

1 **NUMERICAL MODELLING OF NON-UNIFORM CORROSION INDUCED**
2 **CONCRETE CRACK WIDTH**

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12
13 **ABSTRACT**

14
15 Corrosion of reinforced concrete is one of the major deterioration mechanisms which result in
16 premature failure of the reinforced concrete structures. Crack width is often used as an effective
17 criterion to assess the serviceability of concrete structures. However, research on prediction of
18 corrosion-induced concrete crack width, especially by considering the corrosion as a non-
19 uniform process, has still been scarce. This paper attempts to develop a finite element model
20 to predict the crack width for corrosion-affected concrete structures under realistic non-uniform
21 corrosion of the reinforcement. A non-uniform corrosion model was first formulated as a
22 function of time. To simulate arbitrary cracking in concrete, cohesive elements are inserted in
23 the sufficiently fine mesh which is achieved through a script written in Python. The surface
24 crack width is obtained as a function of service time and verification against experimental
25 results from literature is conducted. Accurate prediction of crack width can allow timely
26 maintenance which prolongs the service life of the reinforced concrete structures.

27
28 **Keywords:** corrosion, crack width, cohesive crack model, reinforced concrete, numerical
29 modelling.

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31 1 INTRODUCTION

32 Reinforced concrete (RC) structures have been the most common type of structures used in the
33 civil engineering construction since middle nineteenth century. RC structures have been widely
34 used for buildings, bridges, retaining walls, tunnels, and indeed any physical infrastructure built
35 on and under the ground. Since 1970s, it has become an accepted knowledge that the concrete
36 cover has its limitation on protecting the reinforcing steel from corrosion. As a result, a series
37 of research has been initiated on improving the understanding of the corrosion of steel in
38 concrete (Faustino, et al., 2014, Faustino, et al., 2017, Wilkins and Lawrence, 1980), such as
39 the Concrete in the Oceans research programme in the UK in the 1970s. Furthermore, it appears
40 to be inevitable that RC structures will suffer from reinforcement corrosion in chloride (Cl^-)
41 and carbon dioxide (CO_2) laden environment. Practical experience and experimental
42 observations (Andrade, et al., 1993, Li, 2003, Li, 2005, Otsuki, et al., 2000) suggest that
43 corrosion affected RC structures deteriorate faster in terms of serviceability (e.g., cracking or
44 deflection) than safety (e.g., strength). Therefore, there is a well justified need for a thorough
45 investigation of the cracking process and crack width of concrete, not least bearing in mind that
46 crack width is one of the most important practical parameters for the design and assessment of
47 RC structures.

48

49 To model cracking of concrete, some researchers have resorted to analytical approach, mainly
50 due to the accuracy of the solution and the convenience of its practical application (Bhargava,
51 et al., 2006, Li and Yang, 2011, Pantazopoulou and Papoulia, 2001, Yang, et al., 2017). For
52 example, Li and Yang (2011) developed an analytical model for concrete crack width caused
53 by reinforcement corrosion and applied load, by introducing a stiffness reduction factor to
54 account for the post-cracking quasi-brittle behaviour of concrete. The stiffness reduction factor
55 then modifies the differential equation for obtaining the cracked stress and strain components.

56 Correlations between material corrosion and the structural effects can then be established, e.g.,
57 crack width (Li and Yang, 2011), time to surface cracking (Bhargava, et al., 2006), etc.
58 However, the application of analytical modelling in crack propagation in concrete is limited to
59 some special cases, e.g., particular boundary conditions, and the assumption that the crack is
60 smeared and uniformly distributed in the damaged solid to satisfy the requirement on
61 continuous displacement. Some studies have employed complex functions to formulate the
62 stress development under arbitrary boundary conditions (Ning, et al., 2012, Yang, et al., 2015),
63 i.e., non-uniform corrosion rust distribution; however, they have been limited to elastic solution
64 only.

65

66 In light of the limitation of analytical modelling on crack propagation in concrete, numerical
67 modelling has brought considerable advantages. Depending on the specific application and the
68 scale of the problem, different numerical techniques may be used, e.g., finite element method
69 (FEM) (Hanson and Ingraffea, 2003, Roesler, et al., 2007), discrete element method (DEM)
70 (Beckmann, et al., 2012), boundary element method (BEM) (Aliabadi and Saleh, 2002,
71 Chahrour and Ohtsu, 1992), particle meshfree method (Rabczuk and Belytschko, 2006,
72 Rabczuk and Zi, 2006, Rabczuk and Belytschko, 2007, Rabczuk, et al., 2007, Rabczuk, et al.,
73 2008, Rabczuk, et al., 2008, Rabczuk, et al., 2010) and peridynamics (Gerstle, et al., 2007,
74 Huang, et al., 2015). Amongst these numerical methods, FEM has received the most research
75 interest in solving corrosion-induced reinforced concrete cracking problems. Amongst these
76 numerical models available in literature, most focuses on uniform corrosion. However, Yuan
77 and Ji (2009) conducted accelerated corrosion test by using an artificial environmental chamber
78 on RC specimens and confirmed that the corrosion rust distribution along the reinforcing bar
79 was non-uniform. According to their results, the half of the reinforcing bar in the cover side
80 was corroded and the distribution of the corrosion rust was in a semi-elliptical shape. It is also

81 evident that the localized corrosion can be equivalent to about four to eight times that of the
82 reinforcement under overall corrosion (González, et al., 1995).

