

USING BAYESIAN NETWORKS FOR THE ASSESSMENT OF UNDERWATER SCOUR FOR ROAD AND RAILWAY BRIDGES

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ABSTRACT

Flood-induced scour is by far the leading cause of bridge failures, resulting in loss of lives, traffic disruption and significant economic losses. In Scotland, there are around 2,000 structures, considering both road and railway bridges, susceptible to scour. Scour assessments are currently based on visual inspections, which are expensive, time-consuming, and often the information collected is qualitative and subjective. The two main transport agencies in Scotland, Transport Scotland and Network Rail, spend £2m and £0.4m per annum, respectively, in routine inspections. Nowadays sensor and communication technologies offer the possibility to assess in real time the scour depth at critical bridge locations; yet monitoring an entire infrastructure network is not economically sustainable. A way to overcome this limitation is to install monitoring systems on a limited number of critical locations and use a probabilistic approach to extend this information to the entire population of assets. The state of the bridge stock is represented through a set of random variables and ad-hoc Bayesian networks (BNs) are used to describe their conditional dependencies. The aim of this paper is to develop a probabilistic scour hazard model by building a BN able to estimate the depth of scour in the surrounding of bridge foundations. The BN can estimate, and continuously update, the present and future scour depth using real-time information from monitoring of scour depth and river flow characteristics. In the occurrence of a flood, monitoring observations are used to infer the posterior distribution of the state variables probabilistically, and therefore to give in real-time the best estimate of total scour depth. Bias, systematic and model uncertainties are modelled as nodes of the BN in such a way as the accuracy of predictions can be updated when information from the scour monitoring system is incorporated into the BN. In order to demonstrate the functioning of the BN, bridges managed by TS in South-West Scotland were used to build a small bridge network. They cross the same river (River Nith) and only one of them is instrumented with a scour monitoring system.

1 INTRODUCTION & BACKGROUND

Flood-induced scour is the principal cause of failure of bridges, resulting in significant loss of life, traffic disruption and economic losses (Wardhana & Hadipriono, 2003). Scour can be defined as the excavation and removal of material from the bed of streams around bridge foundations as a result of the erosive action of flowing water. Scour processes are classified according to the circumstances and structures that have caused it. The following types of scour are reported: (i) constriction scour or contraction scour; (ii) local scour; and (iii) natural scour (Kirby et al, 2015). The first two types are associated with the existence of a bridge or hydraulic structures and can be collectively termed as localised scour. Constriction scour is usually the result of confining the width of the river channel, for instance between bridge abutments and piers, while local scour is caused by the interference of individual structural elements, such as piers or abutments, with the flow. The latter type of scour is characterised by the formation of scour holes only in the immediate vicinity of those elements (Lauchlan & Melville, 2001). Natural scour is instead attributable to natural variations in the flow, irrespectively of the presence of a river crossing, and its contribution to the total scour is neglected in the proposed work.

Scour processes occur naturally and are expected to occur at most bridges and any hydraulic structures during their service life (Richardson & Davis, 2001). Bridges, culverts and every hydraulic structure founded on a river bed are prone to scour around their foundations. The main scour mechanisms listed above work additively to give the total scour (Figure 1). A bridge may fail due to a combination of

different scour types; however, one mechanism is often the major cause to bridge failure. When the depth of scour becomes significant, the capacity of abutment or pier foundations of bridges may be severely compromised, leading to structural instability and ultimately catastrophic failure. Development of scour holes can cause damages to bridges, thus posing a potential threat to public safety.

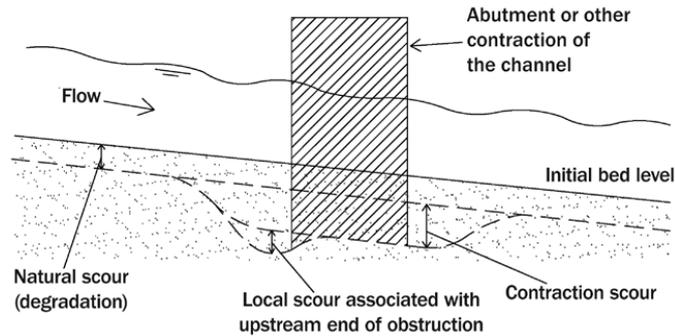


Figure 1. Schematic illustrating total scour (Kirby et al, 2015)

In the UK, there are more than 9,000 major bridges over waterways. According to Van Leeuwen and Lamb (2014), abutment and pier scour was identified as the most common cause of 138 rail bridge failures during the period 1846-2013; in some cases, the failure was associated with fatalities. Almost 95,000 bridge spans and culverts are susceptible to scour processes. Reviews of 1,502 river crossing failures that occurred in the United States in the period 1966 - 2005 revealed flooding and scour were the cause of 58% of the recorded failures (878 bridge failures) (Briaud et al, 2007). Following record daily rainfalls for the UK in November 2009, 20 road bridges across Cumbria were damaged or destroyed and the town of Workington was severed (Cumbria Intelligence Observatory, 2010). Furthermore, the winter storms of 2015 resulted in serious damage/destruction to bridges across Scotland and the north of England (DoT, 2015). This included the Lamington viaduct, which resulted in the closure of the West Coast mainline between Glasgow and London for nearly three months (Network Rail, 2016).

