Performance evaluation of nonhomogeneous hospitals: the case of Hong Kong hospitals

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Abstract Throughout the world, hospitals are under increasing pressure to become more efficient. Efficiency analysis tools can play a role in giving policymakers insight into which units are less efficient and why. Many researchers have studied efficiencies of hospitals using data envelopment analysis (DEA) as an efficiency analysis tool. However, in the existing literature on DEA-based performance evaluation, a standard assumption of the constant returns to scale (CRS) or the variable returns to scale (VRS) DEA models is that decision-making units (DMUs) use a similar mix of inputs to produce a similar set of outputs. In fact, hospitals with different primary goals supply different services and provide different outputs. That is, hospitals are nonhomogeneous and the standard assumption of the DEA model is not applicable to the performance evaluation of nonhomogeneous hospitals. This paper considers the nonhomogeneity among hospitals in the performance evaluation and takes hospitals in Hong Kong as a case study. An extension of Cook et al. (2013) [1] based on the VRS assumption is developed to evaluated nonhomogeneous hospitals' efficiencies since inputs of hospitals vary greatly. Following the philosophy of Cook et al. (2013) [1], hospitals are divided into homogeneous groups and the product process of each hospital is divided into subunits. The performance of hospitals is measured on the basis of subunits. The proposed approach can be applied to measure the performance of other nonhomogeneous entities that exhibit variable return to scale.

Keywords: Data envelopment analysis; hospital efficiency; nonhomogeneity; subunit

1. Introduction

The health care sector plays an important role in promoting individual wellbeing and developing a harmonious society. As a fast growing sector, expenditures on the health care are increasing for the majority of nations. However, the resources for public sector activities are severely limited. The more resources spent on the health care, the fewer can be spent on other public services, such as education, domestic and international public assistance and aid, and basic social security. Moreover, among expenditures on the health care, the hospital expenditure is one of the most important parts. The hospital in particular has been under increasing scrutiny and the debate about restructuring and governance in hospitals is lively and heated in many countries. As a consequence, hospitals have been under increasing pressure to improve their efficiency.

Data envelopment analysis (DEA) as an axiomatic nonparametric programming technique has been increasingly used to measure the relative efficiency of the hospital since the mid-1980s [2]. The rationale for its popularity in efficiency evaluation is its absence of an explicit assumption on the production function and imposed weights on inputs and outputs [3]. In the history of the application of DEA, it has been used in both the public sector and the private sector [4].

In prior DEA literature on the efficiency evaluation of hospitals, Sherman [5] is the first to use DEA to measure efficiencies of hospitals. Since then, studies on DEA-based performance evaluation of hospitals have grown steadily. Some researches focused on using conventional DEA models to measure efficiencies of hospitals. For example, Ozcan [6] applied DEA to measure the aggregate technical efficiencies of acute care hospitals in 39 U.S. metropolitan areas and examined the cost inefficiency and waste of hospitals on the basis of performance. Biørn et al. [7] used DEA models to measure hospitals' efficiencies and then make an empirical study on 48 somatic hospitals over a period of nine years to examine the effect of the activity-based financing of the Norwegian financing system on hospitals' efficiency. Ozcan et al. [8] applied input-oriented envelopment and multiplier DEA models to measure the performance of 30 general hospitals linked to Brazilian Federal Universities, and results generated some suggestions for teaching ratios and public financing for hospitals. Narci et al. [9] investigated the relationship between competition and the efficiency of hospitals in Turkey by means of

employing the Multivariate Tobit regression while the hospital's technical efficiency was evaluated using DEA. The issue of measuring the performance of hospitals using conventional DEA models has been widely researched in the literature (see e.g. [10] for an overview), such as Grosskopf and Valdmains [11], Ozcan et al. [12], Sahin and Ozcan [13], Grosskopf et al. [14], Gruca and Nath [15], Kirigia et al. [16], and Grosskopf et al. [17].

Some researches on DEA-based performance evaluation of hospitals develop extensions of DEA to deal with issues of congestion [18, 19], multifactor efficiency [20], translog cost functions [21], mergers [22], and quality measures [23, 24, 25, 26]. For example, the effect of congestion on efficiency refers to the phenomenon that outputs decline with the increase of inputs. Simões and Marques [18] assessed the performance and the contribution of the congestion effect in Portuguese hospitals and found that half of the 68 inefficient major hospitals displayed to be congested. O'Neill [20] incorporated multifactor efficiency measure into DEA to address the important practical and methodological concern of comparing teaching versus non-teaching hospitals. Banker et al. [21] compared inferences about hospital cost and production correspondences based on the translog cost function to that of DEA method. Harris et al. [22] employed DEA to examine the impact of horizontal mergers on US hospital's technical efficiency and obtained results illustrated that mergers increase the performance of the hospitals. To extend the DEA to deal with quality measures, Khushalani and Ozcan [23] presented a dynamic network DEA with considering the quality of hospitals to measure efficiencies of hospitals.