83

84 Recently, some researchers have started to model the non-uniform corrosion caused cracking
85 in concrete. For example, Jang and Oh (2010) considered a few of non-uniform distributions
86 of the corrosion rust and calculated the stresses in concrete induced by corrosion expansion.

87 Pan and Lu (2012) modelled the crack propagation in concrete in FEM under non-uniform
88 corrosion expansion and assumed concrete as a heterogeneous material. Moreover, Du et al.

89 (2014) established a damage plasticity model to simulate the non-uniform corrosion induced
90 concrete cracking. A number of parameters were investigated on their effects to the surface

91 cracking. Zhao et al. (2011) proposed a Gaussian distribution for the non-uniform corrosion
92 caused displacement and simulated the cracking behaviour of concrete by smeared crack

93 model. The literature review suggests (1) the non-uniform corrosion models derived so far have
94 seldom been based on experimental test results but more on hypothesis and; (2) no model has

95 been developed in modelling discrete crack propagation and calculating the surface crack
96 width, caused by non-uniform corrosion. Crack width is an important parameter regarding the

97 durability of concrete structures while it is still not quite clear how those underlying factors,
98 e.g., corrosion rate, material/mechanical properties of concrete, may quantitatively affect the

99 development of crack width of the concrete. Experimental results will be very useful to help
100 and understand the degradation mechanism; however, existing experimental data are rather

101 limited with regards to the full understanding of the problem. Therefore, it is well justified that
102 a numerical model be developed to predict the non-uniform corrosion induced concrete crack

103 width after corrosion initiation.

104

105 This paper attempts to develop a numerical method to simulate the discrete crack propagation
106 in concrete and predict the surface crack width under the non-uniform corrosion of
107 reinforcement. A non-uniform corrosion model is first formulated as a function of time, based
108 on experimental results. In order to model the arbitrary discrete cracking of concrete, cohesive
109 crack elements are inserted in the sufficiently fine mesh, through an in-house script written in
110 Python. Discussion on choices of the cohesive parameters is presented since they are the key
111 to establishing a rational model with cohesive elements. After the formulation of the model, an
112 example is presented to demonstrate its application. The developed model is then verified by
113 comparing the results with those from experiments. Moreover, a parametric study is carried out
114 to investigate the effects of some basic parameters, e.g., corrosion rate, elastic modulus of
115 concrete, tensile strength of concrete, shape of softening curve, on the development of surface
116 crack width after corrosion initiation.

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118
119

2 CONSTITUTIVE MODEL FOR CRACKS

120 The failure of structures is significantly influenced by the properties of the material used.
121 Concrete is considered as a quasi-brittle material, in which the tensile stress gradually decreases
122 after it reaches its peak value while the tensile strain continues to increase. Such the stiffness
123 degradation behaviour is called strain softening. The concept of strain softening evolves from
124 plasticity where the post-peak decline of the tensile stress is considered as a gradual decrease
125 of the tensile strength, i.e., softening. Concrete exhibits the strain softening behaviour is
126 because there is an inelastic zone developed ahead of its crack tip, often referred to as fracture
127 process zone (FPZ), as shown in Figure 1-a. When a crack propagates in concrete, the cracked
128 surfaces may be in contact and are tortuous in nature (Mindess and Diamond, 1982), due to
129 various toughening mechanisms such as aggregate bridging, void formation or microcrack

130 shielding (Shah, et al., 1995). Therefore, the cracked surfaces may still be able to sustain the
131 tensile stress which is characterized by the softening degradation curve.

132

133 To model the discrete cracks in concrete, cohesive crack model was first developed by
134 Hillerborg et al. (Hillerborg, et al., 1976). In general, the constitutive relation for the cracks can
135 be described as follows:

$$136 \quad \sigma = f_{T-S}(\delta) \quad (1)$$

137 where σ is the local opening stress of the crack, δ is the total displacement of the crack and
138 f_{T-S} is a nonlinear function defining the traction-separation relationship. To prevent mesh
139 sensitivity in FE analysis, the stress-displacement relation is used rather than stress-strain
140 relation. As the distance between the nodes is used as a crack measure rather than a change in
141 strain (which depends on the element length), the mesh dependency is significantly reduced.

142

143 The function f_{T-S} defines the complete mechanical behaviour of a cohesive crack, i.e., linear
144 elasticity and nonlinear fracture property. It is schematically shown in Figure 1-b. The tensile
145 stress linearly increases until its maximum value, i.e., tensile strength f_t' ; such a linearity is
146 determined by a penalty stiffness K_p . After reaching the peak value, the tensile stress
147 decreases, following certain strain softening rules, e.g., linear, bi-linear, exponential, etc. The
148 area underneath the f_{T-S} curve is known as the fracture energy G_f .