1.1 Vulnerability of bridges to scour

Current risk models calculate potential losses by combining specific hazard parameters, quantified and characterised exposed components and their assessed vulnerability. Vulnerability (or fragility) analysis is an important part in the risk assessment because a vulnerability model can define how much a structure is susceptible to failure with respect to a hazard or, in other words, what is the chance of failure due to the impact of a given hazard (Roca & Whitehouse, 2012). There are two fundamental approaches to evaluate the hazard vulnerability of a system, such as a building or a bridge: fragility functions and hazard indexes (Calvi et al, 2006). The first examples of such risk frameworks started to be developed in earthquake engineering (Porter, 2003) but in the following years, risk analyses combining different stages were employed in flood and coastal engineering (FEMA, 2005) as well as hurricane engineering (Barbato et al, 2013).

A few researchers have then applied this breakdown of risk assessment into basic probabilistic analyses to the problem of scour induced by flood (Roca & Whitehouse, 2012; Tubaldi et al, 2017). However, scour bridge vulnerability has not been adequately studied in the past, perhaps due to the lack of a scour damage database. However, in the literature it is possible to find three different approaches to describe the vulnerability of a bridge against scour mechanisms. These proposed methodologies are based on:

- (i) *Structural analyses*. This vulnerability approach involves finite element analysis to model the interaction between all the components and media included (deck, pier, foundation, soil and water) (Hung & Yau, 2014; Klinga & Alipour, 2015; Zampieri et al, 2017);
- (ii) *Geotechnical analyses*. This approach evaluates the vulnerability to scour considering the bearing capacity of bridge foundation changes resulting from scour (De Falco et al, 1997; Federico et al, 2003);
- (iii) *Performance parameters*. The most common parameter involved in this approach is the ratio between the total scour depth D_T at the pier and the foundation depth D_F . The employment of

this performance parameter has led researchers to proposed methodologies to compute a scour vulnerability index (SVI) (Barbetta et al, 2015).

1.2 Bridge scour in the UK

Network Rail (NR) owns and operates around 19,000 underline bridges nationally: approximately 8,700 of these structures are held within a National Scour Database and the projected spend on scour protective works from 2014-2019 is in the region of £27m. For the Scotland Route only, 1,750 structures are routinely inspected for scour and, of those, 58 are considered to be at high risk. Transport Scotland (TS) is responsible for the Scottish trunk road network including 1,567 bridges or culverts over water. Of those, around 8% are currently classified as needing detailed consideration, including possible monitoring and scour protection measures. TS is currently aware of about £3.5m of known scour repairs and scour resilience works to carry out.

The current practice for bridge scour inspection depends on visual checks at regular intervals. NR and TS assess the risks associated with scour and other effects on highway and railway structures during floods using the Procedures BD 97/12 and EX2502, respectively. The total NR Scotland Route spend on scour assessments in 2016/2017 was approximately £440,000. Similarly, TS spends £2m per annum on routine inspections of bridges and other structures and approximately one-third of its total assets are inspected each year. In addition, all bridges over water are visually inspected for scour effects following periods of heavy rainfall. Underwater visual inspections are even more expensive and time-consuming, and often the information collected is qualitative and subjective. Along with rules on how to perform scour visual inspections and to calculate the estimated scour depth, these two procedures provide a scour risk assessment framework based on a SVI.

In this paper, we develop a scour hazard model by building a Bayesian Network (BN) able to estimate the depth of scour surrounding bridge foundations. The BN can estimate, and continuously update, the present and future scour depth using real-time information from monitoring of scour depth, and river flow characteristics. Through a feature of BNs called Bayesian Learning, the observations collected from a scour monitoring system installed on a critical bridge can be spread across the network thus appraising and updating scour at unmonitored bridges. By estimating the depth of the scour hole near the bridge foundation, the bridge vulnerability analysis based on the SVI (defined as the ratio D_T/D_F) can therefore be performed. This work is the first application of BNs to bridge scour risk management, and also the first implemented case where updating of the network is based on real-time information from a monitoring system.

In section 2, we describe the developed BN for scour depth prediction and the two numerical algorithms employed to update the variables involved. Section 3 presents the small bridge network consisting of bridges managed by TS in south-west Scotland. It was built by choosing bridges over the same river (River Nith) to demonstrate the functioning of the BN. Only one bridge is instrumented with a scour monitoring system. In section 4, the results obtained with the two algorithms are reported.

2 METHODOLOGY

Sensor and communication technologies offer nowadays the possibility to monitor in real-time every change in characteristics of a bridge; yet monitoring an entire infrastructure network is economically unsustainable. A way to overcome this limitation is to install monitoring systems at a limited number of critical locations and use a probabilistic approach to extend this information to the entire asset. The idea is to represent the state of the bridge stock through a set of random variables and to use ad-hoc Bayesian Networks to describe their conditional dependencies.

A BN, depicted in Figure 2, is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph comprised of nodes and links (Jensen and Nielsen, 2007). It was created by Judea Pearl in 1985. The presence of a link between two nodes means that the node that appears earlier in the chain has a direct influence on the other connected node. Nodes that are not connected (there is no path from one of the variables to the other in the Bayesian network) represent variables that are conditionally independent of each other. Each node represents a random

variable in the Bayesian sense, i.e., the relation between the two variables is always given by the Bayes' rule:

$$\text{pdf}(\theta | \bar{y}) = \frac{\text{pdf}(\theta) \times \text{pdf}(\bar{y} | \theta)}{\text{pdf}(\bar{y})} \quad (1)$$

where $\text{pdf}(\bar{y} | \theta)$ is the probability distribution function (pdf) known as the likelihood of the observed data \bar{y} , $\text{pdf}(\theta)$ is the prior pdf of parameter θ . $\text{pdf}(\theta | \bar{y})$ is called the posterior probability of θ and the dominator $\text{pdf}(\bar{y})$ is a normalising factor called evidence. Bayes' rule describes how the probability of parameter θ changes given information gained from measured data \bar{y} . In Bayesian network terminology, a node is a parent of a child if there is a link from the former to the latter.