Some researches measure hospitals' performance via using DEA in combination with other methods, such as Stochastic Frontier Analysis (SFA) [27], Malmquist index [28, 29, 30], simulation [31], bootstrap method [32, 33], and Bayesian analysis [34]. Jacobs [27] compared the efficiency ranking from cost indices with that of DEA and SFA, and suggested that the differences in efficiency rankings across different methods were due to random noise. Sahin et al. [28] adopted the Malmquist index with DEA to analyze the operational performance of the 352 Ministry of Health's general hospitals following the implementation of the Health Transformation Program in Turkey. In order to improve the performance of the emergency department of a Jordanian hospital, Al-Refaie et al. [31] used simulation to evaluate each nurse assignment configuration, and applied DEA to identify performance improvement target of

each nurse assignment configuration. Kounetas and Papathanassopoulos [32] employed a bootstrapped DEA to estimate the productive efficiency of each Greek hospital and a bootstrapped truncated regression was applied to explore the impact of environmental factors on hospitals' technical and scale efficiency. Mitropoulos et al. [34] combined stochastic DEA models with Bayesian analysis to enhance statistical inference in DEA and to account for statistic noise that arises from measurement errors. For a comprehensive discussion of the literature on the DEA-based performance of hospitals, see the excellent review of O'Neill et al. [10] which highlights a comparison and taxonomy of DEA-based efficiency researches on hospitals.

Prior literature employing constant returns to scale (CRS) or variable returns to scale (VRS) DEA models to measure hospitals' performance makes the homogeneity assumption that decision-making units (DMUs) use a similar mix of inputs to produce a similar set of outputs. That is, all hospitals provide the same services using the same kinds of inputs to produce the same kinds of outputs. In fact, because of different scales and primary service goals, hospitals specialize in different health services and produce different outputs. In this sense, the assumption of homogeneity among DMUs in the performance evaluation of hospitals may not apply. Hospitals are nonhomogeneous. O'Neill [20] and Mitropoulos et al. [34] also studied the nonhomogeneity among hospitals in performance evaluation. As we have mentioned in the literature review, O'Neill [20] incorporated multifactor efficiency measures into DEA to evaluate performance while the nonhomogeneity among hospitals is between teaching versus non-teaching hospitals. Mitropoulos et al. [34] combined stochastic DEA models with Bayesian analysis to enhance statistical inference in DEA and the nonhomogeneity is the sample of hospitals of different sizes and operational characteristics, such as primary care hospitals, secondary care hospitals, and tertiary care hospitals. However, inputs and outputs for all hospitals are the same in these two papers. Thus, the nonhomogeneity in these two papers is different to the nonhomogeneity of producing different outputs in this paper and methods of these two papers is not applicable to this paper. Hospitals in Hong Kong exemplify this nonhomogeneity in producing different outputs. Before displaying the nonhomogeneity among Hong Kong hospitals, inputs and outputs are presented in Table 1. Hospitals consume two inputs: the number of Full-time Equivalent (FTE) staff and the number of beds, and produce

six outputs: total inpatient length of stay (output 1), total Accident & Emergency attendances (output 2), total Specialist Outpatient attendances (output 3), family medicine specialist clinic attendances (output 4), total allied health outpatient attendance (output 5), and general outpatient attendances (output 6). The number of FTE staff is calculated from the manpower on FTE basis. It includes all full-time and part-time staff in Hospital Authoority's workforce, such as permanent, contract, and temporary staff. The inputs and outputs are derived from the Hospital Authority Annual Report of Hong Kong.

<Insert Table 1 About Here>

It is noteworthy that not all hospitals produce all outputs. As shown in Fig. 1, the Pamela Youde Nethersole Eastern Hospital used these two inputs to produce all of the six outputs while the Tung Wah Eastern Hospital only has output 1, output 3, output 5, and output 6. In this sense, the homogeneity among hospitals does not apply. Hospitals are nonhomogeneous. This is due to the fact that some hospitals have decided not to provide a certain health service, or because of some resource restrictions cannot supply that health service. The performance evaluation on hospitals ignoring the nonhomogeneity is equivalent to penalizing or crediting the hospital for not providing certain health services [1]. Thus, it seems appropriate to take the nonhomogeneity of hospitals into consideration in the efficiency evaluation. This paper uses hospitals in Hong Kong as a case study and focuses on measuring efficiencies of nonhomogeneous hospitals in Hong Kong. The nonhomogeneity problem considered in this paper is that some DMUs simply do not have a particular production process as compared to other DMUs. Hence, outputs produced from the particular process do not exist. The missing value for the output in the nonhomogeneity problem is due to the missing production process. Thus, we take the view that addressing the nonhomogeneity problem requires an approach that addresses the fundamental point that some of the organizations under study are fundamentally different in structure from others. The proposed approach can be employed to other nonhomogeneous hospitals.

<Insert Fig. 1 About Here>

Cook et al. [1] (CHIRZ for short in the following) developed DEA-based models to measure efficiencies of a set of nonhomogeneous DMUs. With the recognition that DMUs are nonhomogeneous, CHIRZ divided outputs into output subgroups and measured efficiencies of output subgroups to obtain efficiencies of nonhomogeneous DMUs. Following CHIRZ, we

focus on measuring subunits' efficiencies to obtain DMUs' efficiencies. Dividing DMUs into subunits is to tackle the nonhomogeneity among DMUs. It is notable that the concept of subunits of a DMU is similar to that of subsystems of a network DMU but no intermediate products exit among subunits. Moreover, subunits of DMUs are determined by an algorithm (in the Appendix 1 of the electronic supplemental material of this paper) on the basis of DMUs' input and output indicators. In this sense, the subunit of DMUs does not correspond to the specific department of hospitals. The subunit of DMUs is a dummy, and the production process of a DMU can be divided into independent processes via organizing DMUs into subunits. For example, outputs of the Tung Wah Eastern hospital consists of output 1, output 3, output 5, and output 6. In terms of the two inputs and six outputs of hospitals in Hong Kong, and using the algorithm for generating subunits, six subunits can be obtained. In addition, each subunit corresponds to one output. Thus, the Tung Wah Eastern hospital can be represented as four subunits and each subunit utilizes the two inputs to produce the particular output of the subunit. That is, each subunit is associated with a vector with two inputs and one output. Our methodology can be summarized as first classifying DMUs into different DMU groups, dividing DMU into subunits, allocating inputs among subunits, measuring the efficiency of each subunit, and finally combining subunits' efficiencies to obtain the DMU's overall efficiency.

