Novel Characterizations for Switched Nonlinear Systems with Average Dwell Time: Further Findings

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***Abstract*—It is well known, present day theory of switched systems is largely based on assuming certain small but finite time interval termed average dwell time. Thus it appears dominantly characterized by some slow switching condition with average dwell time satisfying a certain lower bound, which implies a constraint nonetheless. In cases of nonlinear systems there may well appear non-expected complexity phenomena of particularly different nature when switching becomes no longer. A fast switching condition with average dwell time satisfying an upper bound is explored and established. A comparison analysis of these innovated characterizations via slightly different overview yielded new results on the transient behaviour of switched nonlinear systems, while preserving the system stability. The multiple-Lyapunov functions approach is used in the analysis and switched systems framework is extended shading new light on the underlying, switching caused system complexities.**

Keywords—arbitrary switching; average dwell time; lower bound condition; multiple Lyapunov functions; switched nonlinear systems; stability; upper bound condition.

I. INTRODUCTION

The behaviour of hybrid systems, to which also switched systems also belong, may have remarkably different system dynamics from either of their components [4, 7, 14, 20, 23, 25, 27]. For example, one switched system can be stable although all its components are unstable, also, some inappropriate switching signal may destabilize the overall switched system even though all of its components are stable [4, 5, 8-12, 14, 21]. Naturally, the analytical studies via the approach relying on Lyapunov stability theory and its extensions [1, 3, 4, 7, 8, 14, 17-20, 22, 27-29] were instrumental for building the theory of switched systems and switching based control [6].

Naturally, the issue of system stability is the crucial one, and moreover in the case of switched systems a rather delicate one [8, 12, 15, 16, 17, 22, 23, 27]. Thus, most of the existing literature is focused on the problem of stability under arbitrary switching [16, 14, 28-31. In due times, many important results have been obtained during a few of the past decades since the pioneering contributions of A. S. Morse (1996, 1997); for instance, see [2, 4, 14, 15, 20, 21, 24, 26, 28, 29, 30]. In order to guarantee stability under arbitrary switching, the common Lyapunov function method plays a rather important role (if not the central role because of its conservatism). This is because the existence of a common Lyapunov function implies the global uniform asymptotic stability of the switched system. The importance of common Lyapunov function has been further consolidated by a converse theorem, due to Molchanov and Pyatnitskiy [17], that asserted if the switched system is globally uniformly asymptotically stable (GUAS), then all the subsystems ought to have a common Lyapunov function.

Over time and in particular more recently, the approach exploiting multiple Lyapunov functions [2, 12, 13, 26] and the associated dwell time [17, 18] or average dwell time [9] are recognized as another rather efficient tool in stability studies of switched systems [10-12, 20, 24-28, 27, 29-31]. The concept of average dwell time switching, which was introduced by Hespanha and Morse (1999) in [9], appeared more general than the standard dwell time switching for both stability analysis and related control design and synthesis problems; for instance, see [10-13, 15, 16, 22-26, 31]. It does imply that the number of switching actions in a finite interval is bounded from above while the average time between two consecutive switching actions is not less than a constant [6, 8, 9, 14, 29]. It is believed the multiple Lyapunov function approach per-se reduces the inherent conservatism of the common Lyapunov function approach.

In fact, when confining to linear dynamic systems only, some well-known design procedures for explicit construction of multiple Lyapunov functions have been developed among which the S-procedure and the LMI [1, 3, 14] and the hysteresis switching action [9, 13, 304] have been particularly fruitful as most of references in this paper on control synthesis design and the references therein clearly demonstrated. In multiple Lyapunov functions approaches, it is generally assumed that each Lyapunov-like function associated for each subsystem is increasing (with the first time-derivative decreasing) with time as time elapses. For the first time, Ye and co-authors (1998) in their stability theory for hybrid systems [28] have also studied an approach allowing for a Lyapunov-like function to rise to a limited extent and have established rather interesting property of a class of such functions called weak Lyapunov.

