

Robust Discrete-state-feedback Stabilization of Hybrid Stochastic Systems with Time-varying Delay Based on Razumikhin Technique

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Abstract

This paper deals with the robust stabilization of continuous-time hybrid stochastic systems with time-varying delay by feedback controls based on discrete-time state observations. By employing the Razumikhin technique, delay-independent criteria to determine controllers and time lags are established just under a weaker condition that the time-varying delay should be a bounded function. Meanwhile, for the nondelay system, we obtain a better bound on the duration τ between two consecutive state observations. The new theory developed in this paper improves the existing results. Numerical examples are provided to demonstrate the effectiveness of our results.

Keywords: Hybrid stochastic systems, Time-varying delay, Robust stabilization, Discrete-time feedback control, Razumikhin technique

1. Introduction

Stochastic systems have received a lot of attention as stochastic modeling has played a more and more important role in many branches of science and engineering (see e.g. [1–4]). In practice, the structures and parameters of some stochastic systems may change abruptly due to random failures of components, sudden environment changes, etc. Hybrid stochastic differential equations (SDEs) (also known as SDEs with Markovian switching) have been employed to model such problems (see e.g. [5–7]). An area of particular interest in the study of hybrid SDEs is the automatic control, with subsequent emphasis being placed on the stability analysis. There is an intensive literature in the area (see e.g. [8–12]).

On the other hand, it has been recognized that time delay frequently occurs in various dynamic systems and, very often, it has an unstable effect and leads to poor performance of control systems. Stability and stabilization problems of hybrid stochastic time-delay systems have therefore attracted a lot of interest. A huge number of papers have appeared on these topics. For example, Mao [10] investigated the exponential stability for a class of linear hybrid stochastic delay interval systems. A robust state-feedback controller was designed in [13] to exponentially stabilize a class of bilinear continuous time-delay uncertain stochastic systems with Markovian jumping parameters. In [14], the robust stabilization problem of uncertain stochas-

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tic delay systems was studied and both delay-dependent and delay-independent stabilization criteria were established. We further refer the reader to [15–20] and references therein. It should be pointed out that most of the existing results on the stability of hybrid stochastic delay systems require the time delay to be a constant or a differentiable function whose derivative is bound by a positive constant less than 1, and this is very restrictive.

Moreover, up to 2013, most of the existing papers used the feedback controls based on continuous-time state observations to stabilize unstable stochastic delay systems with Markovian switching. However, it is not only expensive to observe the state continuously in time but also impractical sometimes. In 2013, Mao [21] studied the stabilization problem of hybrid stochastic systems by feedback control based on discrete-time state observations, which developed the corresponding study for deterministic systems (see e.g. [22, 23]). Later Mao et al. in [24] provided us with a better bound on the duration between two consecutive state observations, while You et al. [25] removed the global Lipschitz assumption on coefficients and further investigated the asymptotic stabilization of nonlinear hybrid stochastic systems. Recently, Zhao et al. [26] extended Mao’s work [21] to discuss the p th moment exponential stabilization of continuous-time hybrid stochastic functional differential equations by feedback control based on discrete-time state observations. With the help of an auxiliary system whose control is based on continuous-time state observations and assumed to be p th moment exponentially stable, the criterion for p th moment exponential stability of the discrete-time-state-feedback controlled system was established. This is of course a very general result. However it is due to the general comparison technique used there that the bound on the duration between two consecutive state observations is not very sharp and the theory developed there is a little cumbersome and not easy to use because we must first guarantee and verify the p th moment exponential stability of the auxiliary system.

In this paper, we will work directly on the discrete-time-state-feedback controlled system. By employing the Razumikhin technique, not only the the robust discrete-time-state-feedback stabilization problem for a class of hybrid stochastic systems with a time-varying delay will be settled, but also the restriction that the time-varying delay should be differentiable as in most existing results will be removed. Moreover, a better bound on the duration between two consecutive state observations will be obtained than some existing results. We will present our main results of the linear case in Section 3 after giving some preliminaries in Section 2. Section 4 is devoted to the nonlinear case. The usefulness and applicability of the theory established will be illustrated by a couple of examples in Section 5. Finally, the paper will be concluded in Section 6.

2. Preliminaries and problem formulation

Throughout this paper, we use the following notations. For a matrix A , we let $|A| = \sqrt{\text{trace}(A^T A)}$ be its trace norm and $\|A\| = \max\{|Ax| : |x| = 1\}$ be the operator norm. If A is a symmetric matrix, denote by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ its smallest and largest eigenvalue, respectively. For two symmetric matrices A and B , $A > (<, \geq, \leq)B$ means that $A - B$ is positive definite (negative definite, positive semidefinite, negative semidefinite, respectively). Let $\tau > 0$ and $C([- \tau, 0]; \mathbb{R}^n)$ denote the family of continuous functions φ from $[- \tau, 0]$ to \mathbb{R}^n with the norm $\|\varphi\| = \sup_{-\tau \leq \theta \leq 0} |\varphi(\theta)|$. If both a, b are real numbers, then $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a complete probability space with a filtration

$\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous with \mathcal{F}_0 containing all \mathbb{P} -null sets). Let $w(t) = (w_1(t), \dots, w_m(t))^T$ be an m -dimensional Brownian motion defined on the probability space. Denote by $L^2(\Omega; \mathbb{R}^n)$ the family of all \mathbb{R}^n -valued random variables X such that $\mathbb{E}|X|^2 < \infty$. Let $C_{\mathcal{F}_0}^b([-\tau, 0]; \mathbb{R}^n)$ denote the family of all bounded, \mathcal{F}_0 -measurable $C([-\tau, 0]; \mathbb{R}^n)$ -valued random variables. For $p > 0$ and $t \geq 0$, $L_{\mathcal{F}_t}^p([-\tau, 0]; \mathbb{R}^n)$ stands for the family of all \mathcal{F}_t -measurable $C([-\tau, 0]; \mathbb{R}^n)$ -valued random variables $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau \leq \theta \leq 0} \mathbb{E}|\phi(\theta)|^p < \infty$. Let $r(t), t \geq 0$, be a right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \dots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$\mathbb{P}\{r(t + \Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$ and $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while $\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij}$. We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $w(\cdot)$.