Since input scales of hospitals are very different, this paper extends models of CHIRZ from the CRS version to the VRS framework to measure efficiencies of nonhomogeneous hospitals. This paper employs nonhomogeneous hospitals in Hong Kong as a case study and each hospital considered corresponds to a DMU. All DMUs use the same two kinds of inputs to produce different kinds of six outputs, and the two inputs and six outputs are shown in Table 1. Our proposed approach can obtain the efficiency of each hospital as well as the efficiency of each subunit. On the basis of obtained efficiencies, rankings of hospitals and subunits can be identified.

The rest of the paper is organized as follows. In Section 2, the methodology for efficiency analysis of hospitals is presented. In Section 3, efficiencies of hospitals in Hong Kong are analyzed based on our extended models. Finally, Section 4 contains some concluding remarks.

2. Methodology for efficiency analysis of hospitals taking into account nonhomogeneity

2.1. Efficiency evaluation in nonhomogeneous hospitals

To describe the problem of nonhomogeneity among hospitals, outputs produced by hospitals are depicted in Table 2. According to produced outputs, hospitals in Hong Kong are classified into P groups and each group is denoted as p. Hospitals in the same DMU group produce the same kinds of outputs, although in different quantities. In this paper, we have 37 nonhomogeneous hospitals that are classified into 8 categories, namely $P = 8$ and $p = 1, \cdots, 8$.

<Insert Table 2 About Here>

In each row of Table 2, a tick means that the output of this column is produced in the DMU group of this row. For example, DMUs in $p = 1$ produce output 1, and output 5 while DMUs in $p = 4$ produce output 1, output 3, output 5, and output 6. As shown in Table 2, all DMUs have been organized into 8 different groups. DMUs in different DMU groups possess different kinds of outputs. Hence, according to the argumentation above, it is inappropriate to apply directly the conventional DEA models to nonhomogeneous hospitals.

To address the nonhomogeneity problem, this paper views each hospital as a DMU consisting of several subunits. In terms of the two inputs and six outputs of Hong Kong hospitals and using the algorithm for generating subunits, six different types of subunits can be derived. Each subunit is associated with a vector with three entries representing both inputs and one output of that subunit. Denote the six subunits as subunit 1, subunit 2, subunit 3, subunit 4, subunit 5 and subunit 6 depends on which output dimension is present. That is, subunit 1 can be represented as (No. of FTE staff, No. of beds, output 1), subunit 2 can be represented as (No. of FTE staff, No. of beds, output 2), subunit 3 is (No. of FTE staff, No. of beds, output 3), subunit 4 is (No. of FTE staff, No. of beds, output 4), subunit 5 is (No. of FTE staff, No. of beds, output 5), and subunit 6 can be described as (No. of FTE staff, No. of beds, output 6). We denote the set of subunit types as K, and use the index k to represent each subunit, such that $k = 1$, $k = 2$, $k = 3$, $k = 4$, $k = 5$, and $k = 6$. In general, subunits may have multiple outputs but this is problem specific – see CHIRZ, and Appendix 1 in the electronic supplemental material of this paper for more details.

Further, define L_p as the set of subunits consisting of the DMU in group p. Thus, in our

case, the 8 DMU groups with their subunits can be denoted as follows:

$$L_1 = \{k = 1{,}5\},\ L_2 = \{k = 1,\ 2, 3, 4, 5, 6\},\ L_3 = \{k = 1, 2, 3, 5, 6\},\ L_4 = \{k = 1, 3, 5, 6\}$$

$$L_5 = \{k = 1\},\ L_6 = \{k = 1, 3, 5\},\ L_7 = \{k = 1, 3, 4, 5, 6\},\ L_8 = \{k = 1, 2, 3, 4, 5\}$$

In measuring the efficiency of a DMU, the evaluation should be carried out by conducting a separate DEA analysis for each subunit of the DMU. That is, the efficiency of each subunit k should be evaluated over all DMU groups that contain the subunit k as a member. Specifically, define M_k as DMU groups that contain subunit k as a member, namely $M_k = \{p\ if\ k \in L_p\}$. Corresponding to our hospitals case, the set of DMU groups of each subunit can be presented as follows:

$$M_1 = \{p = 1{,}2{,}3{,}4{,}5{,}6{,}7{,}8\},\ M_2 = \{p = 2{,}3{,}5\},\ M_3 = \{p = 2{,}3{,}4{,}6{,}7{,}8\}$$

$$M_4 = \{p = 2{,}7{,}8\},\ M_5 = \{p = 1{,}2{,}3{,}4{,}6{,}7{,}8\},\ M_6 = \{p = 2{,}3{,}4{,}7\}$$

In the following subsection, models of efficiencies evaluation for nonhomogeneous hospitals are presented.