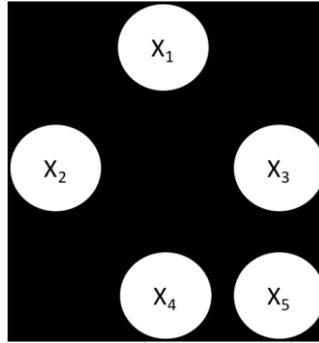


Figure 2. An example of a Bayesian Network

Probabilistic inference in BNs takes two forms: forward (predictive) analysis and backward (diagnostic) analysis. The former type of analysis for the node X_i is based on evidence nodes connected to X_i through its parent nodes and it is also called top-down reasoning. Instead, the diagnostic analysis for the node X_i is based on evidence nodes connected to X_i through its child nodes and it is also called bottom-up reasoning (Ben Gal, 2007). This backward analysis is called Bayesian learning as well.

The true power in using BNs comes from the ease with which they facilitate information updating when a new observation becomes available (Jensen & Nielsen, 2007). When evidence (e.g., information that a node is in a particular state) on one or more variables is entered into the BN, the information propagates through the network to yield updated probabilities in light of the new observations.

For these reasons, Bayesian network frameworks can be merged with monitoring systems to continuously update the risk map of infrastructure systems. This capability of updating is indeed particularly advantageous when the information on which the analysis of a system is based is evolving, as in the case of a real-time monitoring system. If we consider the bridge scour problem, in the occurrence of a flood, monitoring observations are used to probabilistically infer the posterior distribution of all the parent nodes of the network by exploiting features of Bayesian Learning, and to therefore give in real-time the best estimate of scour depth, even in unmonitored bridges.

2.1 Bayesian Network for scour depth estimation

The BN employed in the scour hazard model was developed according to the "Procedure BD 97/12 - The assessment of scour and other hydraulic actions at highway structures" (DoT, 2012). From here the Procedure BD 97/12 will be called just BD 97/12 for brevity. The document provides processes for determining the level of risk associated with scour effects at bridges starting from a design value of river flow. This procedure is based on a two-level assessment and it is used by TS to assess the scour and other hydraulic actions at highway structures. The first level includes simple methods, involving engineering judgement, to identify structures that are not at risk from scour or where the risk is tolerably low. When these conditions are not met, a Level 2 Assessment is performed; it consists of a framework for estimating scour depth at bridge locations, which provides a scour vulnerability analysis based on a SVI defined as the ratio between the total scour depth D_T and the foundation level D_F .

By following all the steps provided by BD 97/12, the scour estimation process was reproduced in the form of a BN. Figure 3a depicts the probabilistic correlation among variables involved in the appraisal of total scour depth D_T . Starting from the river flow characteristics (such as assessment flow Q_A and river level y_B) it is possible to estimate the depth of the two components of scour, constriction scour D_C and local scour D_L , whose sum is equal to the total scour depth. The appraisal of the former type of scour involves variables like the mean threshold velocity $v_{B,C}$ below which scour does not occur and the type of bed material. The phenomenon of constriction scour leads to an increase ΔA in cross-section area of flow that allows estimating an average value of erosion, $D_{C,ave}$. The variable D_C refers to the depth of constriction scour at a particular location along the transverse profile of the watercourse. The local scour principally depends on the shape and width of the pier and on the angle between the flow and the pier. The factor f_y , called depth factor, takes into account the relative depth of the approach flow to the pier width and, for this reason, it depends on the depth of constriction scour D_C itself.

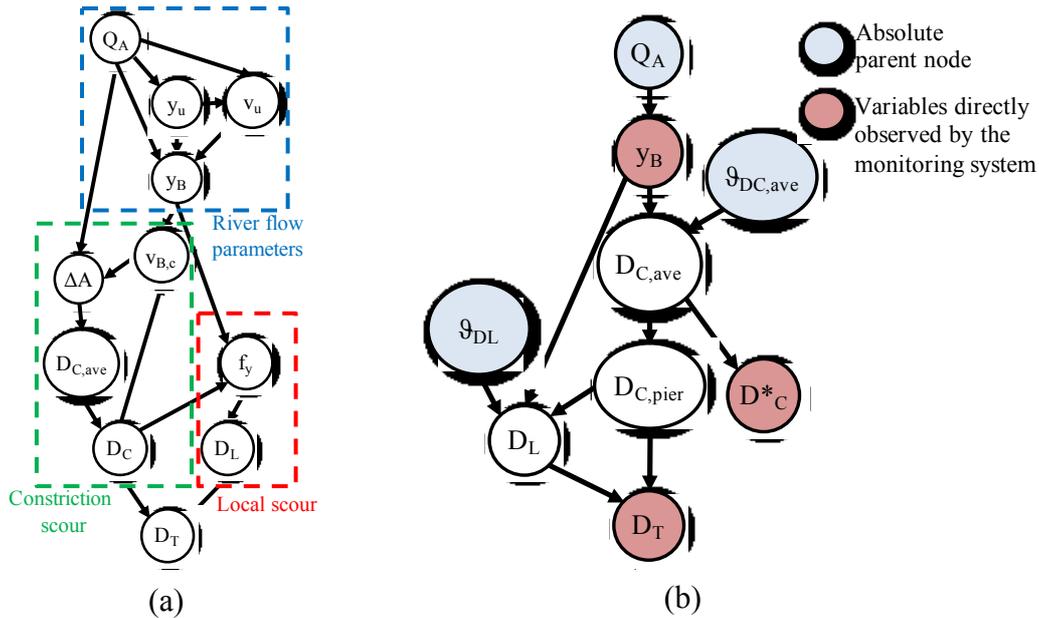


Figure 3. BN for scour depth prediction based on Procedure BD 97/12 (a), and the simplified version (b)

The models implemented into the BN can employ two types of variables relationships: deterministic and probabilistic. The former correlations consist of models being well-established or involving variables can be assumed deterministic not to complicate the resolution of the BN. The latter ones, for their probabilistic nature, must always deal with uncertainties and errors. Models are nothing more than a simplification of the reality, and the “perfect” model does not exist. Therefore, a modified version of the BN is shown in Figure 3b.