149

150 The constitutive relation of the cohesive crack is often divided into elasticity and fracture,
151 respectively. The elastic property of a crack is characterised by K_p and f_t' , whilst the fracture
152 property of the crack is defined by G_f and the softening shape. As such, the crack opening

153 displacement w , neglecting the elastic displacement, can be used to characterise the softening
154 relation only. The constitutive softening relation, expressed as a function of the crack mouth
155 opening displacement, can be expressed as follows:

$$156 \quad \sigma = f(w) = f_t' \exp\left(-\frac{f_t'}{G_f} w\right) \quad (2)$$

157 Equation (2) is based on the exponential softening and related to the real material properties,
158 i.e., the tensile strength f_t' and the fracture energy G_f .

159
160 The key to developing a rational model of cohesive cracking is the choices of the parameters,
161 i.e., the penalty stiffness K_p , the tensile strength f_t' , the fracture energy G_f and the shape of the
162 softening curve. The concept of penalty stiffness evolves from the elastic stiffness which is
163 obtained by dividing the elastic modulus of the concrete by its thickness. Since cohesive
164 interface is normally very thin or even of zero thickness, the elastic stiffness of the cohesive
165 interface approaches infinitesimally large. This makes sense as the interface should be stiff
166 enough prior to the initiation of crack to hold the two surfaces of the bulk concrete together,
167 leading to the same performance as that of no interface existing. This also meets the general
168 condition of cohesive crack model which assumes that the energy required to create the new
169 surfaces is vanishingly small compared to that required to separate them (Bažant and Planas,
170 1998). However, the elastic stiffness cannot be too large as it will cause convergence problems
171 due to ill-conditioning of the numerical solver of the FE programmes (ACI Committee 446,
172 1997). Therefore, the cohesive stiffness becomes a “penalty” parameter, which controls how
173 easily the cohesive interface deforms elastically. As such this stiffness is large enough to
174 provide the same or close response of intact concrete prior to cracking, but not so large as to
175 cause numerical problems.

176

177 The tensile strength f_t' of concrete is used as an important index to determine if a cohesive
178 crack is initiated. For Mode I fracture, once the tensile stress at any point of a structure reaches
179 its tensile strength, a crack is initiated and the material of that point starts to degrade. The
180 fracture energy G_f is the energy absorbed per unit area of crack with the unit of N/mm or N/m
181 usually. It can be regarded as the external energy supply required to create and fully break a
182 unit surface area of the cohesive crack. It may be mentioned that there is an argument as if G_f
183 is a material constant, i.e., independent of boundary condition, etc. G_f is used in the energy
184 balance criterion which controls the stable crack propagation. Further, the shape of softening
185 curve, as illustrated in Figure 1-b, is arguably important. Some researchers believe the shape
186 of the softening is less important than the fracture energy (Elices, et al., 2002). However, there
187 are a range of shapes developed in the last two decades, e.g., linear, bilinear and exponential
188 curves, etc.

189

190 **3 EMBEDDED COHESIVE ELEMENTS**

191 To model the arbitrary cracking in concrete, the cohesive elements are embedded in the mesh.
192 The insertion process of cohesive elements can be shown in Figure 2. Figure 2(a) presents the
193 initial mesh generated, which consists of triangle elements only. First, all individual nodes are
194 replaced by certain number of new nodes at the same location. The number of newly created
195 nodes depends on the number of the elements connecting to the original node. For instance, if
196 a node is connected by 4 triangle elements, the node is replaced by 4 newly created nodes; if a
197 node is connected by 8 triangle elements, the node is replaced by 8 newly created nodes.
198 Subsequently, the 4 nodes at the interface between two triangle elements are identified and
199 linked to form a cohesive element. In this way, all the interfaces between the triangle elements

200 are inserted of cohesive elements. The cohesive elements are shown in red in Figure 2. This
201 insertion process was conducted by a script written in Python.

202

203 The cohesive elements generated in this way are of zero thickness in geometry. The two nodes
204 of a cohesive element, in the thickness direction, share the same coordinates before loading.

205 The constitutive/calculation thickness of the cohesive elements, however, is not zero. In
206 practice, it is usually set as 1.0 for the convenience of transformation between strain and
207 displacement. After the insertion of cohesive elements, the structure can simulate discrete
208 cracking behaviour. As long as the mesh is sufficiently fine, the embedded cohesive elements
209 form a sensible and reasonable network of potential cracking paths.

210

211 The 4-node cohesive element generated from the insertion process has two stress components,
212 i.e., the normal stress and the shear stress, as shown in Figure 2(b). Unlike other solid elements,
213 there are only two stress components for the cohesive elements because the thickness is zero.

214 The cohesive element undergoes linear elastic behaviour prior to the peak load followed by the
215 initiation and evolution of damage, i.e., cracking. The elasticity for the whole cohesive element,
216 in terms of the nominal stresses and nominal displacements, is defined as follows :

217

$$218 \quad \sigma = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \begin{bmatrix} K_n & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (3)$$

219

220 where σ_1 and σ_2 are the normal stress and shear stress respectively, K_n and K_s are the stiffness
221 that relate to the normal stress and shear stress, and δ_1 and δ_2 are the corresponding
222 displacements.