2.2. Models of efficiency evaluation

In measuring efficiencies of nonhomogeneous DMUs, CHIRZ divided outputs into output subgroups and determined an inputs allocation to output subgroups. Then, CHIRZ evaluated the efficiency of each output subgroup and aggregated output subgroups' efficiencies to obtained DMUs' efficiencies. Following the philosophy of CHIRZ, the approach of this paper that we have sketched above requires answering three critical questions:

1. How should inputs be allocated among subunits of each DMU?
2. How can we obtain the efficiency of each subunit?
3. How should efficiencies of constituent subunits be aggregated to come up with an overall efficiency for the entire DMU?

In this subsection, we address these three questions. In order to facilitate easy reading, notations for our model development are summarized in the following Table 3.

<Insert Table 3 About Here>

Question 1. The two inputs considered in this paper are joint inputs that are simultaneously used in all subunits to produce outputs [35]. It is noteworthy that joint inputs of each DMU will have to be assigned to its subunits since we only obtain the overall input for each DMU, and do

not know how resources are allocated internally within the DMU. Suppose the proportion of the i^{th} input allocated to the subunit k of L_p is α_{ikp}. We will suppose that values of α_{ikp} are known to be within a particular interval of $[a_{ikp}, b_{ikp}]$. Also, for each DMU in the group p, the sum of the i^{th} input proportion assigned to all of its subunits is unity. In other words, the restriction of $\sum_{k\in L_p} \alpha_{ikp} = 1$ is imposed on input proportions α_{ikp}. Thus, the i^{th} input of $DMU_j(j \in D_p)$ allocated to its subunit k is $x_{ij}^k = \alpha_{ikp} x_{ij}$. To measure the efficiency of each subunit, one should first to derive the proportion α_{ikp} of the i^{th} input assigned to each subunit k. One reasonable and widely used criterion for determining appropriate values for unknown input proportions α_{ikp} is to choose them so as to obtain the maximum aggregated efficiency of each DMU.

Referring to the expression of DMUs' efficiencies in CHIRZ, this paper considers the overall efficiency of each DMU as a weighted average of efficiencies of its subunits. The idea of describing the overall efficiency as a weighted average of its subunits' efficiencies is similar to the definition of the overall efficiency in Cook and Zhu [36]. In addition, let ω_{kj} denote the weight of the subunit k to DMU_j. Then, weights of subunits of each DMU sum to one, namely $\sum_{k\in L_p} \omega_{kj} = 1$. Thus, the overall efficiency of each DMU is a convex combination of its subunits' efficiencies. The model for determining the split of inputs across subunits in CHIRZ is as follows:

$$
\begin{aligned}
\text{Max } e_d &= \textstyle\sum_{k\in L_p} \omega_{kd} \left[u_k y_{kd} / \left(\sum_i w_i \times \alpha_{ikp^d} x_{id}\right)\right] \\
\text{s.t. } & u_k y_{kj} / (\textstyle\sum_i w_i \times \alpha_{ikp} x_{ij}) \le 1,\ \forall j \in p, k \in L_p, \forall p \\
& \textstyle\sum_{k\in L_p} \alpha_{ikp} = 1\ \ \forall i, p \\
& \textstyle\sum_{k\in L_p} \omega_{kd} = 1 \\
& \alpha_{ikp} \in [\, a_{ikp}, b_{ikp}] \\
& w_i, u_k \ge 0,\ \forall i, k \qquad (1)
\end{aligned}
$$

As we have indicated in the introduction, it is appropriate to use the VRS version of DEA in this context. Thus, the VRS version extension of model (1) that describes inputs allocated in a way of maximizing the aggregated efficiency of DMU_d is as follows:

$$\text{Max } e_d = \sum_{k \in L_p} \omega_{kd} \left[(u_k y_{kd} + u_k^0) / \left(\sum_i w_i \times \alpha_{ikp^d} x_{id} \right) \right]$$

$$\text{s.t. } (u_k y_{kj} + u_k^0) / \left(\sum_i w_i \times \alpha_{ikp} x_{ij} \right) \leq 1, \ \forall j \in p, k \in L_p, \forall p$$

$$\sum_{k \in L_p} \alpha_{ikp} = 1 \ \forall i, p$$

$$\sum_{k \in L_p} \omega_{kd} = 1$$

$$\alpha_{ikp} \in [\, a_{ikp}, b_{ikp}]$$

$$w_i, u_k \geq 0, \ \forall i, k \ u_k^0, free \tag{2}$$

The objective function of model (2) expresses the overall efficiency of DMU_d as a weighted average of its subunits' efficiencies. The variable of u_k^0 denotes the unrestricted variable that used to model each corresponding subunit in a VRS version. The first set of constraints ensures that the efficiency of each subunit of each DMU does not exceed one. The second and the fourth set of constraints guarantee the feasibility of input proportions α_{ikp}. The third constraint ensures the sum of subunits' weight in L_p is also one.