Inspired by the work of Ye and co-authors (1998), recently in 2016 in [6] and [26] the authors extended considerably their findings so as to shade new lights on the underlying switching caused system complexities. This extension of the switched systems theory is considerably interesting as such and appealing too, since intuitively it may well yield reduced conservatism of stability results, which may not be amenable even to the multiple Lyapunov functions approach. They have shown that both slow switching and fast switching can be studied within an appropriately redefined framework. In addition, the implications were further worked out in producing novel results on stability of switched nonlinear systems.

Notation: The notation used in this paper is fairly standard, which may well be inferred from: $\mathbb{R}^n$ represents the n-dimensional Euclidean space; C^2 denotes the space of twice continuously differentiable functions; C^1 once differentiable piece-wise function.

II. Background and Preliminaries

A. A Preliminary Note

It is important to notice that in this paper the advanced Lyapunov stability theory, the full account of which is found in Khalil's monograph (2002) [7] employing both class K and class $\mathcal{KL}$ functions (additional to classic Lyapunov functions) as well as his Comparison Principle. The concepts of these functions are re-stated first for readability reasons.

Definition 1 [7] A continuous function $k:[0,a)\to[0,\infty)$ is aid to belong to class K if it is strictly increasing and has value $k(0)=0$. It is said to belong to class K_∞ if the upper bound of domain is $a=\infty$ and if $\lim_{\rho\to\infty} k(\rho)=\infty$, i.e. if it is $k:[0,\infty]\to[0,\infty]$.

Definition 2 [7] A continuous mapping $k=k(\rho,\sigma)$ defined by function $k:[0,a)\times[0,\infty)\to[0,\infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed σ, the mapping $k=k(\rho,\sigma)$ belong to class K functions with respect to ρ and, for each fixed ρ, the mapping $k=k(\rho,\sigma)$ is decreasing with respect to σ such that $\lim_{\sigma\to\infty} k(\rho,\sigma)=0$.

B. On Basics of Switched Systems Theory

It is well known that, in general terms [5, 7, 14], a controlled nonlinear dynamic system can be represented by means of the state transition and output measurement equations (1). In order such a system to have sustained operability functions its state transition mechanism must have the property $f(0,0)=0$ and output measuring mechanism must have $h(0)=0$ on the grounds of basic natural laws.

$$\left.\begin{cases}\dot{x}(t)=f(x(t),u(t)),\ \ x(t_0)=x_0\\ y(t)=h(x(t)).\end{cases}\right\}\text{for } \forall t\in[t_0,+\infty)\,. \quad (1)$$

In here, the system's quantities denote: $x\in X^n$ the state space, the input space, $y\in Y^m$ the output space; $f:\mathbb{R}^n\times\mathbb{R}^r\to\mathbb{R}^n, h:\mathbb{R}^n\to\mathbb{R}^m$. Upon the synthesis design of a certain controlling infrastructure then it appears $u=u\left(t;t_0,u_0\right),\forall t\in[t_0,\infty)$.

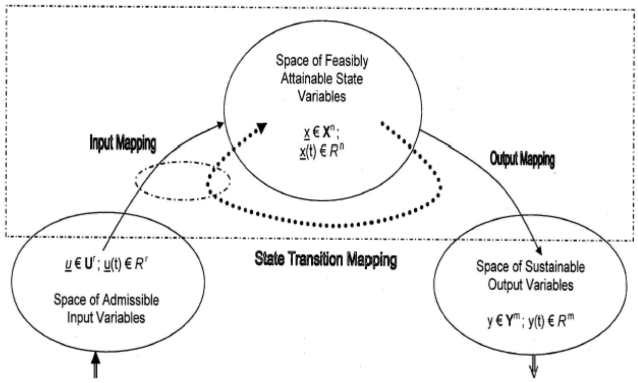


Fig. 1 An illustration of controlled general nonlinear systems in accordance to the fundamental laws of physics [5]; although input, state and output spaces in terms of involved classes of functions can be mathematically defined by a chosen measuring norm, at any fixed time instant all vector-valued variables become real-valued vectors that may be Euclidean ones.