Let $C^{2,1}(\mathbb{R}^n \times [-h, \infty) \times S; \mathbb{R}_+)$ denote the family of all nonnegative continuous functions $V(x, t, i)$ from $\mathbb{R}^n \times [-h, \infty) \times S$ to \mathbb{R}_+ satisfying that for each $i \in S$, $V(x, t, i)$ is continuously twice differentiable in x and once in t . If $V \in C^{2,1}(\mathbb{R}^n \times [-h, \infty) \times S; \mathbb{R}_+)$, define a functional $\mathcal{L}V$ from $C([-h, 0]; \mathbb{R}^n) \times \mathbb{R}_+ \times S$ to \mathbb{R} by

$$\begin{aligned} \mathcal{L}V(\varphi, t, i) = & V_t(\varphi(0), t, i) + V_x(\varphi(0), t, i)f(\varphi, t, i) + \frac{1}{2}\text{trace}[g^T(\varphi, t, i)V_{xx}(\varphi(0), t, i)g(\varphi, t, i)] \\ & + \sum_{j=1}^N \gamma_{ij}V(\varphi(0), t, j), \end{aligned} \quad (2.1)$$

where $V_t(x, t, i) = \frac{\partial V(x, t, i)}{\partial t}$, $V_x(x, t, i) = (\frac{\partial V(x, t, i)}{\partial x_1}, \dots, \frac{\partial V(x, t, i)}{\partial x_n})$, $V_{xx}(x, t, i) = (\frac{\partial^2 V(x, t, i)}{\partial x_i \partial x_j})_{n \times n}$. We emphasize that $\mathcal{L}V$ is thought as a single notation but not \mathcal{L} acting on V .

Let us now consider the linear hybrid stochastic differential delay equations (SDDEs) in the form

$$dx(t) = [A(r(t))x(t) + A_d(r(t))x(t - \delta(t))]dt + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t) \quad (2.2)$$

on $t \geq 0$, with initial data $x_0 = \xi \in C_{\mathcal{F}_0}^b([-h, 0]; \mathbb{R}^n), r(0) = r_0 \in S$. Here $A(i) = A_i, A_d(i) = A_{di}, B^k(i) = B_i^k, B_d^k(i) = B_{di}^k$ are given constant matrices. The time delay $\delta(t)$ is a Borel-measurable function on $t \geq 0$ with $0 \leq \delta(t) \leq h$ for all $t \geq 0$. Assume that the given hybrid SDDE (2.2) is unstable. Our aim is to design a feedback control $u(x(\eta(t)), r(t))$ based on the discrete-time observations of the state $x(t)$ at times $0, \tau, 2\tau, \dots$ in the drift part so that the controlled system

$$\begin{aligned} dx(t) = & [A(r(t))x(t) + A_d(r(t))x(t - \delta(t)) + u(x(\eta(t)), r(t))]dt \\ & + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t) \end{aligned} \quad (2.3)$$

will be mean-square exponentially stable, where u is a mapping from $\mathbb{R}^n \times S$ to \mathbb{R}^n , $\tau > 0$ and $\eta(t) = [t/\tau]\tau$ for $t \geq 0$, in which $[t/\tau]$ is the integer part of t/τ . We choose the structure control, one of the most common linear feedback controls, of the form $u(x, i) = F(i)G(i)x$, where F and G are mappings from S to $\mathbb{R}^{n \times l}$ and $\mathbb{R}^{l \times n}$, respectively, and one of them is given while the other needs to be designed. These two cases are known as (see e.g. [24]):

- State feedback: design $F(\cdot)$ when $G(\cdot)$ is given;
- Output injection: design $G(\cdot)$ when $F(\cdot)$ is given.

Consequently, the controlled system (2.3) becomes

$$\begin{aligned} dx(t) = & [A(r(t))x(t) + A_d(r(t))x(t - \delta(t)) + F(r(t))G(r(t))x(\eta(t))]dt \\ & + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t). \end{aligned} \quad (2.4)$$

The controlled system (2.4) is in fact a hybrid SDDE with mixed bounded variable delays in the form

$$\begin{aligned} dx(t) = & [A(r(t))x(t) + A_d(r(t))x(t - \delta(t)) + F(r(t))G(r(t))x(t - \zeta(t))]dt \\ & + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t), \end{aligned} \quad (2.5)$$

where the other bounded variable delay $\zeta : [0, \infty) \rightarrow [0, \tau]$ is defined by $\zeta(t) = t - v\tau$ for $v\tau \leq t < (v+1)\tau$, where $v = 0, 1, 2, \dots$. It is easy to know that given any initial data $x_0 = \xi \in C_{\mathcal{F}_0}^b([-\gamma, 0]; \mathbb{R}^n)$, where $\gamma = h \vee \tau$, system (2.5) has a unique continuous solution $x(t)$ such that $\mathbb{E}|x(t)|^2 < \infty$ for all $t \geq -\gamma$ (see [11]). As systems (2.4) and (2.5) are equivalent, we will mainly focus on system (2.5) in the rest of this paper as we feel it is more convenient to apply the Razumikhin method to it.