With the recognition that the value of ω_{kd} is unknown in model (2), we define ω_{kd} from the accounting perspective following CHIRZ. That is, the weight of each subunit ω_{kd} should be the share of the aggregate inputs assigned to that subunit. However, since the weight ω_{kd} is not determined exogenously, we have the formulation as follows:

$$\omega_{kd} = \sum_i w_i \times \alpha_{ikp^d} x_{id} / \sum_{k \in L_{p^d}} \sum_i w_i \times \alpha_{ikp^d} x_{id} \tag{3}$$

Based on formula (3), it is obvious that $\sum_{k \in L_p} \omega_{kd} = 1$. Then, the objective function of model (2) can be represented as follows:

$$\begin{aligned} e_d &= \sum_{k \in L_{p^d}} \omega_{kd} \left[(u_k y_{kd} + u_k^0) / \sum_i w_i \times \alpha_{ikp^d} x_{id} \right] \\ &= \sum_{k \in L_{p^d}} \frac{\sum_i w_i \times \alpha_{ikp^d} x_{id}}{\sum_{k \in L_{p^d}} \sum_i w_i \times \alpha_{ikp^d} x_{id}} \times \frac{u_k y_{kd} + u_k^0}{\sum_i w_i \times \alpha_{ikp^d} x_{id}} \\ &= \frac{\sum_{k \in L_{p^d}} u_k y_{kd} + \sum_{k \in L_{p^d}} u_k^0}{\sum_i w_i x_{id}} \end{aligned} \tag{4}$$

Then, the model (2) can be converted into the following model (5):

$$\text{Max } e_d = \sum_{k \in L_{p^d}} (u_k y_{kd} + u_k^0) / \sum_i w_i x_{id}$$

$$\text{s.t. } (u_k y_{kj} + u_k^0) / \left(\sum_i w_i \times \alpha_{ikp} x_{ij} \right) \leq 0, \ \forall j \in p, k \in L_p, \forall p$$

$$\alpha_{ikp} \in [a_{ikp}, b_{ikp}], \quad \forall \mathrm{i}, \mathrm{k} \in L_p, \forall p$$

$$w_i, u_k \geq 0, \quad \forall i, k \;\; u_k^0, free \tag{5}$$

Model (5) is nonlinear since products of unknown input proportions α_{ikp} and unknown input multipliers w_i exist in the model. To convert model (5) into a linear model, replace $w_i \times \alpha_{ikp}$ with a variable z_{ikp}. Then, we have $\sum_{k\in L_p} \alpha_{ikp} = 1 \Longrightarrow \sum_{k\in L_p} w_i \times \alpha_{ikp} = w_i \Longrightarrow \sum_{k\in L_p} z_{ikp} = w_i$. Applying the Charnes-Cooper (C-C) transformation $\mathrm{t} = 1/\sum_i w_i x_{id}$ and defining $\mu_k = tu_k$, $v_i = tw_i$, $\gamma_{ikp} = tz_{ikp}$, $\mu_k^0 = tu_k^0$, then model (5) can be converted to be:

$$\text{Max } e_d = \sum_{k\in L_{pd}} \mu_k y_{kd} + \sum_{k\in L_{pd}} \mu_k^0$$

$$s.t. \quad \sum_i v_i x_{id} = 1$$

$$\mu_k y_{kj} + \mu_k^0 - \sum_i \gamma_{ikp} x_{ij} \leq 0, \quad \forall \mathrm{j} \in \mathrm{p}, \mathrm{k} \in L_p, \forall p$$

$$\sum_{k\in L_p} \gamma_{ikp} = v_i, \quad \forall i, \forall p$$

$$\gamma_{ikp} \in [v_i a_{ikp}, v_i b_{ikp}], \quad \forall i, k \in L_p, \forall p$$

$$v_i, \mu_k \geq 0, \quad \forall i, k \;\; \mu_k^0, free \tag{6}$$

Suppose the optimal solution of model (6) for DMU_d is $(v_i^*, \mu_k^*, \gamma_{ikp^d}^*, \mu_k^{0*})$, then the optimal proportions α_{ikp^d} can be obtained, namely $\alpha_{ikp^d}^* = \gamma_{ikp^d}^* / v_i^*$. Inputs of DMU_d allocated to its subunit k is $x_{id}^{k*} = \alpha_{ikp^d}^* x_{id}$. Thus, the efficiency of each subunit can be evaluated with its inputs and outputs. In addition, the weight of each subunit to each DMU_d can also be derived. Since $\omega_{kd} = \sum_i w_i \times \alpha_{ikp^d} x_{id} / \sum_{k\in L_{pd}} \sum_i w_i \times \alpha_{ikp^d} x_{id}$, and $\sum_{k\in L_{pd}} \sum_i tw_i \times \alpha_{ikp^d} x_{id} = \sum_i v_i x_{id} = 1$, then the weight of each subunit k to DMU_d can be represented as $\omega_{kd}^* = \sum_i tw_i \times \alpha_{ikp^d}^* x_{id} = \sum_i \gamma_{ikp^d}^* x_{id}$.

Moreover, concerning the same model conversion, model (1) is transformed into the following model (7) in CHIRZ.

$$\text{Max } e_d = \sum_{k\in L_{pd}} \mu_k y_{kd}$$

$$s.t. \quad \sum_i v_i x_{id} = 1$$

$$\mu_k y_{kj} - \sum_i \gamma_{ikp} x_{ij} \leq 0, \quad \forall \mathrm{j} \in \mathrm{p} \; \mathrm{k} \in L_p, \forall p$$

$\sum_{k\in L_p}\gamma_{ikp} = v_i, \quad \forall i, \forall p$

$\gamma_{ikp} \in [v_i a_{ikp}, v_i b_{ikp}], \quad \forall i, k \in L_p, \forall p$