223

224 **4 NON-UNIFORM CORROSION MODEL**

225 Concrete with an embedded bar subjected to an internal pressure at the interface between the
226 bar and concrete can be modelled as a thick-wall cylinder (Bazant, 1979, Pantazopoulou and
227 Papoulia, 2001, Tepfers, 1979). As shown in Figure 3, D is the diameter of the bar and d_0 is
228 the thickness of the annular layer of concrete pores at the interface between the bar and
229 concrete, often referred to as interfacial transition zone (ITZ). Usually d_0 is constant once
230 concrete has hardened. The inner and outer radii of the cylinder are considered $a = D/2 + d_0$
231 and $b = C + D/2 + d_0$.

232

233 It is known that the corrosion products of reinforcement usually occupy a few times more space
234 than the original steel. The accumulation process of the corrosion products starts from filling
235 in the ITZ, with its thickness d_0 , but normally do not produce stresses in concrete. There might
236 be arguments that at this stage stress can be induced; but for simplicity, it is assumed stress-
237 free in this paper. As the corrosion products proceed further in concrete, a band of corrosion
238 products forms, as shown in Figure 3(a). Due to the fact that the chlorides, as well as moisture
239 and oxygen, reach the reinforcement surface at different rates through different sides of the
240 concrete structure, it is very rare to have uniform corrosion around the reinforcement. This is
241 particularly the case for offshore RC structures as the surface facing the ocean waves has much
242 faster ingress rate and hence more severe corrosion on this side. It has been found that the
243 front of corrosion products for the half of rebar facing concrete cover is in a semi-elliptical
244 shape, while corrosion of the opposite half of rebar is negligibly small and can be neglected
245 (Yuan and Ji, 2009). Yuan and Ji (2009), amongst the very limited experimental results, is
246 used as the basis of formulating the time-dependent non-uniform corrosion.

247

248

249 As illustrated in Figure 3(b), there may be three bands accommodating the corrosion products:
 250 the semi-elliptical band of corroded steel with maximum thickness d_{co-st} , the porous circular
 251 band d_0 and the semi-elliptical rust band with maximum thickness d_m . The front of the
 252 corrosion is in a semi-elliptical shape with the semi-major axis equal to $D/2 + d_0 + d_m$ and the
 253 semi-minor axis equal to $D/2 + d_0$.

254

255 Based on the geometry, the total amount of corrosion products $W_{rust}(t)$ can be related to these
 256 band thicknesses, shown as follows:

257

$$258 \quad \frac{2W_{rust}(t)}{\pi} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) = Dd_0 + d_0^2 + \frac{D}{2}d_m + d_0d_m \quad (4)$$

259

260 where ρ_{rust} is the density of corrosion products, ρ_{st} is the density of steel and α_{rust} is the
 261 molecular weight of steel divided by the molecular weight of corrosion products. It varies from
 262 0.523 to 0.622 according to different types of corrosion products (Liu and Weyers, 1998).

263

264 By neglecting the small quantities d_0d_m and d_0^2 , d_m can be derived as follows:

265

266

$$267 \quad d_m(t) = \frac{4W_{rust}}{\pi D} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) - 2d_0 \quad (5)$$

268

269 $d_m(t)$ in Equation (4) is the maximum corrosion-induced expansion along the interface to the
 270 concrete cylinder under which the stress will be initiated in the cylinder. $d_m(t)$ determines the
 271 shape of the semi-ellipse which is the boundary condition of the concrete cylinder in deriving
 272 stresses and strains in concrete.

273

274 In Equation (5), $W_{rust}(t)$ is related to the corrosion rate of the steel rebar and can be expressed
 275 as follows (Liu and Weyers, 1998):

$$276 \quad W_{rust}(t) = \sqrt{2 \int_0^t 0.105(1/\alpha_{rust})\pi D i_{corr}(t) dt} \quad (6)$$

277 where i_{corr} is the corrosion current density in $\mu A/cm^2$, which is widely used as a measure of
 278 corrosion rate.

279
 280 To determine the displacement boundary condition of the concrete cylinder, the function of the
 281 semi-ellipse of the corrosion front needs to be derived. It is known that, in rectangular
 282 coordinate system, the function for an ellipse can be expressed as follows:

$$283 \quad \frac{y^2}{\left(\frac{D}{2} + d_0 + d_m\right)^2} + \frac{x^2}{\left(\frac{D}{2} + d_0\right)^2} = 1 \quad (7)$$

284 Equation (7) can be transformed in a polar coordinate system as follows,

$$285 \quad r = \frac{(D + 2d_0 + 2d_m)(D + 2d_0)}{\sqrt{(2D + 4d_0)^2 + 16d_m(D + 2d_0 + d_m)\cos^2\theta}} \quad (8)$$

287
 288 The displacement boundary condition of the concrete cylinder $\delta(\theta, t)$ can be derived as
 289 follows:

$$290 \quad \delta(\theta, t) = \frac{\left[D + 2d_0 + \frac{8W_{rust}(t)}{\pi D} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) - 4d_0 \right] (D + 2d_0)}{\sqrt{(2D + 4d_0)^2 + 32 \left[\frac{2W_{rust}(t)}{\pi D} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) - d_0 \right] \left[D + 2d_0 + \frac{4W_{rust}(t)}{\pi D} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) - 2d_0 \right] \cos^2\theta}} - \frac{D}{2} - d_0 \quad (9)$$

291
 292 where $0 \leq \theta \leq \pi$.