Notice that Figure 1 which depicts a relevant illustration, which is related to this class of systems and the quoted notions. For, it depicts controlled general nonlinear systems in accordance to the fundamental laws of physics on rigid-body energy, matter and momentum of motion, i.e. evolution in time.

Furthermore a controlled nonlinear dynamic system

$$\left.\begin{cases}\dot{x}(t)=f_\sigma(x(t),u(t)),\ \ x(t_0)=x_0\\ y(t)=h_\sigma(x(t)).\end{cases}\right\}\text{for } \forall t\in[t_0,+\infty)\,, \quad (2)$$

where subsystems $f_{\sigma(t)}=f_i, h_{\sigma(t)}=h_i$ with $i\in M$ in an index set M are fixed given models, and the control input $u=u(t)$ is also given (upon its synthesis) is called a switched system. An autonomous switched nonlinear dynamic system thus appears to be defined as follows:

$$\left.\begin{cases}\dot{x}(t)=f_\sigma(x(t)),\ \ x(t_0)=x_0\\ y(t)=h_\sigma(x(t)).\end{cases}\right\}\text{for } \forall t\in[t_0,+\infty)\,. \quad (3)$$

For a causal signal $\sigma:[t_0,+\infty)$, if it is

$$\sigma(t^+)=\Sigma([t_0,t],\sigma([t_0,t]),\{x([t_0,t]),y([t_0,t])\}),\forall t\in[t_0,+\infty) \quad (4)$$

a piece-wise constant time-sequence function such that $\sigma(t^+)=\lim_{\tau\downarrow t}\sigma(\tau)$, for $\tau\geq 0$ in continuous time case, and $\sigma(t^+)=\sigma(t+1)$, for $t\geq 0$ in discrete time case, is called a switching signal.

A switching signal is said to be a switching path if it is defined as mapping of finite, semi-open time interval into the index set *M* such that $\sigma:[t_0,t_1)\to M$ for every $[t_0,t_1)$ with $t_0<t_1<+\infty$. A switching law is called a time-driven switching law if it depends only on time and its past value $\sigma(t^+)=\sum(t,\sigma(t))$. A switching law is called a state-feedback switching law if it depends only on its past value an on the values of state variables at that time $\sigma(t^+)=\sum(\sigma(t),x(t))$ for $\forall t\in[t_0,+\infty)$. A switching law is called a output-feedback switching law if it depends only on its past value an on the values of output variables at that time $\sigma(t^+)=\sum(\sigma(t),y(t))$ for $\forall t\in[t_0,+\infty)$. At present, no other concepts and notions about feasible switching signals matter.

The concept of average dwell time is given below and the respective rather important result is cited too.

Definition 3 (Hespanha & Morse, 1999): For a switching signal σ and any $t_2>t_1>t_0$, let $N_\sigma(t_1,t_2)$ be the number of switching over the interval $[t_1,t_2)$. If the condition $N_\sigma(t_1,t_2)\leq N_0+(t_2-t_1)/\tau_a$ holds for $N_0\geq 1$, $\tau_a>0$, then N_0 and τ_a are called the average dwell time (ADT) and the chatter bound, respectively.

***Theorem* 1** (Hespanha & Morse, 1999): Consider the switched system (3), and let α and μ be given constants. Suppose that there exist smooth functions $V_{\sigma(t)}:\Re^N\to\Re$, $\sigma(t)\in\ell$, and two K_∞ functions k_1 and k_2 such that for each $\sigma(t)=i$, the following conditions hold:

$k_1(|x_t|)\leq V_i(x_t)\leq k_2(|x_t|)$, $\dot V_i(x_t)\leq-\alpha V_i(x_t)$, and for any $(i,j)\in\ell\times\ell$, $i\neq j$, $V_i(x_t)\leq\mu V_j(x_t)$; then the system is globally uniformly asymptotically stable for any switching signal with ADT $\tau_a>\tau_a^*=\dfrac{\ln\mu}{\alpha}$.