3. Main results

This section is devoted to the linear case. We will give the stability criterion first and then provide the method for designing the controller, both the cases of state feedback and output injection included.

Write Eq. (2.5) as

$$\begin{aligned} dx(t) = & [(A(r(t)) + F(r(t))G(r(t)))x(t) + A_d(r(t))x(t - \delta(t)) - F(r(t))G(r(t))(x(t) - x(t - \zeta(t)))]dt \\ & + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t) \end{aligned} \quad (3.1)$$

on $t \geq \gamma$. For each $t \geq \gamma$, let us define an operator $\Phi(t, \cdot) : L_{\mathcal{F}_t}^2([-2\gamma, 0]; \mathbb{R}^n) \rightarrow L^2(\Omega; \mathbb{R}^n)$ by

$$\begin{aligned} \Phi(t, \varphi) = & \int_{t-\zeta(t)}^t [A(r(s))\varphi(s-t) + A_d(r(s))\varphi(s-\delta(s)-t) + F(r(s))G(r(s))\varphi(s-\zeta(s)-t)]ds \\ & + \sum_{k=1}^m \int_{t-\zeta(t)}^t [B^k(r(s))\varphi(s-t) + B_d^k(r(s))\varphi(s-\delta(s)-t)]dw_k(s). \end{aligned} \quad (3.2)$$

Moreover, on $t \geq \gamma$, let $x_t(\theta) = x(t + \theta)$ for $-2\gamma \leq \theta \leq 0$, which is regarded as a $C([-2\gamma, 0]; \mathbb{R}^n)$ -valued stochastic process. Then it is easy to see that $x(t) - x(t - \zeta(t)) = \Phi(t, x_t)$, and hence (2.5) can be further written as

$$\begin{aligned} dx(t) = & [(A(r(t)) + F(r(t))G(r(t)))x(t) + A_d(r(t))x(t - \delta(t)) - F(r(t))G(r(t))\Phi(t, x_t)]dt \\ & + \sum_{k=1}^m [B^k(r(t))x(t) + B_d^k(r(t))x(t - \delta(t))]dw_k(t) \end{aligned} \quad (3.3)$$

on $t \geq \gamma$ with initial data $x(t) = \xi$ for $t \in [-\gamma, 0]$ and $x(t) = x(t, \xi)$ for $t \in [0, \gamma]$.

Let us first present a lemma in order to prove our main result.

Lemma 3.1. *Set*

$$M_A = \max_{i \in S} \|A_i\|^2, \quad M_{A_d} = \max_{i \in S} \|A_{di}\|^2, \quad M_{FG} = \max_{i \in S} \|F_i G_i\|^2,$$

$$M_B = \max_{i \in S} \sum_{k=1}^m \|B_i^k\|^2, \quad M_{B_d} = \max_{i \in S} \sum_{k=1}^m \|B_{di}^k\|^2,$$

and define

$$K_\tau = 6\tau^2(M_A + M_{A_d} + M_{FG}) + 4\tau(M_B + M_{B_d}).$$

Then for all $t \geq \gamma$ and $\varphi \in L_{\mathcal{F}_t}^2([-2\gamma, 0]; \mathbb{R}^n)$, the operator Φ defined by (3.2) has the property that

$$\mathbb{E}|\Phi(t, \varphi)|^2 \leq K_\tau \sup_{-2\gamma \leq \theta \leq 0} \mathbb{E}|\varphi(\theta)|^2.$$

Proof. By Hölder's inequality and Doob's martingale inequality, we can derive from (3.2) that

$$\begin{aligned} \mathbb{E}|\Phi(t, \varphi)|^2 &\leq 2\tau \mathbb{E} \int_{t-\zeta(t)}^t |A(r(s))\varphi(s-t) + A_d(r(s))\varphi(s-\delta(s)-t) + F(r(s))G(r(s))\varphi(s-\zeta(s)-t)|^2 ds \\ &\quad + 2 \sum_{k=1}^m \mathbb{E} \int_{t-\zeta(t)}^t |B^k(r(s))\varphi(s-t) + B_d^k(r(s))\varphi(s-\delta(s)-t)|^2 ds \\ &\leq 6\tau \mathbb{E} \int_{t-\tau}^t (M_A |\varphi(s-t)|^2 + M_{A_d} |\varphi(s-\delta(s)-t)|^2 + M_{FG} |\varphi(s-\zeta(s)-t)|^2) ds \\ &\quad + 4\mathbb{E} \int_{t-\tau}^t (M_B |\varphi(s-t)|^2 + M_{B_d} |\varphi(s-\delta(s)-t)|^2) ds \\ &\leq K_\tau \sup_{-2\gamma \leq \theta \leq 0} \mathbb{E}|\varphi(\theta)|^2 \end{aligned}$$

as required.

Next we establish sufficient conditions to guarantee the robust exponential stability of system (3.3).