Let us focus on the quantities that can be monitored, that is, river level and depths of scour, and the used models. The water level y_B is measured by gauging stations; an observation of y_B updates the water flow Q_A . The model employed is assumed to be deterministic using the well-known Manning’s equation that connects river flow and level. A scour monitoring system can provide data about the scour depth, for instance, in the middle of the channel (constriction scour, D^*_C) and at a pier location (total scour, D_T). Observations of these variables cannot update the absolute parent node Q_A because the path is blocked by the observation of y_B . In order to exploit these scour observations within the BN, two new variables, $\theta_{DC,ave}$ and θ_{DL} , were therefore included; they are model uncertainties added to the mathematical models used to estimate the variables $D_{C,ave}$ and D_L , respectively. These new absolute parent nodes are named not-fixed model uncertainty because they are updated every time new observation of D^*_C and D_T entering the network. Through their employment, the value of scour depths obtained with the empirical formulas provided by BD 97/12 is corrected thanks to observations from scour monitoring system.

Let us summarise the three steps for solving the network and updating the posterior pdf of the nodes once observations about some variables become available:

- (i) the BN starts with the prior pdfs of the parent nodes: flow Q_A and the not-fixed model uncertainties $\theta_{DC,ave}$ and θ_{DL} . Observations of river level y_B , constriction scour in the middle of the river D^*_C and total scour D_T enter into the network (Figure 4a);
- (ii) the BN is figuratively split into three sub-networks because there are three different updating: the observation of y updates Q_A ; the observation of D^*_C and the updated pdf of y update not-fixed model uncertainty $\theta_{DC,ave}$; and the observation of D_T , the updated pdf of y and $D_{C,pier}$ update the not-fixed model uncertainty θ_{DL} (Figure 4b);
- (iii) descendant nodes are updated through the models provided by BD 97/12 exploiting updated information given by evidence on the parent nodes (Figure 4c).

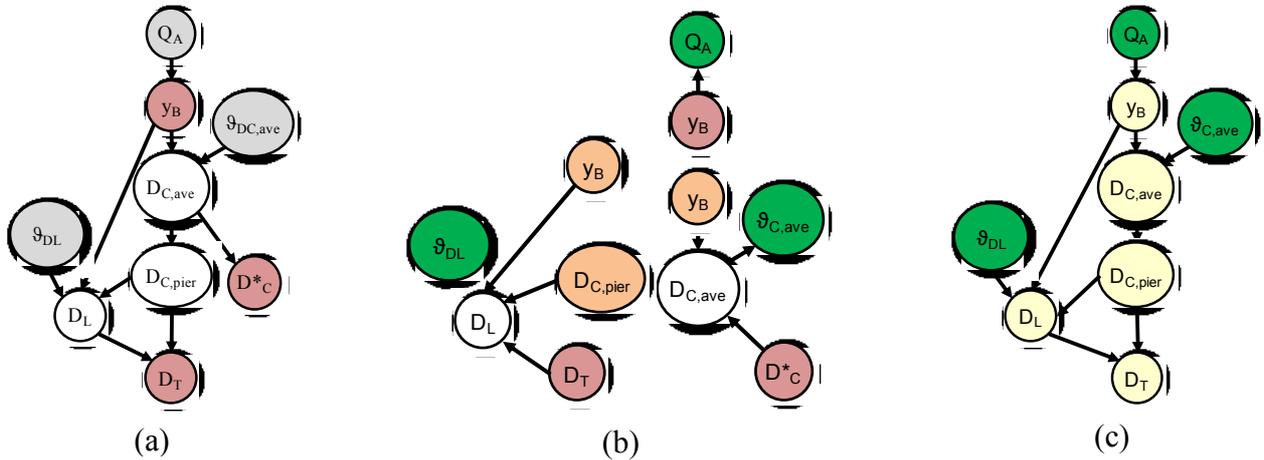


Figure 4. Starting with prior pdfs (a), updating of parent nodes (b), and updating of descendant nodes (c)

By following the same stages described above in the construction of the BN, we can develop a network on a bigger scale. For instance, Figure 5 shows a BN for correlating the total scour depth prediction at two different bridges, each of them with N piers. The estimation of the scour at the second bridge is based on the models corrected by the model uncertainty updated by direct observations of D^*_C and D_T at the first bridge. The two not-fixed model uncertainties are parent nodes of both sub-network because the models used to estimate scour depth are the same for any bridge. Consequently, uncertainties and error are correlated at all bridges.