Theorem 1 considers multiple Lyapunov functions with “jump” on switching boundary. An extension due to (Ye et al, 1998) in [29], the underlying essentialities of which were further highlighted by Zhang and Gao (2010) in [31], allows the Lyapunov-like function to rise to a limited extent, in addition to the jump on switching boundary. This is the so-called week Lyapunov functions, and it allows both the jump on the switching boundary and the increase over any interval. Now consider $\sigma(t)=i$ and within the interval $[t_i,t_{i+1})$, denote the unions of scattered subintervals during which the week Lyapunov function is increasing and decreasing by $T_r(t_i,t_{i+1})$ and $T_d(t_i,t_{i+1})$, respectively. Hence $[t_i,t_{i+1})=T_r(t_i,t_{i+1})\bigcup T_d(t_i,t_{i+1})$. Further use $T_r(t_{i+1}-t_i)$ and $T_d(t_{i+1}-t_i)$ to represent the length of $T_r(t_i,t_{i+1})$ and $T_d(t_i,t_{i+1})$ correspondingly. Then the following important result can be obtained:

Then the following important result has been derived:

***Theorem* 2** (Ye et al, 1998; Zhang & Gao, 2010): Consider the switched system $\dot x_t=f_\sigma(x_t)$, and let $\alpha>0,\beta>0$ and $\mu>1$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)}:\Re^n\to\Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t)=i$, the following conditions hold:

$$k_1(|x_t|)\leq V_i(x_t)\leq k_2(|x_t|),$$

$$\dot V_i(x_t)\leq\begin{cases}-\alpha V_i(x_t) & over \quad t\in T_d(t_i,t_{i+1})\\ \beta V_i(x_t) & over \quad t\in T_r(t_i,t_{i+1})\end{cases},$$

$$V_i(x_t)\leq\mu V_j(x_t) \qquad \forall\big(\sigma(t)=i \quad \& \quad \sigma(t^-)=j\big),$$

Then the system is GUAS for any switching signal with ADT $\tau_a>\tau_a^s=\dfrac{(\alpha+\beta)T_{\max}+\ln^\mu}{\alpha}$, $T_{\max}=\max T_r(t_{i-1},t_i)$, $\forall i$.

It may well be seen that the result above actually includes Theorem 1 as a special case. Namely, $\beta=0$ implies no increase over the interval and hence $T_{\max}=0$, then the ADT condition reduces to the ADT condition $\tau_a>\tau_a^*=\dfrac{\ln\mu}{\alpha}$, (5) in Theorem 1. It is this generality of the weak Lyapunov functions that has given incentives to explore the alternatives on fast and slow switching rules (Wang and co-authors, 2016).

III. On Fast And Slow Swithing: Recent Dicoveries

In the sequel the subsequently derived novel results, which are subject to further exploration in this paper, are presented first. In what follows, following notations from Branicky (1998) are adopted throughout [2]. In particular, a general arbitrary switching sequence is expressed by

$$\sum=\{x_0;(i_0,t_0),(i_1,t_1),\ldots,(i_j,t_j),\ldots,|\,i_j\in\overline{M},j\in N\} \quad (5)$$

in which t_0 is the initial time, x_0 is the initial state, (i_k,j_k) means that the i_k-th subsystem is activated for $t\in[t_k,t_{k+1})$.

Therefore, when $t \in [t_k, t_{k+1})$, the trajectory of the switched system (1) is produced by the i_k-th subsystem. Thus, for any $j \in \overline{M}$, the set

$$\Sigma_t(j) = \Big\{[t_{j_1}, t_{j_1+1}), [t_{j_2}, t_{j_2+1}), \ldots, [t_{j_n}, t_{j_n+1}) \ldots, \quad \sigma(t) = j, t_{j_k} \le t \le t_{j_k+1}, k \in N\Big\} \tag{6}$$

denotes the sequence of switching times of the j-th subsystem, in which the j-th subsystem is switched on at t_{j_k} and switched off at t_{j_k+1}.