$v_i, \mu_k \geq 0, \quad \forall i, k \quad (7)$

Suppose the optimal solution of model (7) for DMU_d is $(v_i'^*, \mu_k'^*, \gamma_{ikp^d}'^*)$. Then the optimal proportions $\alpha_{ikp^d}'^*$ can be obtained, namely $\alpha_{ikp^d}'^* = \gamma_{ikp^d}'^* / v_i'^*$. Inputs of the subunit k of DMU_d in CHIRZ is $x_{id}^{k\prime *} = \alpha_{ikp^d}'^* x_{id}$. Moreover, the weight of each subunit can be derived. It is noteworthy that $\omega_{kd} = \sum_i w_i \times \alpha_{ikp^d} x_{id} / \sum_{k\in L_{p^d}} \sum_i w_i \times \alpha_{ikp^d} x_{id}$, and $\sum_{k\in L_{p^d}} \sum_i tw_i \times \alpha_{ikp^d} x_{id} = \sum_i v_i x_{id} = 1$. Therefore, the weight of each subunit k to DMU_d in CHIRZ can be calculated as $\omega_{kd}'^* = \sum_i tw_i \times \alpha_{ikp^d}'^* x_{id} = \sum_i \gamma_{ikp^d}'^* x_{id}$.

Question 2. Note that the efficiency of each subunit is evaluated over all corresponding subunits with the same kind of inputs and outputs. That is, when measuring the efficiency of the subunit k, the evaluation is undertaken over all DMU groups that contain k as a member, namely p ∈ M_{k^d}. The model for measuring the efficiency of each subunit k^d of DMU_d(d ∈ p^d) in CHIRZ is:

Max $e_{k^d d} = \mu_k y_{kd}$

$s.t. \quad \sum_i v_i \times x_{id}^{k^d\prime *} = 1$

$\mu_k y_{kj} - \sum_i v_i \times x_{ij}^{k'*} \leq 0, \quad \forall j \in p, for\ p \in M_{k^d}$

$v_i, \mu_k \geq 0, \quad \forall k, i \quad (8)$

Therefore, we extend the model for evaluating the efficiency of each subunit in CHIRZ from CRS framework to VRS version as follows:

Max $e_{k^d d} = \mu_k y_{kd} + \mu_{k^d}^0$

$s.t. \quad \sum_i v_i \times x_{id}^{k^d *} = 1$

$\mu_k y_{kj} + \mu_{k^d}^0 - \sum_i v_i \times x_{ij}^{k*} \leq 0, \quad \forall j \in p, for\ p \in M_{k^d}$

$v_i, \mu_k \geq 0, \quad \forall k, i\ \mu_{k^d}^0, free \quad (9)$

Question 3. Suppose the optimal objective function of model (8) and model (9) are $e_{k^d d}'^*$

and $e^{*}_{k^d d}$, respectively. Thus, efficiencies of each subunit under both the CRS framework and the VRS framework are derived. In addition, we have obtained the weight of subunits' performance to DMUs' performance, then the overall efficiency of each DMU can be computed by taking a weighted average of its subunits' efficiencies. That is, for DMU_d, its CRS efficiency is $e'^{*}_{d} = \sum_{k \in L_{pd}} \omega'^{*}_{kd} \times e'^{*}_{k^d d}$ and it is the efficiency from CHIRZ models. The VRS efficiency of DMU_d of our proposed approach is $e^{*}_{d} = \sum_{k \in L_{pd}} \omega^{*}_{kd} \times e^{*}_{k^d d}$.

3. Results of Hong Kong hospitals

This section presents the data source and details of efficiency analysis results. The linprog algorithm in the Matlab on an i7-4600U 2.1 GHz 8GB PC is used to calculate the results. Inputs and outputs data of Hong Kong hospitals are displayed in Table 1 of Appendix 2 in the electronic supplemental material of this paper. Recall that the purpose of solving the first problem in the proposed approach is to obtain inputs and weights of subunits that make up the DMU. In order to make the paper more concise, results of the proportion of the two joint inputs allocated to each subunit and the weight of each subunit are provided in Table 2, Table 3, and Table 4 of Appendix 2 in the electronic supplemental material. The structure of this section is as follows. The overview of inputs and outputs data is presented briefly in section 3.1. The most significant issue is efficiencies of subunits and DMUs, and these are reported in section 3.2. Section 3.3 identifies correlations among subunits and DMUs with a view to shedding light on different improvement strategies. Section 3.4 describes the comparison among different definitions of DMU efficiency.

3.1. Overview of data

Our data set consists of 37 hospitals in Hong Kong. Data of inputs and outputs are collected from the Hospital Authority Annual Report for the fiscal year 2012 - 2013 and are presented in Table 1 of Appendix 2 in the electronic supplemental material. According to outputs produced, the 37 DMUs are grouped into 8 DMU groups in which each DMU can be considered as consisting of subunits. Each subunit has the same two kinds of inputs and its specific output. Each DMU group consists of different DMUs, such as $p = 1$ consists of DMUs 1, 25, and 29,

and $p = 6$ consists of DMUs 7, 8, 9, 10, 14, 15, 17, 21, 28, 32, 33, and 34. Moreover, DMUs corresponding to hospitals in the same group have some common features. Hospitals of the group $p = 1$ are characterized by nursing homes or rehabilitation hospitals, and usually provide continuing nursing care. These of the group $p = 2$ are large acute general hospitals and operate on a considerable scale. Some hospitals are community-based and provide medical service on the basis of community, such as hospitals in group 4. Hospitals in the group 6 are tertiary institutions focused on some particular medical field such as children, mental disorder, ophthalmology, and thoracic medicine.