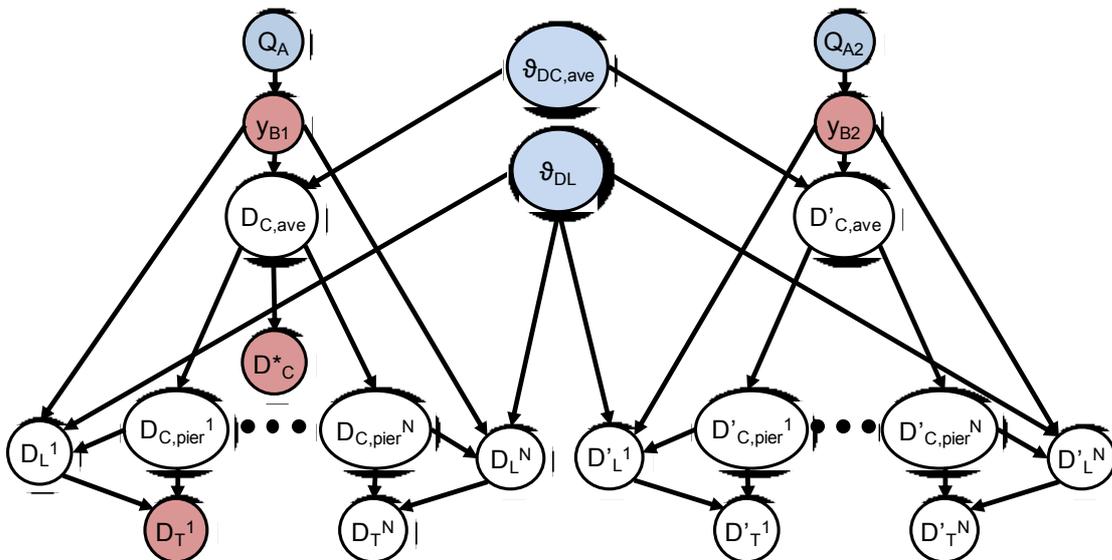


Figure 5. Bayesian Network for two different bridges, both with N piers

2.2 Numerical algorithms for model updating

Correlations present in a BN are expressed in Bayesian terms so Eq. (1) is always the basis of Bayesian statistic inference, but in most of the cases it is hard to know how the evidence at the denominator can be

described; a closed form to calculate it exists only in a few simple cases. To solve Eq. (1) and find the shape and estimators of posterior distribution we need a numerical algorithm for Bayesian inference.

In the past few years, computer algorithms have been developed to draw an (approximate) random sample from the posterior distribution, without having to evaluate it. We can approximate the posterior distribution to any accuracy we wish by taking a large enough random sample from it. Examples of sampling methods are the Markov Chain Monte Carlo (MCMC) and the Metropolis-Hasting (MH).

In this section we will present two different algorithms to solve numerically Eq. (1) into a Bayesian Network. The two numerical algorithms are based on, respectively, the Hessian Matrix method and the Transitional Markov Chain Monte Carlo (TMCMC) method.

2.2.1 Linear Gaussian Bayesian Networks

The first developed algorithm can solve any Linear Gaussian Bayesian Network (LGBN) by updating parent nodes' pdfs when data or observations about one of their child nodes enter into the BN. LGBN is a Bayesian network where the involved variables can be described only by Normal (Gaussian) or Log-Normal pdfs and with linear relationships among the variables. This means that, if the model employed between two variables is non-linear, a linear interpolation that fits with the non-linear relationship must be found. Uncertainties from models, variables, and observations and can be implemented into the algorithm.

In mathematics, the Hessian matrix or Hessian (H) is a square matrix of second-order partial derivatives of a scalar-valued function. This algorithm is based instead on another definition of H: if we define the variable LH as the negative logarithm of the likelihood, in the word of statistics H is the inverse of likelihood covariance matrix. The basic equations to calculate estimators of posterior pdf($\theta | \bar{y}$) are given below:

$$LH = -\log[g(\bar{y} | \theta)] \quad (2)$$

$$H = (\sigma_{\bar{y}|\theta}^2)^{-1} = \left[\frac{\partial^2 LH}{\partial \theta^2} \right] \quad (3)$$

$$(\sigma_{\theta|\bar{y}}^2)^{-1} = (\sigma_{\theta}^2)^{-1} + (\sigma_{\bar{y}|\theta}^2)^{-1} \quad (4)$$

$$\mu_{\theta|\bar{y}} = \sigma_{\theta|\bar{y}}^2 \left[\mu_{\theta} (\sigma_{\theta}^2)^{-1} + \bar{y} (\sigma_{\bar{y}|\theta}^2)^{-1} \right] \quad (5)$$

where estimators (i.e., mean value and standard deviation) with $\theta | \bar{y}$ as a subscript refer to the posterior pdf, with θ to the prior pdf, with $\bar{y} | \theta$ to the likelihood, and $g(\bar{y} | \theta)$ indicates the likelihood function.

2.2.2 Transitional Markov Chain Monte Carlo method

The MCMC method can simulate random samples from a target pdf that can only be evaluated up to a scaling constant. From the Bayesian point of view, the target pdf is the posterior pdf, and the scaling constant represents the evidence appearing at the denominator of Bayes' Theorem. The most popular MCMC method is the MH algorithm. MH algorithm can draw samples from the target pdf without knowing the model evidence, but it cannot evaluate it (Ching & Wang, 2015).

In 2007, a modified version of the MCMC method was proposed, called the Transitional Markov Chain Monte Carlo algorithm (Ching & Chen, 2007). The TMCMC algorithm is a marriage between the MH algorithm and the sampling-importance-resampling (SIR) method and it was motivated by the Adaptive MCMC (Beck & Au, 2002). Similar to the MH algorithm, the TMCMC algorithm can draw samples from the target pdf without the knowledge of the model evidence. Nonetheless, it can estimate the model evidence, without extra computation cost. TMCMC algorithm is more complicated and more different to code than the MH algorithm, but there is no need to specify the proposal pdf, no need to determine the burn-in period, the convergence issue is minimised, and the computational time is extremely reduced.

More details about the method, including the steps to code the algorithm, can be found in (Ching & Chen, 2007; Ching & Wang, 2015).