293

294 5 FINITE ELEMENT SIMULATION

295 For concrete with embedded reinforcing steel bar, it is widely accepted to be modelled as a
296 thick-wall cylinder (Pantazopoulou and Papoulia, 2001, Tepfers, 1979). Due to the symmetry
297 of the loading and the structure, only half of the cylinder is modelled. Two elements are
298 employed in this study, i.e., 4-node cohesive elements at all interfaces between the triangle
299 elements, and 3-node plane strain element for the bulk concrete. Reduced integration is used
300 for the plane strain element. As a result, the damage evolution of the cohesive element is
301 combined with the elastic deformation of the bulk concrete in the overall response. Very fine
302 mesh is initially generated for the sake of inserting sufficient number of cohesive elements.
303 There are 26,784 solid triangle elements plus 40,011 cohesive elements inserted, for half of the
304 cylinder.

305

306 Crack initiation marks the beginning of degradation or damage of concrete at a point. Crack is
307 assumed to initiate when the maximum nominal tensile stress reaches the tensile strength of
308 the concrete for the Mode I fracture – opening mode, expressed as follows,

309

$$310 \quad \langle \sigma_1 \rangle = f_t' \quad (10)$$

311

$$312 \quad \text{where } \langle \sigma_1 \rangle = \begin{cases} \sigma_1 & \text{for } \sigma_1 > 0 \\ 0 & \text{for } \sigma_1 < 0 \end{cases}$$

313

314 The operation $\langle \sigma_1 \rangle$ is to ensure that a crack will not initiate under compression.

315

316 After cracking is initiated, the cohesive element is damaged and the normal stress of this
317 element softens in a manner as defined. The failure of the element is governed by the softening
318 curve. To calculate the residual stress after its peak stress, a damage parameter D is introduced
319 into the stress calculation as follows:

320
321
$$\sigma = (1 - D)\sigma_u \quad (11a)$$

322
$$\sigma_u = K_p \delta \quad (11b)$$

323
324 where σ_u is the undamaged stress as shown in Figure 4.

325
326 Convergence is usually a problem in the execution of FE programmes for materials exhibiting
327 softening behaviour under implicit scheme as in most FE programmes. Sudden dissipation of
328 energy will make the computation more dynamical. An artificial viscosity is therefore used to
329 overcome the convergence difficulties by making the stiffness matrix of the material positive.
330 This viscosity regularizes the constitutive relation of the cohesive element by modifying the
331 stiffness reduction variable D as follows,

332
$$\dot{D}_v = \frac{D - D_v}{\mu} \quad (12)$$

333 where μ is the viscosity parameter which can be specified in the property of cohesive element
334 and D_v is the viscous stiffness degradation variable. Once μ and D are known, D_v can be
335 determined. A small viscosity value μ helps improve the rate of convergence without
336 compromising results.

337
338
339 **6 WORKED EXAMPLE**

340 As a demonstration of the application of the developed numerical method and techniques in
341 solving non-uniform corrosion induced concrete cracking, an example is carried out. The
342 values for all the basic parameters are shown in Table 1, together with their sources. The
343 boundary condition to the concrete cylinder caused by corrosion is first calculated from
344 Equation (9). Figure 5 shows the displacement $d_m(t)$, a key parameter controlling the whole

345 inner boundary condition, during the first 10 years' service life. It can be seen that the corrosion
346 does not cause any displacement of concrete, until about 0.2 year. During this initial period,
347 the corrosion products fill the band of the ITZ, of which the process is assumed stress-free.
348 According to the inputs in Table 1, this process takes 0.19 year; afterwards, the corrosion will
349 cause deformation of concrete and hence stresses. The displacement d_m steadily increases over
350 the service time and reaches about 0.33mm after 10 years. Once d_m is known, the whole inner
351 boundary condition of the concrete cylinder is determined.

352

353 The mesh grid of numerical example is shown as Figure 6. Under the time-varying expansion
354 caused by corrosion, the concrete cylinder undergoes a short period of elasticity, followed by
355 cracking initiation and propagation. Figure 7 shows the stress plot (i.e., the maximum principal
356 stress) for the concrete cylinder at 6%, 13%, 21%, and 75% of the displacement at 10 years.
357 From the loading stage up to 21% of the 10-year displacement, the stress concentration can be
358 clearly seen which moves from the inner boundary to the outer boundary of the cylinder.
359 Meanwhile, a discrete crack was initiated at about 10^0 counter-clockwise from the bottom point
360 of the inner circular boundary. This discrete crack propagates outwards as the loading increases
361 and finally reach the concrete cover. Moreover, from the insets of Figure 7, it is clearer to see
362 the crack and the distance between the true crack tip and the stress concentration. Such a
363 distance represents the length of the fracture process zone and will be discussed in detail later.

