On Solving the Load Flow Problem as an Optimization Problem

Waqquas A. Bukhsh
Dept. of Electronic and Electrical Engineering,
University of Strathclyde,
Glasgow, United Kingdom.

Abstract

The load flow problem is usually solved as a system of nonlinear equations using gradient based iterative methods. This technical note explains a way in which the load flow problem can be posed in a form of an optimization problem. The proposed formulation of the load flow problem allows a user flexibility and control over a solution e.g. enforcing bounds on variables. The proposed approach also makes it easier to identify sources of infeasibility, which otherwise are quite difficult to determine when using traditional approaches. The optimization based formulation of load flow presented in this note is implemented in an open source toolbox OATS [1].

Nomenclature

Sets

$\mathscr{B}$	Buses, indexed by b.
$\mathscr{B}^{\mathrm{V}\delta}$	Slack buses, indexed by b.
$\mathscr{B}^{\mathrm{PV}}$	Generator buses, indexed by b.
$\mathscr{B}^{\mathrm{PQ}}$	Demand buses, indexed by b.
$\mathscr{G}$	Generators, indexed by g.
$\mathscr{D}$	Loads, indexed by d.
$\mathscr{L}$	Lines between zones, indexed by l.

Parameters

$P_d^{\mathrm{D}},\ Q_d^{\mathrm{D}}$	Real, reactive power demand of load d.
$G_b^{\mathrm{B}},\ B_b^{\mathrm{B}}$	Shunt conductance, susceptance at bus b.
$G_{bb'},\ B_{bb'}$	Conductance, susceptance of a line from bus b to bus b'.
$\mathrm{P}_g^{\mathrm{LB}},\ \mathrm{P}_g^{\mathrm{UB}}$	Min., max. real power generation output of generator g.
$\mathrm{Q}_g^{\mathrm{LB}},\ \mathrm{Q}_g^{\mathrm{UB}}$	Min., max. reactive power generation output of generator g.
V_b^{Tgt}	Voltage magnitude target at bus b.
P_g^{Tgt}	Real power target at generator g.

Variables

$v_b,\ \theta_b$	Voltage magnitude and angle at bus b.
$p_g^{\mathrm{G}},\ q_g^{\mathrm{G}}$	Real, reactive power generation of generator g.
$p_{bb'}^{\mathrm{L}},\ q_{bb'}^{\mathrm{L}}$	Real, reactive power flow on a line from bus b to bus b'.
ϵ	Variables modelling constraint violations.

(e-mail: waqquas.bukhsh@strath.ac.uk)

I. Introduction

The load flow problem is a well know problem in power systems analysis [2]. The solution of a load flow problem gives complete operating knowledge of a system i.e. complex voltages at each bus and power flow on each branch of an electrical network.

The power balance and power flow equations in polar coordinates can be written as follows [3]:

$$\sum_{g\in\mathscr{G}_b} p^{\mathrm{G}}_g = \sum_{d\in\mathscr{D}_b} P^{\mathrm{D}}_d + \sum_{b'\in\mathscr{B}_b} p^{\mathrm{L}}_{bb'} + G^{\mathrm{B}}_b v_b^2 \tag{1a}$$

$$\sum_{g\in\mathscr{G}_b} q^{\mathrm{G}}_g = \sum_{d\in\mathscr{D}_b} Q^{\mathrm{D}}_d + \sum_{b'\in\mathscr{B}_b} q^{\mathrm{L}}_{bb'} - B^{\mathrm{B}}_b v_b^2 \tag{1b}$$

$$p^{\mathrm{L}}_{bb'} = v_b^2 G_{bb} + v_b v_{b'}\left(G_{bb'}\cos(\theta_b-\theta_{b'}) + B_{bb'}\sin(\theta_b-\theta_{b'})\right) \tag{1c}$$

$$q^{\mathrm{L}}_{bb'} = -v_b^2 B_{bb} + v_b v_{b'}\left(G_{bb'}\sin(\theta_b-\theta_{b'}) - B_{bb'}\cos(\theta_b-\theta_{b'})\right) \tag{1d}$$