A. Novel Insights into the Complexity of Switching

It should be noted, Theorem 2 is a slow switching result in the sense that it is characterized by a lower bound on the average dwell time. The three recent results have emanated from the novel insights into the switching complexity.

***Theorem* 3 [6, 26]** Consider the switched system $\dot{x}_t = f_\sigma(x_t)$, and let $\alpha > 0, \beta > 0$ and $\mu > 1$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)} : \Re^n \to \Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t) = i$, the following conditions hold:

$$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|),$$
$$V_i(x_t) \le \mu V_j(x_t) \quad \forall(\sigma(t) = i \quad \& \quad \sigma(t^-) = j),$$
$$\dot{V}_i(x_t) \le \begin{cases} -\alpha V_i(x_t) & over \quad t \in T_d(t_i, t_{i+1}) \\ \beta V_i(x_t) & over \quad t \in T_r(t_i, t_{i+1}). \end{cases}$$

Then the system is GUAS for any switching signal with ADT

$$\tau_a < \tau_a^f = \frac{(\alpha+\beta)T_{\min} - \ln^\mu}{\beta}, \; T_{\min} = \min T_d(t_{i-1}, t_i), \quad \forall i.$$

Intuitively, a switched nonlinear will achieve induced stability by arbitrary switching if the upper bound for the fast switching is larger than the lower bound for slow switching, which is the essence of the subsequent theorem.

***Theorem* 4 [6, 26]** Consider the switched system $\dot{x}_t = f_\sigma(x_t)$, and let $\alpha > 0, \beta > 0$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)} : \Re^n \to \Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t) = i$, the following conditions hold:

$$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|),$$
$$V_i(x_t) = V_j(x_t) \quad \forall(\sigma(t) = i \quad \& \quad \sigma(t^-) = j),$$
$$\dot{V}_i(x_t) \le \begin{cases} -\alpha V_i(x_t) & over \quad t \in T_d(t_i, t_{i+1}) \\ \beta V_i(x_t) & over \quad t \in T_r(t_i, t_{i+1}). \end{cases}$$

Then the system is GUAS for arbitrary switching signal if the following condition are fulfilled:

$$\frac{T_{\max}}{T_{\min}} \le \frac{\alpha}{\beta}, T_{\max} \equiv \max T_r(t_{i-1}, t_i), T_{\min} \equiv \min T_d(t_{i-1}, t_i), \quad \forall i.$$

***Theorem* 5 [6, 26]** Consider the switched system $\dot{x}_t = f_\sigma(x_t)$ and let $\alpha > 0$, $\mu > 1$ be given constants. Suppose that there exist C^1 functions $V_{\sigma(t)} : \Re^N \to \Re$, $\sigma(t) \in \ell$, and two K_∞ functions k_1 and k_2 such that $\forall \sigma(t) = i, k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$, $\dot{V}_i(x_t) \le -\alpha V_i(x_t)$, and $\forall (i, j) \in \ell \times \ell$, $i \ne j$, $V_i(x_t) \le \mu V_j(x_t)$; then the system is GUAS for any switching signal *if and only if* the ADT satisfies the condition $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$.

The poof details of these results are found in [6]. Notice that the slow switching condition reduces to the average dwell time of Hespanha and Morse (1999).

B. Further Supporting Evidence via Simulations

At this point, let recall the expression of a general arbitrary switching sequence (5):

$$\sum = \{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_j, t_j), \ldots, | \, i_j \in \overline{M}, j \in N\} \tag{5}$$

Also, notice that t_0 is the initial time, x_0 is the initial state, (i_k, j_k) means that the i_k-th subsystem is activated for $t \in [t_k, t_{k+1})$. Duplicate the template file by using the Save As command, and use the naming convention prescribed by your conference for the name of your paper.