364

365 Other than the principal discrete crack propagating from the inner boundary to the outer cover
366 surface of the concrete, there are also a number of small cracks or damage zones around this
367 principal crack. Figure 8 shows the cohesive elements whose damage parameter D is between
368 0.1 and 1.0. It should be mentioned that the value 0 of D represents the no damage and 1.0
369 represents the state of fully damaged/cracked. These damage zones will never be cracked fully,

370 as there is only one discrete crack propagating. In Figure 8, the black lines represent the
371 partially damaged cohesive elements whose damage value is bigger than 0.1 but smaller than
372 1.0, i.e., $0.1 < D < 1.0$; the green lines represent the fully cracked cohesive elements, i.e., $D =$
373 1.0. The fully cracked cohesive elements form a discrete crack propagating from inner
374 boundary to the cover surface, whilst the partially damaged cohesive elements around the
375 discrete crack can be treated as damaged zones around the main crack. Such damaged zones
376 are believed considerably affected by the displacement boundary condition applied at the inner
377 boundary. For example, once the discrete crack is initiated at the inner boundary, there may be
378 some sliding between the concrete cylinder and the steel, according to the frictional property
379 of the rust. In modelling, such a phenomenon has not been considered and the sliding is
380 prohibited. How much this may affect the results needs to be investigated and confirmed in
381 future research.

382

383 Once the crack approaches the outer cover surface of the concrete, the crack starts to be visible,
384 known as longitudinal crack, on the surface of concrete structures. Since the theoretical
385 thickness of the cohesive element is set to be 1.0, the strain of the cohesive element is equal to
386 its displacement. Upon measuring the distance between the nodes of the last cohesive element
387 at the outer boundary of the cylinder, the surface crack width can be expressed in a function of
388 time, shown in Figure 9. In Figure 9, it can be seen that the crack propagates to the surface at
389 about 1.20 years when the steel loss ratio is about 0.29% after which the crack width gradually
390 increases with time. The abrupt increase in the crack width corresponds to the sudden energy
391 release when the crack approaches the cover surface. At 10 years, the steel loss ratio is about
392 0.94% and the crack width reaches about 0.33mm.

393

394 **7 VERIFICATION**

395 To verify the proposed numerical method, the results are compared with those from (Vidal, et
396 al., 2004). Vidal et al. (Vidal, et al., 2004) have experimentally investigated the relationship
397 between the degree of corrosion and the surface crack width; more importantly, non-uniform
398 pitting corrosion was naturally produced over periods of 14 and 17 years. In their research, the
399 results of the “beam B in concrete tensile zone” were taken, because the corrosion distribution
400 (i.e., Figure 8 in (Vidal, et al., 2004)) of this sample is very similar to that proposed in this
401 paper, i.e., the semi-elliptical shape. The inputs from the test (Vidal, et al., 2004) were used in
402 the numerical simulation, as presented in Table 2. The experimental results of crack width were
403 expressed as a function of the pit penetration; however, it is transformed to crack width versus
404 maximum displacement, d_m , for the sake of comparison with the numerical results. The
405 transformation procedure is presented below before the comparison is provided.

406 The values of the pit penetration and its corresponding crack width are directly extracted from
407 the Figure 10 in (Vidal, et al., 2004) and $\Delta A_s / A_s$ calculated. To correlate the corrosion extent
408 $\Delta A_s / A_s$ to the corrosion front displacement d_m in the developed model of this research, the
409 loss of the cross section of rebar is first determined as follows:

$$410 \quad \Delta A_s = \frac{\alpha_{rust} W_{rust}}{\rho_{st}} \quad (13)$$

411 where W_{rust} is a function of the corrosion rate and can be determined via Equation (6). α_{rust}
412 and ρ_{rust} are material constants and have been defined earlier. The corrosion extent $\Delta A_s / A_s$
413 can therefore be derived as follows:

$$414 \quad \frac{\Delta A_s}{A_s} = \frac{\alpha_{rust} W_{rust}}{\rho_{st} A_s} \quad (14)$$

415 Based on Equation (5), d_m can be obtained as a function of the corrosion extent $\Delta A_s / A_s$,
416 presented as follows:

$$417 \quad d_m(t) = \frac{4\rho_{st}A_s}{\alpha_{rust}\pi D} \left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}} \right) \frac{\Delta A_s}{A_s} - 2d_0 \quad (15)$$

418 The comparisons of the crack width from the developed numerical model and the experiments
419 are illustrated in Figure 10. It can be seen that the simulated results are in good agreement with
420 the experimental values. It is also interesting to see that the crack width growth is in a roughly
421 linear relationship with the maximum corrosion-caused displacement d_m .

