z_w w + Z_{\dot{w}}\dot{w} + Z_q q + Z_z z + Z_h h \\ I_{yy}\dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_z z + M_h h \end{aligned}\right\} \tag{10}$$

In addition, the following kinematic condition is added:

$$\left.\begin{aligned} \dot{h} &= -w + U\theta \\ \dot{z} &= w \\ \dot{\theta} &= q \end{aligned}\right\} \tag{11}$$

By defining a state space vector, the system of equations can be written in matrix form as follows:

$$M\dot{\eta} = A\,\eta \tag{12}$$

where M is the mass matrix; A is the state matrix, η is the state vector,

$$M = \begin{bmatrix} m - X_{\dot{u}} & -X_{\dot{w}} & -X_{\dot{q}} & 0 & 0 & 0 \\ -Z_{\dot{u}} & m - Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 & 0 \\ -M_{\dot{u}} & -M_{\dot{w}} & I_{yy} - M_{\dot{q}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} X_u & X_w & X_q & 0 & -mg & X_h \\ Z_u & Z_w & Z_q & -Z_Z & -Z_\theta & Z_h \\ M_u & M_w & M_q & -M_Z & -M_\theta & M_h \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & U & 0 \end{bmatrix}$$

$$\eta = \begin{bmatrix} u & w & q & z & \theta & h \end{bmatrix}^T$$

In cruise, the hydrodynamic force F_h=0, and the state equations can be expressed as,

$$\left.\begin{aligned} m\dot{u} &= X_u u + X_w w - mg\theta + X_q q + X_{\dot{w}}\dot{w} + X_h h \\ m(\dot{w} - qU) &= Z_u u + Z_w w + Z_{\dot{w}}\dot{w} + Z_q q + Z_h h \\ I_{yy}\dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_h h \\ \dot{h} &= -w + U\theta \\ \dot{\theta} &= q \end{aligned}\right\} \tag{13}$$

In matrix form as follows:

$$M_{\dot{\eta}} = A_{\eta} \tag{14}$$

$$M = \begin{bmatrix} m & -X_{\dot{w}} & 0 & 0 & 0 \\ 0 & (m - Z_{\dot{w}}) & 0 & 0 & 0 \\ 0 & -M_{\dot{w}} & 0 & I_{yy} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} X_u & X_w & -mg & X_q & X_h \\ Z_u & Z_w & 0 & (Z_q + mU) & Z_h \\ M_u & M_w & 0 & M_q & M_h \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & U & 0 & 0 \end{bmatrix}$$

$$\eta = \begin{bmatrix} u & w & \theta & q & h \end{bmatrix}^T$$

LONGITUDINAL STATIC STABILTY

Static stability in cruise

In order to study the longitudinal stability characteristics of WIG, the characteristic equation of the system needs to be evaluated. This can be done by taking the Laplace transform of the matrix form. The characteristic equation Eq. (14) is a fifth order form:

$$A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{15}$$

where A_i is coefficients and s is the Laplace variable. According to the Routh-Hurwitz criterion, with $A_5 = 1$, the static stability is assured when:

$$A_0 > 0 \tag{16}$$

and,

$$A_0 = [U\left(M_h X_u Z_w - M_h X_w Z_u - M_u X_h Z_w + M_u X_w Z_h + M_w X_h Z_u - M_w Z_h X_u\right) + mg(M_h Z_u - M_u Z_h)] / mI_{yy}(Z_{\dot{w}} - m)$$

Usually, M_u is negligible and $m >> z_{\dot{w}}$, so Eq. (16) can be written as,

$$M_h Z_u (X_w - \frac{mg}{U}) + X_u (M_w Z_h - M_h Z_w) > 0 \tag{17}$$

In Eq. (17), the derivative $X_w < C_L$, so $X_w - \frac{mg}{U} < 0$. For a stabilized WIG, $Z_u < 0$, $M_h < 0$, $X_u < 0$, $Z_w < 0$ and $Z_h < 0$. Finally, we can get Irodov and Staufenbiel's static height stability criterion,

$$\frac{M_w}{Z_w} - \frac{M_h}{Z_h} < 0$$

$$\text{or,} \quad x_h - x_\theta > 0 \tag{18}$$

where x_θ, corresponding to the aerodynamic center in airvehicle stability, is the aerodynamic center in pitch for WIG vehicle. x_h is the aerodynamic center in height. The axis *x* is directed upstream for body axis system and the symbols are non-dimensional.