Equations (1c-1d) could be substituted in Equations (1a-1b) to obtain $2n^{\mathrm{B}}$ nonlinear equations. The resulting system is a system of $2(n^{\mathrm{G}}+n^{\mathrm{B}})$ variables in $2n^{\mathrm{B}}$ equations. Traditional approach of solving load flow problem is to fix all demands at demand buses(referred to as PQ buses), and the voltages at all generator buses, and the real power generation outputs at all generator buses (referred to as PV buses) except one (referred to as slack bus), and also fix the phase angle at the slack bus, then we get $2n^{\mathrm{B}}$ nonlinear equations in $2n^{\mathrm{B}}$ variables. This system can be solved for the remaining variables by numerical methods. It is well known that the load flow problem can have 0, 1 or multiple solutions [3].

II. Solving Load flow problem as an optimization problem

This section presents a way to solve the load flow problem as an optimization problem. Let us partition the set of buses into mutually exclusive and collectively exhaustive sets as follows:

$$\mathscr{B} = \mathscr{B}^{\mathrm{V}\delta} \cup \mathscr{B}^{\mathrm{PV}} \cup \mathscr{B}^{\mathrm{PQ}} \tag{2}$$

where $\mathscr{B}^{\mathrm{V}\delta}$ is a set of slack buses, $\mathscr{B}^{\mathrm{PV}}$ is set of generator buses and $\mathscr{B}^{\mathrm{PQ}}$ is a set of demand buses. Note that with this formulation it is possible to have multiple slack buses. The phase angle will only be fixed at one of the buses.

The load flow problem can be formulated as an optimization problem as follows:

$$\min \mathbf{0}, \tag{3a}$$

subject to

$$\left.\begin{aligned} \sum_{g\in\mathscr{G}_b} p^{\mathrm{G}}_g &= \sum_{d\in\mathscr{D}_b} P^{\mathrm{D}}_d + \sum_{b'\in\mathscr{B}_b} p^{\mathrm{L}}_{bb'} + G^{\mathrm{B}}_b v_b^2, \\ \sum_{g\in\mathscr{G}_b} q^{\mathrm{G}}_g &= \sum_{d\in\mathscr{D}_b} Q^{\mathrm{D}}_d + \sum_{b'\in\mathscr{B}_b} q^{\mathrm{L}}_{bb'} - B^{\mathrm{B}}_b v_b^2, \end{aligned}\right\}, \forall\ b\in\mathscr{B}, \tag{3b}$$

$$\left.\begin{aligned} p^{\mathrm{L}}_{bb'} &= v_b^2 G_{bb} + v_b v_{b'}\left(G_{bb'}\cos(\theta_b-\theta_{b'}) + B_{bb'}\sin(\theta_b-\theta_{b'})\right), \\ q^{\mathrm{L}}_{bb'} &= -v_b^2 B_{bb} + v_b v_{b'}\left(G_{bb'}\sin(\theta_b-\theta_{b'}) - B_{bb'}\cos(\theta_b-\theta_{b'})\right), \end{aligned}\right\}, \forall\ (b,b')\in\mathscr{L}, \tag{3c}$$

$$\theta_{b_0} = 0, \quad b_0 \in \mathscr{B}^{\mathrm{V}\delta} \tag{3d}$$

$$V^{\mathrm{Tgt}}_b \le v_b \le V^{\mathrm{Tgt}}_b, \quad \forall\ b\in\mathscr{B}^{\mathrm{V}\delta}\cup\mathscr{B}^{\mathrm{PV}}, \tag{3e}$$

$$P^{\mathrm{Tgt}}_g \le p^{\mathrm{G}}_g \le P^{\mathrm{Tgt}}_g, \quad \forall\ b\in\mathscr{B}^{\mathrm{PV}} \tag{3f}$$

where Equation 3(a) is the zero objective function, which means that we are seeking *a* feasible solution of the problem. Constraints (3b-3c) are power balance and power flow equations, respectively. Constraint 3d is a constraint on fixing one of the slack bus to zero. This constraint removes the redundancy in the solution by fixing phase angle at one of the slack bus. Constraints 3e fix the voltage magnitudes and real power generation set-points to given targets for generator buses.