Theorem 3.2. *Assume that there exist positive definite matrices $Q_i (i \in S)$ such that*

$$\bar{Q}_i := Q_i(A_i + F_i G_i) + (A_i + F_i G_i)^T Q_i + \sum_{k=1}^m (B_i^k)^T Q_i B_i^k + \sum_{j=1}^N \gamma_{ij} Q_j \quad (3.4)$$

are all negative-definite matrices. Set

$$\begin{aligned} \lambda_M &= \max_{i \in S} \lambda_{\max}(Q_i), \quad \lambda_m = \min_{i \in S} \lambda_{\min}(Q_i), \quad \lambda = \max_{i \in S} \lambda_{\max}(\bar{Q}_i), \quad M_{QA_d} = \max_{i \in S} \|Q_i A_{di}\|^2, \\ M_{QFG} &= \max_{i \in S} \|Q_i F_i G_i\|^2, \quad \bar{M} = \max_{i \in S} \left(\sum_{k=1}^m \|(B_i^k)^T Q_i B_{di}^k\|^2 \right), \quad \beta = \max_{i \in S} \lambda_{\max} \left(\sum_{k=1}^m (B_{di}^k)^T Q_i B_{di}^k \right) \end{aligned}$$

(and of course $\lambda < 0$). If τ is sufficiently small for

$$\lambda + 2\sqrt{\frac{\lambda_M M_{QA_d}}{\lambda_m}} + 2\sqrt{\frac{\lambda_M M_{QFG} K_\tau}{\lambda_m}} + 2\sqrt{\frac{\lambda_M \bar{M}}{\lambda_m}} + \frac{\beta \lambda_M}{\lambda_m} < 0, \quad (3.5)$$

then the solution of (3.3) satisfies

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log(\mathbb{E}|x(t; \xi)|^2) \leq -\frac{\log(q)}{\gamma},$$

where $q > 1$ is the unique root to the following equation

$$\lambda + 2\sqrt{\frac{q \lambda_M M_{QA_d}}{\lambda_m}} + 2\sqrt{\frac{q \lambda_M M_{QFG} K_\tau}{\lambda_m}} + 2\sqrt{\frac{q \lambda_M \bar{M}}{\lambda_m}} + \frac{\beta q \lambda_M}{\lambda_m} = -\lambda_M \frac{\log(q)}{\gamma}. \quad (3.6)$$

Proof. The proof is an application of the Razumikhin-type theorem (Theorem 8.9 on page 311 of [11]) with $p = 2$. Define $V \in C^{2,1}(\mathbb{R}^n \times [-\gamma, \infty) \times S; \mathbb{R}_+)$ by $V(x, t, i) = x^T Q_i x$. Obviously,

$$\lambda_m |x|^2 \leq V(x, t, i) \leq \lambda_M |x|^2.$$

In the next, we need to show that

$$\mathbb{E}[\max_{i \in S} \mathcal{L}V(\varphi, t, i)] \leq -\frac{\log(q)}{\gamma} \mathbb{E}[\max_{i \in S} V(\varphi(0), t, i)] \quad (3.7)$$

for all $t \geq \gamma$ and those $\varphi = \{\varphi(\theta) : -2\gamma \leq \theta \leq 0\} \in L^2_{\mathcal{F}_t}([-2\gamma, 0]; \mathbb{R}^n)$ satisfying

$$\mathbb{E}[\min_{i \in S} V(\varphi(\theta), t + \theta, i)] < q \mathbb{E}[\max_{i \in S} V(\varphi(0), t, i)], \quad \forall \theta \in [-2\gamma, 0]. \quad (3.8)$$

For this purpose, we compute $\mathcal{L}V(\varphi, t, i)$ as follows

$$\begin{aligned} \mathcal{L}V(\varphi, t, i) &= 2\varphi^T(0)Q_i[(A_i + F_i G_i)\varphi(0) + A_{di}\varphi(-\delta(t)) - F_i G_i \Phi(t, \varphi)] + \sum_{j=1}^N \gamma_{ij} \varphi^T(0)Q_j \varphi(0) \\ &+ \sum_{k=1}^m [B_i^k \varphi(0) + B_{di}^k \varphi(-\delta(t))]^T Q_i [B_i^k \varphi(0) + B_{di}^k \varphi(-\delta(t))] \\ &\leq 2\varphi^T(0)Q_i(A_i + F_i G_i)\varphi(0) + \alpha_1 |\varphi(0)|^2 + \frac{M_{QA_d}}{\alpha_1} |\varphi(-\delta(t))|^2 \\ &+ \alpha_2 |\varphi(0)|^2 + \frac{M_{QFG}}{\alpha_2} |\Phi(t, \varphi)|^2 + \sum_{j=1}^N \gamma_{ij} \varphi^T(0)Q_j \varphi(0) \\ &+ \varphi^T(0) \left(\sum_{k=1}^m (B_i^k)^T Q_i B_i^k \right) \varphi(0) + \alpha_3 |\varphi(0)|^2 + \frac{\overline{M}}{\alpha_3} |\varphi(-\delta(t))|^2 + \beta |\varphi(-\delta(t))|^2 \\ &\leq (\lambda + \alpha_1 + \alpha_2 + \alpha_3) |\varphi(0)|^2 + \left(\frac{M_{QA_d}}{\alpha_1} + \frac{\overline{M}}{\alpha_3} + \beta \right) |\varphi(-\delta(t))|^2 + \frac{M_{QFG}}{\alpha_2} |\Phi(t, \varphi)|^2. \end{aligned} \quad (3.9)$$

It follows from (3.8) that

$$\mathbb{E}|\varphi(\theta)|^2 < \frac{q\lambda_M}{\lambda_m} \mathbb{E}|\varphi(0)|^2, \quad \forall \theta \in [-2\gamma, 0]. \quad (3.10)$$