3 CASE STUDIES

The functioning of the developed BN was demonstrated using a small bridge network, consisted of bridges managed by TS in South-West Scotland. It was built by choosing bridges over the same river (River Nith) with only the first bridge being instrumented with a scour monitoring system. Consequently, the aim is to exploit observations on Bridge 1 in order to predict scour depth at other bridge locations. Figure 6 depicts the map of River Nith with bridges chosen for the network.

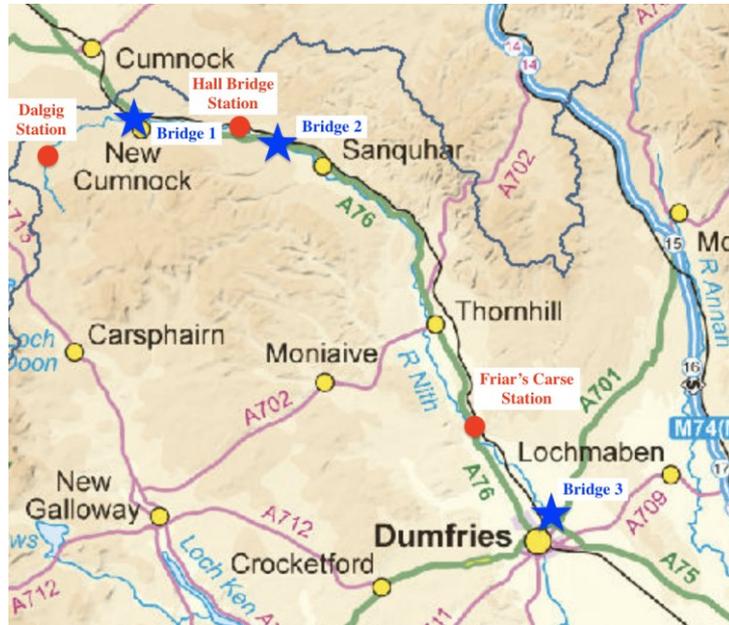


Figure 6. Small network of bridges over the River Nith. Red circles represent SEPA's gauging stations

Three bridges were chosen from the TS scour database; they have been checked through a Level 2 scour assessment in the last few years because they all have experienced significant scour events in the past. In the following, some information and details about the three bridges are reported:

- Bridge 1: A76 200 Bridge on River Nith in New Cumnock
It is a 3-span (9.1m, 10.7 m and 9.1 m) stone-masonry arch bridge, with two piers in the riverbed, carrying an 8.5m-width carriageway. Abutments and piers are all founded on spread footings on the natural riverbed.

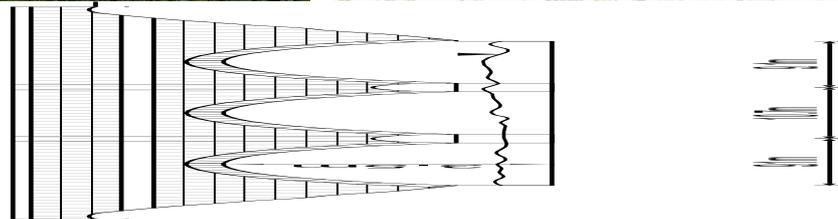
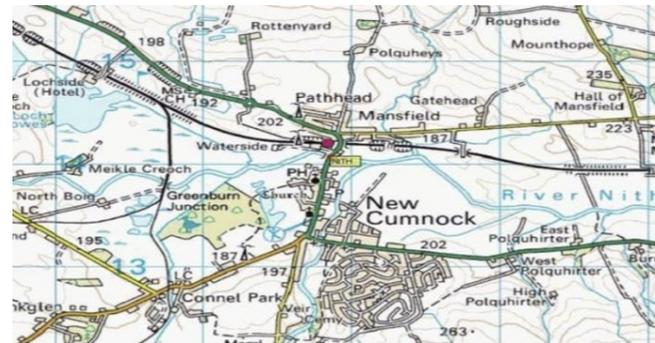


Figure 7. Picture, location plan and transversal section of Bridge 1

- Bridge 2: A76 120 Guildhall bridge on River Nith in Kirkconnel
 It is a 3-span (8.8m, 11.3 m and 11.3 m) masonry arch bridge, with one piers in the riverbed. Abutments and piers are all founded on spread footings on natural ground except one abutment's spread footing that is founded on rock.