The basic descriptive statistics of inputs and outputs are summarized in Table 1, including the maximum level, the minimum level, the average level, the range defined by the lowest and highest observed value of each input and output, and the standard deviation of each input and output. It shows that variables vary substantially. Taking the number of FTE staffs as an example, the value of this variable range from 57.02 to 5870.16 and the standard deviation is 1718.55. The same phenomenon occurs in other variables. The large ranges and standard deviations of variables reveal that operating scales of hospitals have big differences. This observation justifies our use of VRS assumption in proposed models.

3.2. Efficiency results of subunits and DMUs

From utilizing the proportion of each input assigned to each subunit, the corresponding adjusted inputs for each subunit can be calculated. Following the methodology in section 2, model (6) is used to calculate the input proportion of subunits. It is noteworthy that limits on input proportion are specific. We have consulted the Hospital Authority of Hong Kong, and it was suggested that the limits A in the following Table 4 are used.

<Insert Table 4 About Here>

The limit on $p = 1$ communicates the idea that a minimum of 0.15 and a maximum of 0.8 of each input can be allocated to each subunit for DMUs in $p = 1$. The other limits have the same effect of restricting effect on input proportions. Also, it can found that the more subunits the DMU group has, the narrower the limit on its input proportion. For example, the DMU group $p = 1$ consists of two subunits $(1, 5)$, and the limit on its input proportion is [0.15, 0.80]. The DMU group $p = 2$ consists of six subunits $(1, 2, 3, 4, 5, 6)$, then limit on the

proportion of inputs is [0.10, 0.60] that is narrower than that of $p = 1$. That is, the limit on the input proportion is related to the number of subunits the DMU group has. The proportion of the two joint inputs allocated to each subunit and the weight of each subunit are presented in Appendix 2 in the electronic supplemental material of this paper. Recall that efficiencies of subunits can be computed from model (9). Then, combining subunits' efficiencies with their corresponding weight in the way that mentioned in Section 2, the overall efficiency of the DMU is obtained. Results of efficiencies of each subunit and each DMU are shown in the following Table 5.

<Insert Table 5 About Here>

As shown in Table 5, three DMUs are efficient overall, namely DMU_4, DMU_{14}, and DMU_{28}. All subunits of the three efficient DMUs are efficient. For example, DMU_4 is efficient overall, and so it performs efficiently in all of its subunits $(1,2,3,5,\text{and } 6)$. However, DMU_{37} is efficient in subunits of $1, 2, 3, 5, \text{and } 6$, but only inefficient in subunit 4. Then, DMU_{37} has an imperfect performance overall. Thus, a DMU is efficient overall if and only if all of its subunits are efficient. This suggests a focus for management action: hospitals such as DMU_{37} can have perfect performance overall by improving the performance of its inefficient subunit.

It can be found from Table 5 that overall efficiencies of all DMUs are between the highest efficiency and the lowest efficiency of its subunits. Except four DMUs $(DMU_{20}, DMU_{22}, DMU_{24}, DMU_{26})$, all DMUs in $p = 2$ perform efficiently on at least one of its subunits. The majority of DMUs perform preferably than other DMUs on one of its subunits, but on other subunits, they perform worse. However, some DMUs dominate other DMUs, namely they have better performance on all subunits as well as the overall system. For instance, in $p = 2$, DMU_{18} has better efficiencies on all subunits and the overall system than DMU_{20}, DMU_{22}, and DMU_{24}. In $p = 6$, DMU_{10} dominates DMU_8, DMU_{17}, DMU_{21}, DMU_{32}, and DMU_{33}. For DMUs being dominated, they can improve efficiencies by taking DMUs that dominate them as targets. That is, DMU_{20}, DMU_{22}, and DMU_{24} can improve efficiencies based on the performance of DMU_{18} while DMU_8, DMU_{17}, DMU_{21}, DMU_{32}, and DMU_{33} can based on the performance of DMU_{10}. For example, DMU_{18}, DMU_{20}, DMU_{22}, and DMU_{24} are all large acute general hospitals and produce all outputs. To improve performance

by taking DMU_{18} as a benchmark, the three hospitals should divide more resource of the FTE staff on subunit 6 to produce more output of this subunit, because DMU_{18} allocates a relative high proportion of the FTE staff on subunit 6 and the output from subunit 6 in DMU_{18} is large while the three hospitals have low proportion of this input and bad performance on subunit 6. Moreover, DMU_{20}, DMU_{22}, and DMU_{24} can divide more beds to subunit 4 to improve performance since DMU_{18} divides the highest proportion of beds to subunit 4. For DMU_{8}, DMU_{17}, DMU_{21}, DMU_{32}, and DMU_{33} to improve performance based on DMU_{10}, all of the five hospitals should allocate more beds to subunit 3. The reason for this suggestion is that the proportion of beds to subunit 3 is the highest in DMU_{10}, but it is not the case for the five hospitals. In addition, the inputs scale of DMU_{18} as compared to that of DMU_{20}, DMU_{22}, and DMU_{24} is relatively small. The same phenomenon occurs in the case of DMU_{10} as compared to DMU_{8}, DMU_{17}, DMU_{21}, DMU_{32}, and DMU_{33}. Thus, DMU_{20}, DMU_{22}, and DMU_{24} in $\mathrm{p} = 2$, and DMU_{8}, DMU_{17}, DMU_{21}, DMU_{32}, and DMU_{33} in $\mathrm{p} = 6$ should control their input scale and take advantage of input scale.