422

423 **8 ANALYSIS AND DISCUSSION**

424 Under the non-uniform expansion, it would be useful to plot the hoop stress distribution along
425 the inner boundary and the outer boundary of the concrete cylinder. The hoop stress determines
426 the cracking initiation state and the inner and outer boundaries represent the two key locations
427 where internal cracking and surface cracking start. It can be seen from Figure 11, the hoop
428 stress changes along the inner boundary. At loading stage of 6% of the 10-year displacement,
429 the whole inner boundary is in tension in the hoop direction except for a small region around
430 the location of 90° ; such a small region at the bottom of the circular boundary is in bi-axial
431 compression state, although the compression in the hoop direction is small. Moreover, the
432 tensile stress is concentrated around the location of 10° of the inner boundary, which indicates
433 the location of the start of the crack. As the expansive load is increased and meanwhile the
434 crack is propagated, the hoop stress (tensile) at the location of 10° drops to 0 and the hoop stress
435 at location of 90° increases up to about 10MPa (in compression). The top half of the boundary
436 of the concrete cylinder is dominantly in tension in the hoop direction. Clearly, there are two

437 regions to be of interest, i.e., one around the location of 10^0 where the tensile stress is
438 concentrated and the other around the location of 90^0 where the compression is localised.

439

440 Figure 12 illustrates the hoop stress distribution along the outer boundary of the concrete
441 cylinder. It is very interesting to see that, under non-uniform corrosion expansion, the outer
442 boundary of the concrete is not all in tension in the hoop direction from the beginning of
443 loading. For example, under the 13% of the 10-year displacement, the area around the 10^0
444 location is in compression; however, it gradually changes to tension and increases to its peak
445 stress before it drops to 0 which marks the cover surface cracking. The three curves in Figure
446 12 represent the typical stress states of the outer boundary of the concrete: (1) tension-
447 dominated with a small compression concentration area around the location of 10^0 , at minor
448 loading level; (2) all in tension with a strong tension concentration around the location of 10^0
449 at cracking loading level; and (3) nearly stress-free at post-surface cracking loading level.

450

451 Figure 13 shows the hoop stress distribution along the discrete crack path in the 10^0 direction
452 outwards. It represents the tensile stress distribution along the discrete crack. It can be seen
453 that, as the loading increases, the peak stress (i.e., tensile strength) moves along the crack. After
454 the peak stress moves over a point, the stress of that point starts to decrease. Such a
455 phenomenon is called strain softening, as explained earlier. From the point where the tensile
456 stress softens to zero to the point where the tensile stress reaches its maximum value, the region
457 is known as Fracture Process zone (FPZ). In Figure 13, it has been schematically demonstrated
458 the length of the FPZ in this problem is 7.05mm. This FPZ has not been fully developed until
459 the loading of 11.4% of the 10-year displacement; afterwards, the constant length of FPZ will
460 move, ahead of the true crack, towards the cover surface, as shown in Figure 7.

461

462 To investigate the sensitivity of some underlying factors, a parametric study is conducted.
463 Figure 14 shows the effect of corrosion rate i_{corr} on the crack width as a function of time. Four
464 corrosion rates have been chosen, i.e., the varying corrosion rate (as presented in Table 1),
465 $i_{corr} = 0.5 \mu A / cm^2$, $i_{corr} = 1.0 \mu A / cm^2$ and $i_{corr} = 5.0 \mu A / cm^2$ (broomfield, 2007). It can be seen
466 that, as the corrosion rate increases, the time to surface cracking is advanced significantly.
467 Further, the crack width under higher corrosion rate is considerably larger than that under lower
468 corrosion rate. It is also interesting to find out that the varying corrosion rate used (e.g., in
469 Table 1) is equivalent to a corrosion rate between $i_{corr} = 1.0 \mu A / cm^2$ and $5.0 \mu A / cm^2$ but more
470 close to $i_{corr} = 1.0 \mu A / cm^2$. Nevertheless, the initial sudden increases in the surface crack width
471 for different corrosion rates are comparable.

472

473 Figure 15 shows the effect of Young's modulus of concrete on the development of the surface
474 crack width. Quite a large range of Young's modulus was selected, i.e., from 4GPa to 28GPa.
475 It has been found that the change of Young's modulus of concrete does not change much the
476 long-term increase of the crack width; however, it does affect the time to surface cracking and
477 the initial increase of crack width. It is surprising to see the larger the elastic modulus of the
478 concrete, the earlier the surface is cracked. This is mainly because, under the displacement
479 boundary condition, the larger elastic modulus will cause higher stress and thus earlier
480 cracking. Figure 16 shows the effect of the tensile strength of concrete on the surface crack
481 width. Four curves have been produced regarding the crack width development over time for
482 the tensile strengths of 3MPa to 6MPa. It can be found that, as the tensile strength changes, the
483 crack width does not change significantly, especially after the surface cracking. However, there
484 is some effect on the time to surface cracking; for example, when the tensile strength is
485 increased from 3MPa to 6MPa, the time to surface cracking is delayed from about 1.16 years
486 to 1.27 years (i.e., 0.11 year delay). Therefore, the tensile strength and Young's modulus of

487 concrete have very little effect on long-term crack width development, which is consistent with
488 experimental results from (Wang and Zheng, 2009, Williamson and Clark, 2000).