Setting $\alpha_1 = \sqrt{\frac{q\lambda_M M_{QA_d}}{\lambda_m}}$, $\alpha_2 = \sqrt{\frac{q\lambda_M M_{QFG} K_\tau}{\lambda_m}}$, $\alpha_3 = \sqrt{\frac{q\lambda_M \overline{M}}{\lambda_m}}$, applying Lemma 3.1 and combining (3.10) with (3.9) yield

$$\begin{aligned} &\mathbb{E}[\max_{i \in S} \mathcal{L}V(\varphi, t, i)] \\ &< (\lambda + \alpha_1 + \alpha_2 + \alpha_3) \mathbb{E}|\varphi(0)|^2 + \frac{q\lambda_M}{\lambda_m} \left(\frac{M_{QA_d}}{\alpha_1} + \frac{\overline{M}}{\alpha_3} + \beta \right) \mathbb{E}|\varphi(0)|^2 + \frac{q\lambda_M}{\lambda_m} \frac{M_{QFG}}{\alpha_2} K_\tau \mathbb{E}|\varphi(0)|^2 \\ &= (\lambda + 2\alpha_1 + 2\alpha_2 + 2\alpha_3 + \frac{\beta q \lambda_M}{\lambda_m}) \mathbb{E}|\varphi(0)|^2. \end{aligned}$$

From (3.6), we find that $\lambda + 2\alpha_1 + 2\alpha_2 + 2\alpha_3 + \frac{\beta q \lambda_M}{\lambda_m} < 0$. Thus

$$\begin{aligned} \mathbb{E}[\max_{i \in S} \mathcal{L}V(\varphi, t, i)] &\leq \frac{1}{\lambda_M} \left(\lambda + 2\alpha_1 + 2\alpha_2 + 2\alpha_3 + \frac{\beta q \lambda_M}{\lambda_m} \right) \mathbb{E}[\max_{i \in S} V(\varphi(0), t, i)] \\ &= -\frac{\log(q)}{\gamma} \mathbb{E}[\max_{i \in S} V(\varphi(0), t, i)], \end{aligned}$$

which is the required inequality (3.7). The proof is therefore complete.

The following two theorems provide us with the LMI method to design the controllers based on discrete-time observations of state to stabilize the unstable hybrid stochastic delay system (2.2). Theorems 3.3 and 3.4 demonstrate the cases of state feedback and output injection, respectively.

Theorem 3.3. Assume that $G_i(i \in S)$ are given and also that there are solutions $Q_i = Q_i^T > 0$ and $Y_i(i \in S)$ to the following LMIs

$$Q_i A_i + Y_i G_i + A_i^T Q_i + G_i^T Y_i^T + \sum_{k=1}^m (B_i^k)^T Q_i B_i^k + \sum_{j=1}^N \gamma_{ij} Q_j < 0. \quad (3.11)$$

Then by setting $F_i = Q_i^{-1} Y_i$, the controlled system (3.3) will be exponentially stable in mean square if $\tau > 0$ is small enough such that (3.5) holds.

Proof. Recalling $F_i = Q_i^{-1} Y_i$, we find that (3.11) is equivalent to the condition that the matrices in (3.4) are all negative-definite. So the required assertion follows directly from Theorem 3.2.

Theorem 3.4. Assume that $F_i(i \in S)$ are given and also that there are solutions $P_i = P_i^T > 0$ and $Y_i(i \in S)$ to the following LMIs

$$\Pi_i = \begin{pmatrix} \Pi_{11i} & \Pi_{21i}^T & \Pi_{31i}^T \\ \Pi_{21i} & -\Pi_{22i} & 0 \\ \Pi_{31i} & 0 & -\Pi_{33i} \end{pmatrix} < 0, \quad (3.12)$$

where $\Pi_{11i} = A_i P_i + F_i Y_i + P_i A_i^T + Y_i^T F_i^T + \gamma_{ii} P_i$, $\Pi_{21i} = (P_i (B_i^1)^T, \dots, P_i (B_i^m)^T)^T$, $\Pi_{22i} = \text{diag}(P_i, \dots, P_i)$, $\Pi_{31i} = (\sqrt{\gamma_{i1}} P_i, \dots, \sqrt{\gamma_{i,i-1}} P_i, \sqrt{\gamma_{i,i+1}} P_i, \dots, \sqrt{\gamma_{iN}} P_i)^T$, $\Pi_{33i} = \text{diag}(P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_N)$. Then by setting $Q_i = P_i^{-1}$ and $G_i = Y_i P_i^{-1}$, the controlled system (3.3) will be exponentially stable in mean square if $\tau > 0$ is small enough such that (3.5) holds.

Proof. We first observe that by the well-known Schur complements (see e.g.[11]), the LMIs (3.12) are equivalent to the following matrix inequalities

$$A_i P_i + F_i Y_i + P_i A_i^T + Y_i^T F_i^T + \gamma_{ii} P_i + \sum_{k=1}^m P_i (B_i^k)^T P_i^{-1} B_i^k P_i + \sum_{j \neq i}^N \gamma_{ij} P_i P_j^{-1} P_i < 0. \quad (3.13)$$

Recalling that $G_i = Y_i P_i^{-1}$ and $P_i = P_i^T$, multiplying P_i^{-1} from left and then from right, and noting $Q_i = P_i^{-1}$, we see that the matrix inequalities (3.13) are equivalent to the following matrix inequalities

$$Q_i A_i + Q_i F_i G_i + A_i^T Q_i + G_i^T F_i^T Q_i + \sum_{k=1}^m (B_i^k)^T Q_i B_i^k + \sum_{j=1}^N \gamma_{ij} Q_j < 0, \quad (3.14)$$

which means that the matrices in (3.4) are all negative-definite. Again, the required assertion follows directly from Theorem 3.2.

From the above theorems we can see that, to get the robust controllers, we should first find solutions for (3.11) or (3.12) and then obtain small τ from condition (3.5) after calculating all related quantities.