III. Soft Constrained Load Flow Problem

If the voltage or real power generation targets are not set carefully, then the load flow problem could be infeasible. This is a common issue while solving load flow problems. In order to overcome this issue, and investigate the cause of infeasibility, the formulation presented in (3) could be relaxed by using *soft constraints*. A soft constrained version of the load flow problem is given as follows:

$$\min \sum_{i=g,v,l} \epsilon_i^2, \tag{4a}$$

subject to

$$\left.\begin{aligned} \sum_{g\in\mathcal{G}_b} p_g^{\mathrm{G}} &= \sum_{d\in\mathcal{D}_b} P_d^{\mathrm{D}} + \sum_{b'\in\mathcal{B}_b} p_{bb'}^{\mathrm{L}} + G_b^{\mathrm{B}} v_b^2, \\ \sum_{g\in\mathcal{G}_b} q_g^{\mathrm{G}} &= \sum_{d\in\mathcal{D}_b} Q_d^{\mathrm{D}} + \sum_{b'\in\mathcal{B}_b} q_{bb'}^{\mathrm{L}} - B_b^{\mathrm{B}} v_b^2, \end{aligned}\right\}, \forall\ b \in \mathcal{B}, \tag{4b}$$

$$\left.\begin{aligned} p_{bb'}^{\mathrm{L}} &= v_b^2 G_{bb} + v_b v_{b'} \left(G_{bb'} \cos(\theta_b - \theta_{b'}) + B_{bb'} \sin(\theta_b - \theta_{b'})\right), \\ q_{bb'}^{\mathrm{L}} &= -v_b^2 B_{bb} + v_b v_{b'} \left(G_{bb'} \sin(\theta_b - \theta_{b'}) - B_{bb'} \cos(\theta_b - \theta_{b'})\right), \end{aligned}\right\}, \forall\ (b,b') \in \mathcal{L}, \tag{4c}$$

$$\theta_{b_0} = 0, \quad b_0 \in \mathcal{B}^{\mathrm{V}\delta} \tag{4d}$$

$$V_b^{\mathrm{Tgt}} \le v_b \le V_b^{\mathrm{Tgt}}, \quad \forall\ b \in \mathcal{B}^{\mathrm{V}\delta} \cup \mathcal{B}^{\mathrm{PV}}, \tag{4e}$$

$$P_g^{\mathrm{Tgt}} \le p_g^{\mathrm{G}} \le P_g^{\mathrm{Tgt}}, \quad \forall\ b \in \mathcal{B}^{\mathrm{PV}} \tag{4f}$$

$$V_b^{\mathrm{LB}} - \epsilon_b^2 \le v_b \le V_b^{\mathrm{UB}} + \epsilon_b^2, \quad \forall\ b \in \mathcal{B}, \tag{4g}$$

$$\left.\begin{aligned} P_g^{\mathrm{LB}} - \epsilon_g^2 \le p_g \le P_g^{\mathrm{UB}} + \epsilon_g^2, \\ Q_g^{\mathrm{LB}} - \epsilon_g^2 \le q_g \le Q_g^{\mathrm{UB}} + \epsilon_g^2, \end{aligned}\right\}, \forall\ g \in \mathcal{G}, \tag{4h}$$

$${p_{bb'}^{\mathrm{L}}}^2 + {q_{bb'}^{\mathrm{L}}}^2 \le (S_{bb'}^{\max})^2 + \epsilon_l^2, \quad \forall\ (b,b') \in \mathcal{L}, \tag{4i}$$

where (3a) is the objective function that minimizes the violation of constraints (4g-4i). Note that with this formulation it is possible to add extra constraints on the generation bounds and power flows as modelled using constraints (4h-4i).

References

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