489

490 There have been arguments as if the shape of the softening curve of the tensile stress-
491 displacement constitutive relation of concrete will affect the fracture behaviour of concrete
492 structures. In this study, we tested three softening curves, i.e., linear, bi-linear and exponential,
493 as shown in Figure 17(a). Although the three curves are different in their softening shapes, the
494 areas underneath the curves are identical, i.e., the fracture energy is the same. $G_f = 50N/m$
495 and $f_t = 5MPa$ are used for all softening curves. The kink point for the bi-linear curve is

496 determined from Petersson (Petersson, 1981). Accordingly, $\sigma_s = \frac{f_t}{3}$ and $W_s = \frac{4W_0}{9}$. The

497 development of the crack width over time for these three cases is shown in Figure 17(b). It can
498 be seen that there is almost no difference amongst these curves. Therefore, it is proved that the
499 shape of the softening curve is less important than the actual value of fracture energy, in terms
500 of surface crack width development.

501

502 Other than material properties, some key numerical parameters are also investigated to
503 determine their sensibility on the results. Figure 18 shows the effects of the viscosity and the
504 penalty stiffness on the surface crack width development. It can be found that, the smaller the
505 viscosity is, the surface crack time is smaller and more simulation steps/time are needed.
506 However the effect of viscosity becomes insignificant when the viscosity is smaller than $1e-4$.
507 For viscosity $1e-6$, the calculation has convergence problem after about 9 years. After the trial
508 and error analysis, the viscosity in this study is set $1e-5$. As discussed, the stiffness of the
509 cohesive interface should be infinitesimally large in theory. For the penalty stiffness of $10GPa$,
510 the elastic deformation of cohesive element has a significant effect on the crack width

511 evolution, as shown in Figure 18b. However, when the penalty stiffness is larger than 100GPa,
512 the crack width development is close to each other, although there are still slight differences in
513 the initial crack width increase. The penalty stiffness in this study is set 1,000GPa. The sudden
514 increase of crack width after formation of surface crack is caused by a deformation of cohesive
515 elements around the reinforcing bar.

516

517

518 **9 CONCLUSIONS**

519 In this paper, a numerical model has been developed to predict the crack width caused by non-
520 uniform reinforcement corrosion. A non-uniform corrosion model was first formulated as a
521 function of the time. Under the non-uniform expansion to the concrete cover, a fracture model
522 was established to simulate the crack initiation and propagation. In formulating arbitrary
523 cracking, cohesive crack elements were inserted at all the boundaries of the mesh, based on an
524 in-house script written in Python. A worked example was presented to demonstrate the
525 application of the derived model and comparisons with the experimental results from literature
526 were made. It has been found that the numerical results are in good agreement with the
527 experimental results. A comprehensive parametric study was also carried out to investigate the
528 sensitivities of some key parameters mainly on the crack width growth. It has also been found
529 that the discrete crack tends to be initiated at the 10^0 location and the corrosion rate is the most
530 influencing single parameter that controls both the initiation and the long-term development of
531 the crack width; however, the elastic modulus and the tensile strength of concrete only affect
532 the initial crack width and the time to surface cracking but have little effect on the long-term
533 development. Moreover, the shape of the softening curve from the constitutive stress-
534 displacement relation of concrete has found to have nearly no effect on the crack width
535 development. It can be concluded that the numerical method presented in the paper can predict
536 the concrete crack width induced by the realistic non-uniform reinforcement corrosion.

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671 2. Values of some variables used for comparison and validation

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Table 1 Values of basic variables used in the example

Description	Symbol	Values	Sources
Inner radius	a	6mm	(Li, 2003)
Outer radius	b	37mm	(Li, 2003)
Effective modulus of elasticity	E_{ef}	7.05 GPa	(Li, 2003)
Poisson's ratio	ν_c	0.18	(Li, 2003)
Shear modulus	G	$E/[2(1+\nu)]$	(Timoshenko and Goodier, 1970)
Density of rust	ρ_{rust}	3600 kg/m ³	(Liu and Weyers, 1998)
Density of steel	ρ_{st}	7850 kg/m ³	(Liu and Weyers, 1998)
Corrosion current density	i_{corr}	$0.3686\ln(t)+1.1305$ $\mu A/cm^2$	(Li, 2003)
Tensile strength	f_t'	5MPa	(Li, 2003)
Fracture energy	G_f	50N/m	(Ren, et al., 2015)
Penalty Stiffness	K_p	100GPa	N/A
Viscosity	μ	1e-5	N/A

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Table 2 Values of some variables used for comparison and validation

Description	Symbol	Values
Inner radius	a	6mm
Outer radius	b	22mm
Modulus of elasticity	E	32 GPa
Creep	c_p	2.0
Effective modulus of elasticity	E_{ef}	$\frac{E}{1+c_p}$
Poisson's ratio	ν_c	0.18
Tensile strength	f_t'	4.7MPa
Fracture energy	G_f	50N/m

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