Let us now consider the nondelay system

$$dx(t) = A(r(t))x(t)dt + \sum_{k=1}^m B^k(r(t))x(t)dw_k(t), \quad (3.15)$$

the same as that considered in [24], which can be regarded as a special case of system (2.2) with $A_d(i) = 0$ and $B_d^k(i) = 0$. Also, we aim to design a discrete-time state feedback control of the form $u(x(\eta(t)), r(t)) = F(r(t))G(r(t))x(\eta(t))$ in the drift part so that the controlled system

$$dx(t) = [A(r(t))x(t) + F(r(t))G(r(t))x(\eta(t))]dt + \sum_{k=1}^m B^k(r(t))x(t)dw_k(t) \quad (3.16)$$

will be mean-square exponentially stable, where $F(i)$, $G(i)$ and $\eta(t)$ are defined the same as those in the previous. By the similar argument, we can obtain the following corollaries which provide the stability criterion of system (3.16) and the method for designing the controllers, respectively.

Corollary 3.5. *Assume that there exist positive definite matrices $Q_i (i \in S)$ such that matrices in (3.4) are all negative-definite. Let λ_M , λ_m , λ , M_A , M_B , M_{FG} , M_{QFG} be the same as those in Lemma 3.1 and Theorem 3.2. If τ is sufficiently small for*

$$\lambda + 2\sqrt{\frac{\lambda_M M_{QFG} K_\tau}{\lambda_m}} < 0, \quad (3.17)$$

where $K_\tau = 4\tau^2(M_A + M_{FG}) + 2\tau M_B$, then the solution of (3.16) satisfies

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log(\mathbb{E}|x(t)|^2) \leq -\frac{\log(q)}{\tau},$$

where $q > 1$ is the unique root to the following equation

$$\lambda + 2\sqrt{\frac{q\lambda_M M_{QFG} K_\tau}{\lambda_m}} = -\lambda_M \frac{\log(q)}{\tau}. \quad (3.18)$$

Corollary 3.6. *Given $G_i (i \in S)$, if the inequalities in (3.11) have their solutions $Q_i = Q_i^T > 0$ and $Y_i (i \in S)$, then by setting $F_i = Q_i^{-1} Y_i$, the controlled system (3.16) will be exponentially stable in mean square if $\tau > 0$ is small enough such that (3.17) holds.*

Corollary 3.7. *Given $F_i (i \in S)$, if the inequalities in (3.12) have their solutions $P_i = P_i^T > 0$ and $Y_i (i \in S)$, then by setting $Q_i = P_i^{-1}$ and $G_i = Y_i P_i^{-1}$, the controlled system (3.16) will be exponentially stable in mean square if $\tau > 0$ is small enough such that (3.17) holds.*

4. Stabilization of nonlinear hybrid SDDEs

In this section, we shall extend our theory to cope with the more general nonlinear problem.

Given an unstable nonlinear hybrid stochastic differential equation with time-varying delay

$$dx(t) = f(x(t), x(t - \delta(t)), t, r(t))dt + g(x(t), x(t - \delta(t)), t, r(t))dw(t) \quad (4.1)$$

on $t \geq 0$ with initial data $x_0 = \xi \in C_{\mathcal{F}_0}^b([-h, 0]; \mathbb{R}^n)$. Assume that both $f : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S \rightarrow \mathbb{R}^{n \times m}$ are locally Lipschitz continuous and obey the linear growth condition. Also, we suppose $f(0, 0, t, i) = 0$ and $g(0, 0, t, i) = 0$ for all $i \in S$ and $t \geq 0$, then $x = 0$ is an equilibrium point for (4.1). We now aim to design a linear feedback control $F(r(t))G(r(t))x(\eta(t))$ based on discrete-time state observations in the drift part so that the controlled system

$$dx(t) = [f(x(t), x(t - \delta(t)), t, r(t)) + F(r(t))G(r(t))x(\eta(t))]dt + g(x(t), x(t - \delta(t)), t, r(t))dw(t) \quad (4.2)$$

will be mean-square exponentially stable. In fact, system (4.2) can be further written as

$$\begin{aligned} dx(t) = & [f(x(t), x(t - \delta(t)), t, r(t)) + F(r(t))G(r(t))x(t - \zeta(t))]dt \\ & + g(x(t), x(t - \delta(t)), t, r(t))dw(t), \end{aligned} \quad (4.3)$$

recalling the definition of ζ in Section 2. It is easy to know that given any initial data $x_0 = \xi \in C_{\mathcal{F}_0}^b([-\gamma, 0]; \mathbb{R}^n)$, where $\gamma = h \vee \tau$, system (4.3) has a unique continuous solution $x(t)$ such that $\mathbb{E}|x(t)|^2 < \infty$ for all $t \geq -\gamma$ (see [11]). As before, next we will focus on system (4.3).

In order to stabilize a nonlinear system by a linear controller, we impose the following conditions.

Assumption 4.1. *For each $i \in S$, there are symmetric matrices Q_i, R_i and S_i with Q_i being positive-definite such that for all $(x, y, t, i) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S$,*

$$2x^T Q_i f(x, y, t, i) + g^T(x, y, t, i) Q_i g(x, y, t, i) \leq x^T R_i x + y^T S_i y.$$

Assumption 4.2. *There are four positive constants $\theta_1, \theta_2, \theta_3$ and θ_4 such that*

$$|f(x, y, t, i)|^2 \leq \theta_1 |x|^2 + \theta_2 |y|^2, \quad |g(x, y, t, i)|^2 \leq \theta_3 |x|^2 + \theta_4 |y|^2$$

for all $(x, y, t, i) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S$.