Next, let recall that by means of Jacobians the basic linearization of general nonlinear systems, such as the class of time-varying nonlinear plant processes to be controlled, is as follows:

$$\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t), \vec{u}(t); t), \; \vec{x}(t_0) = \vec{x}_0, \; \vec{y}(t) = \vec{g}(\vec{x}(t); t) \; . \tag{6}$$

Upon linearization of nonlinear functions $\vec{f}, \vec{g}$ in the vicinity neighborhood of a certain operating state, e.g. such as a *steady-state* operating point *EP* ($\vec{u}_c$, $\vec{x}_c$, $\vec{y}_c$) with the *steady-states* $\vec{x}_c, \vec{y}_c$ which may happen under some *steady control input* $\vec{u}_c$, model (6) yields

$$\begin{cases} \dot{\vec{x}}(t) = A(t)_{n\times n}\vec{x}(t) + B(t)_{n\times r}\vec{u}(t), \\ \vec{y}(t) = C(t)_{m\times n}\vec{x}(t), \end{cases} \quad \vec{x}(t_0) = \vec{x}_0, \tag{7 a}$$

where matrices $A(t)_{n\times n}=\partial\vec{f}/\partial\vec{x}_{|x_c,u_c}$, $B(t)_{n\times r}=\partial\vec{f}/\partial\vec{u}_{|x_c,u_c}$, $C(t)_{n\times r}=\partial\vec{g}/\partial\vec{x}_{|x_c,u_c}$. Should the *steady-state* operating point *EP* ($\vec{u}_c$, $\vec{x}_c$, $\vec{y}_c$) is an *desired equilibrium* (which usually is in practice) that can be achieved under a certain *synthesized equilibrium control* vector $\vec{u}_c=\vec{u}_c^e=const$ defining the desired *equilibrium state* vector $\vec{x}_c=\vec{x}_c^e=const$ hence the desired steady-state *equilibrium output* vector $\vec{y}_c=\vec{y}_c^e=const$, then model (6) yields

$$\dot{\vec{x}}=A_{n\times n}\vec{x}(t)+B_{n\times r}\vec{u}(t),\ \vec{x}(t_0)=x_0;\ y(t)=C_{n\times m}\vec{x}(t). \quad (7\text{ b})$$

It is therefore that the *Lyapunov asymptotic stability* requirement on the system’s steady-state equilibrium is indispensable and rigorous requirement. It is this requirement precisely which is being enhanced by involving a switching law in addition to the synthesized feedback control. The presented illustrations by mean of simulated time-responses further below, along with the above theoretical results, highlight the concepts essence of both the fast and slow switching as well as the average dwell time and the importance of achieving asymptotic Lyapunov stability under arbitrary switching. In any case, these finding guarantee in the close loop the ultimately uniform bounded operation of the plant at desired equilibrium steady-state shall be reached despite the possible uncertainties of the plant.

For this purpose let consider the application of the arbitrary switching sequence (5) a second-order two-input-two-output uncertain nonlinear plant system of class (6) whose states are detectable and measurable (if they are not, then state estimator ought to be employed). It is assumed its Jacobian-linearized system is both observable and controllable. Thus, consider example for which a uncertain plant of class (6) yields system (7 b) having the following system matrices:

$$A_{11}=\begin{bmatrix}-5 & 4\\ 0 & 2\end{bmatrix},\ A_{12}=\begin{bmatrix}-15 & 1\\ 0 & -10\end{bmatrix},\ A_{21}=\begin{bmatrix}2 & 0\\ 1 & -5\end{bmatrix},\ A_{22}=\begin{bmatrix}-15 & 0\\ -5 & -4\end{bmatrix},$$

$$B_{11}=B_{12}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\ B_{21}=B_{22}=\begin{bmatrix}3 & 1\\ 1 & 5\end{bmatrix},$$

$$C_{11}=C_{12}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},C_{11}=C_{12}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}.$$