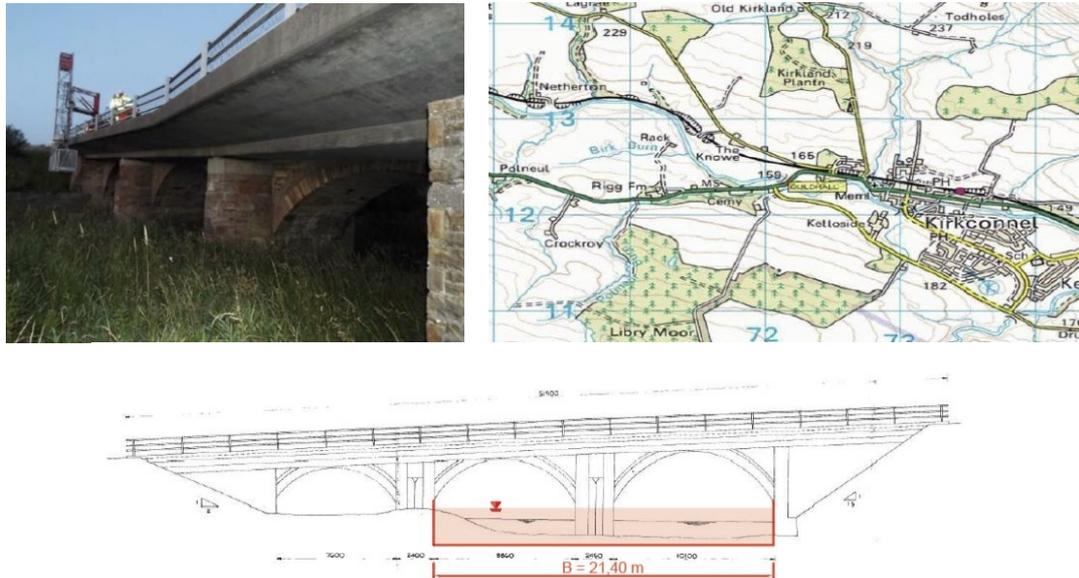


Figure 8. Picture, location plan and transversal section of Bridge 2

- Bridge 3: A75 300 Dalscone bridge on River Nith in Dumfries
 It is a 7-span (spans of 35 m and two of 28 m) steel-concrete composite bridge, with three piers in the riverbed. Abutments are founded on spread footings on made up ground, while piers are all founded on spread footings on natural ground.



Figure 9. Picture, location plan and transversal section of Bridge 3

As can be seen from Picture 6, a SEPA's gauging station precedes every bridge of the network. Consequently, there is no need to set the assessment flow Q_A as an absolute parent node in common for all the bridges. Q_A represents one of the parent node of each sub-network reproducing the scour depth prediction at any single bridge. The whole BN for the estimation of scour depth at every pier of A76, Guildhall and Dalscone bridge is depicted in Figure 10.

4 RESULTS

Normal distributions were employed for every variable except for the river flow; a log-normal distribution was adopted because the discharge cannot be negative. According to Figure 4, the first step in the resolution of the BN is the definition of the prior pdfs of absolute parent nodes. The pdfs of the not-

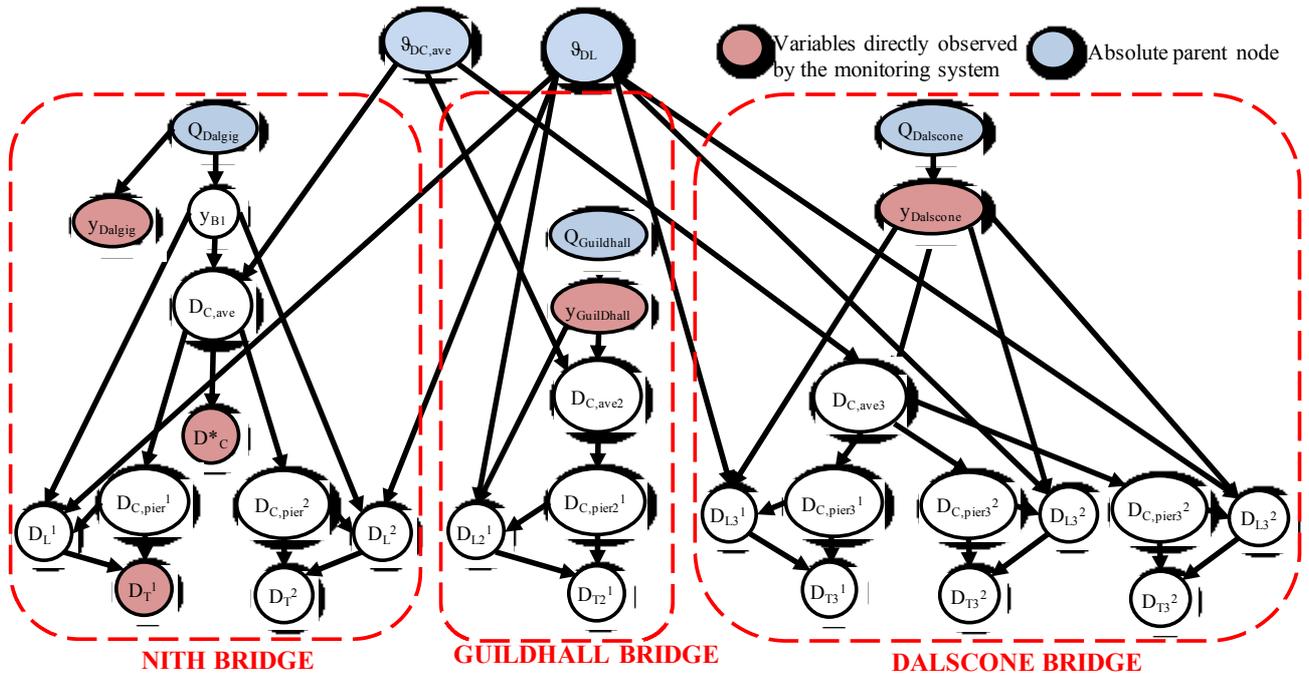


Figure 10. Final BN for depth of scour estimation at three bridges in the South-West of Scotland

fixed model uncertainties were set as Normal distributions with zero mean and a standard deviation of 1 m. Regarding river flow nodes, the parameters of the log-normal pdf were obtained from the data recorded by SEPA's gauging station of last ten years.

Let us focus now on the observations collected from monitoring systems that are entering the BN. Scour is induced by a flood event, consequently, the peak value of river level was chosen to simulate a heavy river flood condition. Table 1 shows these peak values.

Table 1. Case scenario for river level observations

SEPA station	Bridge	Water level [m] 30/12/2013
Dalgig	A76	1.879
Hall Bridge	Guildhall	3.015
Friar's carse	Dalscone	1.512

The maximum value recorded by the monitoring station at Dalgig was collected on 30th December 2013 and, in order to simulate an extreme event during the time, the other two values are the maximum in the following hours on the same day. Scour data entering the network were hypothesised to represent a critical situation since the monitoring system had not yet been installed at the time of this analysis. These hypothesised values are 20 cm for constriction scour depth D_c^* and 45 cm for total scour depth D_T .