For each $t \geq \gamma$, define an operator $\Psi(t, \cdot): L_{\mathcal{F}_t}^2([-\gamma, 0]; \mathbb{R}^n) \rightarrow L^2(\Omega; \mathbb{R}^n)$ by

$$\begin{aligned} \Psi(t, \varphi) &= \int_{t-\zeta(t)}^t [f(\varphi(s-t), \varphi(s-\delta(s)-t), s, r(s)) + F(r(s))G(r(s))\varphi(s-\zeta(s)-t)] ds \\ &\quad + \int_{t-\zeta(t)}^t g(\varphi(s-t), \varphi(s-\delta(s)-t), s, r(s)) dw(s), \end{aligned} \quad (4.4)$$

then the operator has the following property.

Lemma 4.3. *Let Assumption 4.2 hold. Set*

$$\theta_5 = \max_{i \in S} \|F_i G_i\|^2 \quad \text{and} \quad H_\tau = 4\tau^2(\theta_1 + \theta_2 + \theta_5) + 2\tau(\theta_3 + \theta_4),$$

then for all $t \geq \gamma$ and $\varphi \in L_{\mathcal{F}_t}^2([-\gamma, 0]; \mathbb{R}^n)$, we have

$$\mathbb{E}|\Psi(t, \varphi)|^2 \leq H_\tau \sup_{-2\gamma \leq \theta \leq 0} \mathbb{E}|\varphi(\theta)|^2.$$

The proof is similar to that of Lemma 3.1 and so is omitted.

Theorem 4.4. *Let Assumptions 4.1 and 4.2 hold. If the following LMIs*

$$U_i = R_i + Q_i F_i G_i + G_i^T F_i^T Q_i + \sum_{j=1}^N \gamma_{ij} Q_j < 0, \quad i \in S \quad (4.5)$$

have their solutions Q_i and F_i (in the case of feedback control) or G_i (in the case of output injection). Set

$$\bar{\lambda} = \max_{i \in S} \lambda_{\max}(U_i), \quad \lambda_S = \max_{i \in S} \lambda_{\max}(S_i),$$

and λ_M, λ_m and M_{QFG} have been defined in Theorem 3.2. If τ is sufficiently small for

$$\bar{\lambda} + 2\sqrt{\frac{\lambda_M M_{QFG} H_\tau}{\lambda_m}} + \frac{\lambda_S \lambda_M}{\lambda_m} < 0, \quad (4.6)$$

then the solution of (4.3) satisfies

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log(\mathbb{E}|x(t; \xi)|^2) \leq -\frac{\log(q)}{\gamma},$$

where $q > 1$ is the unique root to the following equation

$$\bar{\lambda} + 2\sqrt{\frac{q \lambda_M M_{QFG} H_\tau}{\lambda_m}} + \frac{q \lambda_S \lambda_M}{\lambda_m} = -\lambda_M \frac{\log(q)}{\gamma}. \quad (4.7)$$

Proof. Like Theorem 3.2, the proof is also an application of the Razumikhin-type theorem, so we only give the key steps. Define $V(x, t, i) = x^T Q_i x$, then

$$\begin{aligned} \mathcal{L}V(\varphi, t, i) &\leq \varphi^T(0)U_i\varphi(0) + \varphi^T(-\delta(t))S_i\varphi(-\delta(t)) - 2\varphi^T(0)Q_iF_iG_i\Psi(t, \varphi) \\ &\leq (\bar{\lambda} + \varepsilon)|\varphi(0)|^2 + \lambda_S|\varphi(-\delta(t))|^2 + \frac{M_{QFG}}{\varepsilon}|\Phi(t, \varphi)|^2. \end{aligned} \quad (4.8)$$

Setting $\varepsilon = \sqrt{\frac{q\lambda_M M_{QFG} H_\tau}{\lambda_m}}$, by applying Lemma 4.3 we have

$$\begin{aligned} \mathbb{E}\left[\max_{i \in S} \mathcal{L}V(\varphi, t, i)\right] &< (\bar{\lambda} + \varepsilon)\mathbb{E}|\varphi(0)|^2 + \lambda_S \frac{q\lambda_M}{\lambda_m} \mathbb{E}|\varphi(0)|^2 + \frac{M_{QFG}}{\varepsilon} \frac{q\lambda_M}{\lambda_m} H_\tau \mathbb{E}|\varphi(0)|^2 \\ &= (\bar{\lambda} + 2\varepsilon + \lambda_S \frac{q\lambda_M}{\lambda_m})\mathbb{E}|\varphi(0)|^2 \\ &\leq -\frac{\log(q)}{\gamma} \mathbb{E}\left[\max_{i \in S} V(\varphi(0), t, i)\right]. \end{aligned}$$

So the required assertion follows directly from the Razumikhin-type theorem.

Remark. We have first considered the robust discrete-time-state-feedback stabilization for the linear hybrid stochastic delay differential equations (SDDEs) (2.2) and then generalized the theory to the nonlinear hybrid SDDEs (4.1). It should be pointed out that by a similar argument, the theory can be further generalized to the hybrid stochastic functional differential equations (SFDEs). Moreover, if we choose the Lyapunov function as $V(x, t, i) = (x^T Q_i x)^{\frac{p}{2}}$, we can realize the p th moment exponential stabilization for systems (2.2), (4.1) and the SFDEs robustly by feedback control based on discrete-time state observations. And also, we can obtain a better bound on the duration τ between two consecutive state observations compared with [26]. However, due to the page limit here, we will report the results elsewhere.

5. Numerical examples

In this section, for the purpose of illustrating the effectiveness of the theory developed in this paper, we present two numerical examples.