At the same time, a WIG vehicle should be stable in pitch like an airplane. The WIG vehicle should respond by a negative increment of the pitching moment to a positive increment of the angle of attack, mathematically given as follows:

$$C_{m,\theta} < 0 \tag{19}$$

where C_m is the moment coefficient with respect to the Center of Gravity (CG). According to this criterion, the aerodynamic center in pitch should locate behind the center of gravity.

In Eq. (18), we can see that the position of the center of gravity does not influence the height static stability of WIG vehicle. It is worthwhile keeping in mind that certain behavior of vehicle can be achieved by locating the center of gravity. The location of center of gravity in relation to the location of aerodynamic centers is important. Favorable position of the center of gravity can provide a properly designed WIG vehicle with sufficient stability, the center of gravity should be located between the aerodynamic centers in pitch and height and close to the center in height (Kornev and Matveev, 2003). To make things a little bit more challenging the positions of both center in pitch and center in height are dependent on height and pitch angle.

Static stability in takeoff and landing

The characteristic equation considering hydrodynamic forces is a sixth order form:

$$A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{20}$$

where A_i is coefficients and s is the Laplace variable. According to the Routh-Hurwitz criterion, with A_6=1, the static stability is assured when:

$$A_0 > 0 \tag{21}$$

and,

$$\begin{aligned} A_0 = {} & U[X_h(M_u Z_z - Z_u M_z) + X_u(Z_z M_h - M_z Z_h)] / [(I_{yy} - M_{\dot{q}})[m^2 - m(X_{\dot{u}} + Z_{\dot{w}}) + \\ & X_{\dot{u}} Z_{\dot{w}} - Z_{\dot{u}} X_{\dot{w}}] - (m - X_{\dot{u}}) M_{\dot{w}} Z_{\dot{q}} - (m - Z_{\dot{w}}) M_{\dot{u}} X_{\dot{q}} - M_{\dot{w}} Z_{\dot{u}} X_{\dot{q}} - M_{\dot{u}} X_{\dot{w}} Z_{\dot{q}}] \end{aligned}$$

The equation is relatively complex and it is difficult to evaluate the stability characteristics. It is possible to get a reduced order system for longitudinal motion. A useful approximation for the mode can thus be developed by setting $\dot{u} = 0$, Therefore, the characteristics equation is fifth order form:

$$A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{22}$$

and,

$$A_0 = \frac{U(Z_z M_h - M_z Z_h)}{(-M_{\dot{q}} + I_{yy})(-Z_{\dot{w}} + m) - M_{\dot{w}} Z_{\dot{q}}}$$

where $Z_z = Z_{a,z} + Z_{h,z}$, $M_h = M_{a,h} + M_{h,h}$, $M_z = M_{a,z} + M_{h,z}$, $Z_h = Z_{a,h} + Z_{h,h}$, $M_{\dot{q}} = M_{a,\dot{q}} + M_{h,\dot{q}}$, $Z_{\dot{q}} = Z_{a,\dot{q}} + Z_{h,\dot{q}}$, $M_{\dot{w}} = M_{a,\dot{w}} + M_{h,\dot{w}}$, $Z_{\dot{w}} = Z_{a,\dot{w}} + Z_{h,\dot{w}}$. By making some assumptions and eliminating the factors that can be negligible, the static longitudinal stability criterion for WIG vehicle when considering hydrodynamic forces, the Eq. (22) can be expressed as,

$$Z_z M_h - M_z Z_h > 0$$

or
$$\frac{M_{a,h}}{Z_{a,h}} - \frac{M_{h,z}}{Z_{h,z}} > 0 \tag{23}$$

This static stability criterion is the same as that for Aerodynamically Alleviated Marine Vehicle (AAMV). It can be seen that the term $M_{a,h}/Z_{a,h} = x_h$ is the aerodynamic center in height of Irodov and Staufenbiel's static height stability criterion for WIG vehicle in cruise. The term $M_{h,z}/Z_{h,z} = x_{hyd}$ can be called hydrodynamic center in heave. So, the static stability of WIG vehicle when considering hydrodynamic forces can be expressed as: the aerodynamic center in height x_h should be located ahead of the hydrodynamic center in heave x_{hyd}.