Next, let define

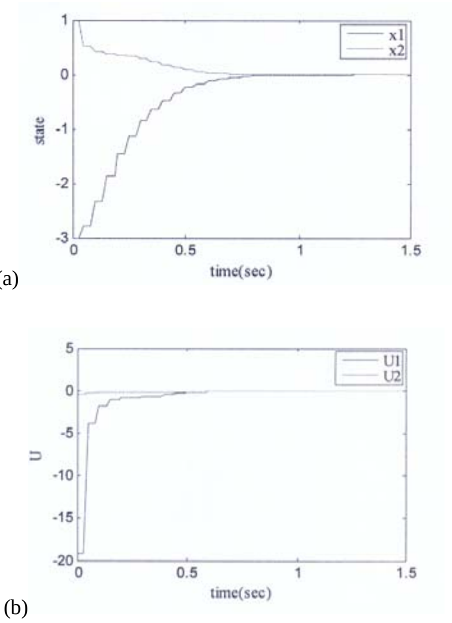
$$\Omega_1=\{x(t)\in R^n \,|\, x(t)^{\mathrm{T}}(P_1-P_2)x(t)\geq 0, x(t)\neq 0\},$$

$$\Omega_2=\{x(t)\in R^n \,|\, x(t)^{\mathrm{T}}(P_2-P_1)x(t)\geq 0, x(t)\neq 0\},$$

where matrices P_1, P_2 are designed in the course of stabilizing state feedback control synthesis. It is therefore that system’s state space is such that $\Omega_1\bigcup\Omega_2=R^n\setminus\{0\}$. Thus the closed-loop system should asymptotically stable or at least uniformly bounded due to the switching law

$$\sigma(t)=\begin{cases}1 & x(t)\in\Omega_1\\ 2 & x(t)\in\Omega_2\setminus\Omega_1\end{cases}$$

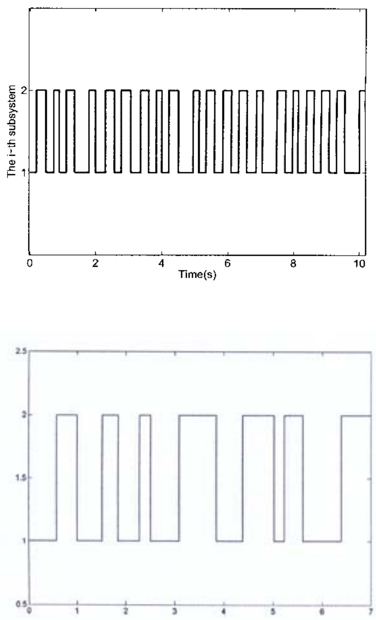
regardless of the plant uncertainty. The simulation results (via Matlab-Simulink of MathWorks, Inc. 2000 [1], [32]) for the state and the control vectors, when the initial condition on states are $x(0)=[-3,1]^T$, are found depicted in Figures 1 and 2, respectively. Figure 3 depicts three switching sequences.



(a)

(b)

Fig. 2 Fast evolution of the time responses of both the state (a) and the control (b) vectors in closed loop under switching based state feedback control.



(a)

(b)

Fig. 3 Evolution of the time responses for three cases of the considered class of switching sequences demonstrating the fast (a) and the slow switching modes with the average-dwell-time strategy in conjunction with the state feedback control.

IV. Concluding Remarks

This paper has further supported the novel characterization of nonlinear switched systems via adopting constrained switching through slow switching and fast switching in [6], [25]. A fast switching rule may even guarantee globally uniform asymptotic stability of the desired steady-state equilibrium to which some synthesized state-feedback control has driven the plant. Thus these finding prove that the standard average dwell time condition associated with the multiple Lyapunov functions, in fact, appears to be an if and only if condition. The three results thus have extended the existing stability theory of switched nonlinear systems.

Acknowledgment

This research has been generously supported by China Postdoctoral Science Foundation (grant No. 20100471336) and NUAA Research Funding Projects (grants No. S0985-021 and No. 1002-56YAH11011). In part, it was also supported by Fund for Science of Dogus University Istanbul and by Ministry of Education & Science of R. Macedonia, Skopje.

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