Example 5.1. We first consider the same example as discussed in [21, 24, 25], namely the nondelay linear hybrid SDE

$$dx(t) = A(r(t))x(t)dt + B(r(t))x(t)dw(t) \quad (5.1)$$

on $t \geq t_0$. Here $w(t)$ is a scalar Brownian motion; $r(t)$ is a Markov chain on the state space $S = \{1, 2\}$ with the generator

$$\Gamma = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix},$$

and the system matrices are

$$A_1 = \begin{bmatrix} 1 & -1 \\ 1 & -5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -5 & -1 \\ 1 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}.$$

The system is not mean square exponentially stable. Let us now design a discrete-time state feedback control to stabilize the system. Assume the controlled hybrid SDE has the form

$$dx(t) = [A(r(t))x(t) + F(r(t))G(r(t))x(\eta(t))]dt + B(r(t))x(t)dw(t), \quad (5.2)$$

where $G_1 = (1, 0), G_2 = (0, 1)$. Applying Corollary 3.6, we can show that if we set

$$F_1 = \begin{bmatrix} -10 \\ 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 \\ -10 \end{bmatrix},$$

and make sure that $\tau < 0.0142$, then the controlled hybrid SDE (5.2) is mean-square exponentially stable.

It should be pointed out that it is required for $\tau < 0.0000308$ in Mao [21], $\tau < 0.0046$ in Mao et al. [24] and $\tau < 0.0074$ in You et al. [25], while applying our new theory we only need $\tau < 0.0142$. In other words, our new theory has improved the existing results.

Example 5.2. Let us consider a two-dimensional controlled linear hybrid SDE with time-varying delay

$$\begin{aligned} dx(t) = & [A(r(t))x(t) + A_d(r(t))x(t - \delta(t)) + F(r(t))G(r(t))x(\eta(t))]dt \\ & + [B(r(t))x(t) + B_d(r(t))x(t - \delta(t))]dw(t). \end{aligned} \quad (5.3)$$

Here $w(t)$ is a scalar Brownian motion and $r(t)$ is a Markov chain on the state space $S = \{1, 2\}$ with the same generator as that in Example 5.1. And all the coefficients are given by

$$\begin{aligned} A_1 = \begin{bmatrix} -1.9 & -3.3 \\ 1.6 & -1.3 \end{bmatrix}, & A_2 = \begin{bmatrix} -5.4 & 3.9 \\ 0 & -3.2 \end{bmatrix}, & A_{d1} = \begin{bmatrix} -0.04 & 0.01 \\ 0.03 & -0.02 \end{bmatrix}, & A_{d2} = \begin{bmatrix} 0.03 & 0.02 \\ -0.02 & -0.01 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, & B_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, & B_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, & B_{d2} = \begin{bmatrix} 0.13 & 0 \\ 0 & 0.1 \end{bmatrix}. \end{aligned}$$

In this example, we discuss the case of output injection. Now we know $F_1 = (0.1, 0.7)^T, F_2 = (0.2, 0.5)^T$ and our aim is to design G_1 and G_2 so that the controlled system (5.3) is mean-square exponentially stable, independent of the delay. By solving LMIs (3.12), we find the feasible solution

$$\begin{aligned} P_1 = \begin{bmatrix} 0.4931 & 0.0537 \\ 0.0537 & 0.5003 \end{bmatrix}, & P_2 = \begin{bmatrix} 0.2938 & 0.1684 \\ 0.1684 & 0.4157 \end{bmatrix}, \\ Y_1 = \begin{pmatrix} 1.5545 & -0.5156 \end{pmatrix}, & Y_2 = \begin{pmatrix} -0.9331 & 0.4965 \end{pmatrix}. \end{aligned}$$

By Theorem 3.4, we can obtain

$$G_1 = \begin{pmatrix} 3.3035 & -1.3856 \end{pmatrix}, \quad G_2 = \begin{pmatrix} -5.0276 & 3.2312 \end{pmatrix}.$$

A further calculation shows that

$$\begin{aligned} \lambda_M = 5.6928, \quad \lambda_m = 1.8163, \quad \lambda = -5.6859, \quad M_{Q_{A_d}} = 0.058, \quad M_{Q_{FG}} = 52.0852, \quad \bar{M} = 0.0079, \\ \beta = 0.101, \quad M_A = 48.4465, \quad M_{A_d} = 0.0029, \quad M_{FG} = 10.358, \quad M_B = 0.04, \quad M_{B_d} = 0.0524. \end{aligned}$$

It is easy to show that (3.5) holds whenever $\tau < 0.0082$. So by Theorem 3.4, if we set $G_i (i = 1, 2)$ as above and make sure $\tau < 0.0082$, then the controlled system (5.3) is mean-square exponentially stable. Moreover, to obtain the upper bound for the second Lyapunov exponent, we set $\tau = 0.001$ and $\gamma = h \vee \tau = 0.1$, and then it is easy to show that Eq. (3.6) becomes

$$1.8543\sqrt{q} + 0.3166q - 5.6859 = -56.928 \log(q),$$

which has a unique root $q = 1.0623$ on $(1, \infty)$. Hence the solution of (5.3) has the property

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log(\mathbb{E}|x(t; \xi)|^2) \leq -0.6044.$$

6. Conclusion

In this paper, we have showed that unstable hybrid stochastic systems with time-varying delay (linear and nonlinear) can be stabilized by feedback control based on discrete-time state observations. Applying the Razumikhin technique, the mean-square exponential stability criteria have been established, just requiring the time-varying delay to be a bounded variable rather than a differentiable function. Methods for designing the robust controllers have also been developed. Particularly, based on the Razumikhin method, we have obtained a better bound on τ for the nondelay system and this is supported by Example 5.1.

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