Hydrodynamic center in heave x_{hyd}

For a WIG vehicle, the derivative $M_{a,h} < 0$, $Z_{a,h} < 0$, and $Z_{h,z} > 0$ considering hydrodynamic characteristics. Analyzing Eq. (22), the following conclusions can be derived:

1) If the x_{hyd} is located ahead of CG, $M_{h,z} > 0$, then $M_{h,z}/Z_{h,z} > 0$. Both terms in Eq. (22) are positive, which means that in order to satisfy Eq. (22) a positive number minus another one should be positive. In addition, although ground effect can enhance the aerodynamic component, hydrodynamic forces acting on WIG vehicle during takeoff, landing and floating might have higher order magnitude of aerodynamic forces because of great difference between density of water and air. Consequently, Eq. (22) can not be fully satisfied.

2) If the x_{hyd} is located behind the CG, $M_{h,z} < 0$, then $M_{h,z}/Z_{h,z} < 0$. The left of Eq. (22) is a positive number minus a negative one, Eq. (22) is satisfied naturally no matter what mode it is.

These conclusions lead to formulating the following requirement for the position of hydrodynamic center in heave x_{hyd}: the hydrodynamic center in heave should be located not only behind aerodynamic center in height, but also behind center of gravity.

Influence of hydrofoil location

One of the problems that developers of WIG vehicle have to resolve is to reduce the power requirement and distance for take-off and landing. The drag during taking-off consists of several contributions, the most significant being the hydrodynamic drag. WIG vehicle needs high lift at low speed during take-off and landing. Several powered-lift schemes have been devised and tested for this problem. One of them is called Power Augmented Ram (PAR for short), which enhances the air cushion by blowing high pressure air under wing. Hydrofoil is another effective approach to improve WIG vehicle performance, especially for takeoff of WIG Hydrowing. In this section, influence of hydrofoil location along the fuselage on static stability of WIG vehicle was investigated.

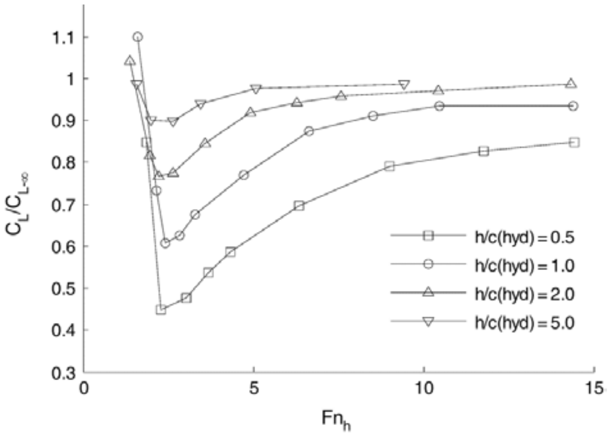


Fig. 3 Free surface effect of 2D hydrofoil (Faltinsen, 2005).

A hydrofoil is a wing-like structure in water that provides lift for boat (Fig. 2). It can lift the boat partially out of the water during forward motion, in order to reduce the hydrodynamic drag. Fig. 3 shows the numerically predicted lift coefficient of a 2D hydrofoil, C_L, as a function of submergence Froude number Fn_h for different *h/c(hyd)* (Faltinsen, 2005).