4.1 LGBN

The employed models have to be linearised in order to apply the algorithm that solves LGBN. The variable scale was changed to logarithm scale, which allows overcoming problems with exponents or products. To linearise more complicated models, such as the relationship between constriction scour D_c and river level y_B shown in Eq. (6), a simple linear regression was performed to find the linear function that predicts the dependent variable values (constriction scour) as a function of the independent variable (river level). In Eq. (6), which is provided by BD 97/12, Manning's equation was employed to describe Q_A as a function of y_B , while the mean threshold velocity $v_{B,C}$ at the denominator was calculated using the Colebrook-White equation (Kirby et al, 2015).

$$D_c = \frac{\Delta A}{B_B} = \frac{Q_A}{B_B v_{b,c}} - y_B = \frac{1/n \cdot B^{5/3} \cdot y_B^{5/3} \cdot (B + 2y_B)^{-2/3} \cdot s^{1/2}}{B_B \left[-\sqrt{32} u_0 \cdot \log_{10} \left(\frac{d}{12y_B} + \frac{0.222\nu}{y_B \cdot u_0} \right) \right]} - y_B = D_c(y_B) \quad (6)$$

Table 2 depicts the results obtained by solving the LGBN. Mean values and standard deviations of constriction and total scour depth at piers of every bridge are reported. It is worth recalling that the BN starts from observations about D_c^* and D_T on Pier 1 of A76 200 bridge.

Table 2. Mean values and standard deviations of scour depth obtained by solving the LGBN

	A76 200		Guildhall		Dalscone	
	Pier 1 (M)	Pier 2 (E)	Pier 1 (E)	Pier 1 (E)	Pier 2 (E)	Pier 3 (E)
μ_{DC} [m]	0.20	0.191	0.654	0.478	0.464	0.471
σ_{DC} [m]	-	0.247	0.609	0.621	0.614	0.619
μ_{DT} [m]	0.45	0.438	0.953	0.791	0.807	0.794
σ_{DT} [m]	-	0.436	0.632	0.763	0.761	0.758

M: Measured, E: Estimated

4.2 TCMCMC

The prior pdfs and the hypothesised values chosen were the same used with the previous method. Table 3 shows the results obtained in the form of mean values and standard deviations of constriction and total scour depth.

Table 3. Mean values and standard deviations of scour depth obtained with TCMCMC

	A76 200		Guildhall		Dalscone	
	Pier 1 (M)	Pier 2 (E)	Pier 1 (E)	Pier 1 (E)	Pier 2 (E)	Pier 3 (E)
μ_{DC} [m]	0.20	0.199	0.607	0.421	0.420	0.432
σ_{DC} [m]	-	0.137	0.225	0.192	0.190	0.187
μ_{DT} [m]	0.45	0.452	0.932	0.802	0.798	0.805
σ_{DT} [m]	-	0.194	0.248	0.243	0.240	0.238

M: Measured, E: Estimated

As it can be seen by making a comparison between the two tables, estimations of the mean value of scour depth are consistent between the two algorithms whereas the TCMCMC method obtains lower values (from 45% to 65% lower than LGBN results) of standard deviations. It is worth remembering that the variance values (i.e., the square of standard deviation) is inversely proportional to the accuracy of a measurement/estimation. This can be explained by TCMCMC algorithm's capacity to handle non-linear models and relationships among variables; the need to linearise strong non-linear models in order to build a LGBN has significantly increased the uncertainties and reduced the accuracy of variable estimations.

5 CONCLUSIONS

In this paper we presented a BN able to estimate the depth of scour surrounding a bridge foundation. The BN can estimate, and continuously update, the present and future scour depth using real-time information from monitoring of scour depth and river flow characteristics. Once an observation collected from a scour monitoring system installed on a critical bridge enters into the BN, its information can be spread across the network thus appraising and updating scour depth at unmonitored bridges. This work is the first application of BNs to bridge scour risk management, and also the first implemented case where updating of the BN is based on real-time information from a monitoring system.

The resolution of the BN starts by defining the prior pdfs of parent nodes. The parent nodes consist of the uncertainty of the model for the prediction of total scour depth so that they can guarantee correlations among every bridge since the estimation models are employed for every bridge of the network. In order to make inference by updating the parent nodes, observations of river level and scour depth are entered into the network.

Two different algorithms were developed to solve the Bayes' rule, the basis of Bayesian statistic inference and, in turn, of BNs. The two numerical algorithms are based on, respectively, the Hessian Matrix method and the TMCMC method. The former algorithm can model any LGBN, which is a Bayesian network employing only Normal or Log-Normal pdfs and with linear relationships among variables. TMCMC method can easily handle any pdf and non-linear models.

The functioning of the developed BN was demonstrated using a small bridge network, consisting of bridges managed by TS in south-west Scotland. It was built by choosing bridges over the same river (River Nith), with only the first bridge being instrumented with a scour monitoring system. A flood event was simulated using river level data from SEPA's gauging stations. Scour depths were hypothesised since the monitoring system had not yet been installed at the time of this analysis.

Both methods led to similar results of the first estimator (i.e., mean value) of scour depth posterior pdfs. In contrast, using the TMCMC algorithm results in lower standard deviations (the second estimators) for all the cases because it allows the implementation of any models and variable relationships (i.e., linear and non-linear). This decrease ranges from 45% to 65% with respect to LGBN results. A lower value of standard deviation means a higher accuracy in the estimation of the variable.

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