$$Fn_h = \frac{U}{\sqrt{gh_{hyd}}} \tag{24}$$

The derivative of moment with respect to h for WIG vehicle can be expressed as,

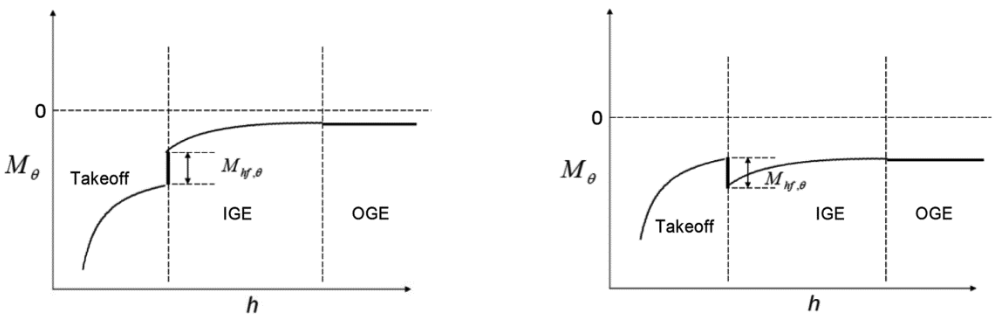
$$M_h = M_{a,h} + M_{h,h} \text{ and } M_{h,h} = M_{hb,h} + M_{hf,h} \tag{25}$$

For WIG vehicle with hydrofoil in water, increase of height for WIG vehicle means decrease of submergence height for hydrofoil, which results in decrease of lift for hydrofoil, $Z_{hf,h} < 0$. WIG vehicle will undergo diving when approaching to the ground, for M_h is positive. From quasi-static force considerations, the diving tendency increases if the hydrofoil is placed after the CG, and the hydrofoil can balance the WIG by compensating for the decreasing moment if it is placed in front of CG.

On the other hand, the derivative of the moment with respect to θ for WIG vehicle can be expressed as

$$M_\theta = M_{a,\theta} + M_{h,\theta} \text{ and } M_{h,\theta} = M_{hb,\theta} + M_{hf,\theta} \tag{26}$$

For a stabilized WIG vehicle, the derivative M_θ should be negative, $M_{a,\theta}$ is negative for pitch stability in cruise, and $M_{hb,\theta}$ is negative based on hydrodynamics. The location of hydrofoil relative to CG will define the magnitude of pitch stability margin, $M_\theta = M_{a,\theta} + M_{hb,\theta} + M_{hf,\theta}$. Fig. 4 shows the influence of hydrofoil location.



(a) Behind CG. (b) In front of CG.

Fig. 4 Influence of hydrofoil location.

If the hydrofoil is located behind CG, the derivative $M_{hf,\theta}$ is negative. It is easy to provide sufficient pitch stability margin of M_θ when floating or taking off for WIG vehicle. Increasing the height in takeoff, the derivatives $M_{a,\theta}$ and $M_{hb,\theta}$ increases. So, the derivative M_θ is approaching to zero. When WIG vehicle finishes takeoff mode, the component of $M_{hf,\theta}$ disappears in M_θ and there is a rapid increase for M_θ. This may lead to a stability margin in the cruising in ground effect phase not being sufficient, or even unstable. If the hydrofoil is located in front of CG, the derivative $M_{hf,\theta}$ is positive. There is a rapid decrease for M_θ when the WIG vehicle finishes takeoff, which will compensate for the increasing of M_θ and ensure sufficient stability margin for WIG vehicle cruising in and out of ground effect.

CONCLUSIONS

The mathematical model of WIG vehicle longitudinal motion was proposed considering both hydrodynamics and aerodynamics. The model is complex and its characteristic equation is in sixth order form in take-off and fifth order in cruise.

Analysis of the equations and the analysis of the static stability criterion shows that the hydrodynamic center in heave should be located not only downstream of the aerodynamic center in height, but also downstream of the center of gravity for a WIG vehicle in water. It is recommended for WIG vehicle with hydrofoil that the hydrofoil should be placed in front of center of gravity to ensure the longitudinal static stability in take off and wing in ground effect cruising phases. And, the aerodynamic center in height should be upstream of the aerodynamic center in pitch for a WIG vehicle cruising in near ground region.

To study the static stability in takeoff and landing regimes, some assumptions were made which limit the practical applicability, as vehicle is highly dynamic in those regimes. And, it can be acceptable for some small WIG craft to be unstable in transient regimes and stable in cruise regime, if the instability does not have time to develop and time interval of takeoff and landing are very short.

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