To further study the performance of subunits, the descriptive statistics on subunits' efficiencies is summarized in Table 6. The statistical indicators include the number of DMUs that contain the corresponding subunit, the number of DMUs that perform efficiently in the subunit, the maximum efficiency value, the minimum efficiency value, the efficiency range of each subunit, and the standard deviation of each subunit's efficiency.

<Insert Table 6 About Here>

It can be seen from Table 6 that for each subunit, some DMUs perform efficiently and thus the maximum efficiency of each subunit is 1. It is a property of the DEA method that many DMUs will be measured as efficient when the number of input and output indicator is large as compared to the total number of DMUs [24]. For example, 8 DMUs are efficient in the subunit 1 over 37 DMUs in M_1. The maximum efficiency of subunit 1 is 1 while the minimum efficiency is 0.13 (DMU_{36}). The range of the efficiency of subunit 1 is 0.7 and the standard deviation is 0.28. The large efficiency range and standard deviation of subunit 1 illustrate the fact that different DMUs are good at providing different health services. It is interesting to note that the efficiency ranges of all subunits are large. The reason for this situation is partly because these nonhomogeneous DMUs have quite different service models. The maximum standard

deviation is 0.36 and it is for efficiencies of subunit 5. In this sense, the performance of subunit 5 for different DMUs varies greatly. It is notable that the subunit 5 provides allied health service for outpatients. Some hospitals are good at providing this service. However, some hospitals, such as hospitals in group 6 are tertiary hospitals that focus on particular medical field of mental illness, ophthalmology, and thoracic care, and do not focus on this service. Thus, for hospitals in group 6, the performance of subunit 5 may be bad while some hospitals have good performance on this subunit. Therefore, the performance of DMUs will be ranked differently based on different subunits.

It is noteworthy that a property of VRS models is that DMUs with the smallest inputs or the biggest outputs are always efficient [37]. In this paper, DMU_4 (smallest inputs) in subunit 1, subunit 2, subunit 5, and subunit 6, DMU_{23} (smallest inputs) in the subunit 4, DMU_{28} (smallest inputs) in the subunit 5 are evaluated as efficient because of this property. In the output aspect, DMU_{37} (biggest outputs) in subunit 1, subunit 2, and subunit 6, DMU_{11} (biggest outputs) in the subunit 3, DMU_2 (biggest outputs) in the subunit 4, DMU_{16} (biggest outputs) in the subunit 5 are other examples of this case.

3.3. Correlations among subunits and DMUs

Since the number of subunits included in each DMU group is different and efficiencies of DMUs in DMU groups vary differently, it is interesting to explore the correlation between the efficiency of the DMU and the number of subunits. Based on the correlation relationship, the implication for economies of scope of hospitals can be figured out. Fig. 2 shows the correlation between the average efficiency of the DMU and the number of subunits.

<Insert Fig. 2 About Here>

As shown in Fig. 2, the horizontal axis is the number of subunits, and the vertical axis is the average efficiency of the DMUs that contain the corresponding number of subunits. It can been found from Fig. 2 that the average efficiency of DMUs increases as the number of subunits increase when the number of subunits is less than 6. That is, when the number of subunits of the hospital is less than 6, the more subunits the hospital has the more efficient it is. In this sense, the implication for economies of scope is that when the hospital do not produce all of the 6 outputs, hospitals are scope economies. However, when the number of subunits increases

from 5 to 6, the average efficiency of the DMU decreases. This result suggests that the hospital is scope diseconomies when the provided outputs expand from 5 to 6.

Moreover, efficiencies of each subunit and each DMU (the overall system) have been obtained, and the efficiency evaluation method of DEA is a non-parametric technique, the Spearman rank correlation coefficient is calculated to measure correlations among efficiencies. Three DMU groups $(p = 3, 7, 8)$ only have one DMU, the Spearman rank correlation coefficients among efficiencies in these three groups cannot be calculated. Thus, we display the Spearman rank correlation coefficients among efficiencies for other five DMU groups as shown in Table 7.

<Insert Table 7 About Here>

The correlation coefficient in Table 7 represents the correlation of the efficiency of the subunit in the column to the DMU's efficiency of the group in the row. For example, the value of 0.5 in the third row expresses the idea that the Spearman rank correlation coefficient between subunit 1's efficiency and the DMU's efficiency in group 2 is 0.5. The value of the Spearman rank correlation coefficient represents the degree of correlation. In this sense, the higher the value, the higher degree of correlation. The values of 1 and -1 illustrate the DMU's efficiency in the group can be expressed as a monotone function for the corresponding subunit's efficiency. Moreover, the value of 1 for the Spearman rank correlation coefficient represents the change trend of the subunit's efficiency and that of the DMU's efficiency are in the same direction. However, the value of -1 represents the change trend of the two kinds of efficiency is in the opposite direction. The output of subunit 1 in group 4 is total inpatient length of stay. The improvement in the efficiency of subunit 1 in group 4 means more resources have been divided into subunit 1, such as beds, or the output of total inpatient length of stay has been expanded. This is not conform to the goal and scale of hospitals in group 4 that hospitals in group 4 are community-based institution and provide regular medical service. Thus, the characteristic of community-based of hospitals in group 4 may be the reason for the negative correlation coefficient between efficiencies of subunit 1 and the overall system.

On the basis of the correlation coefficient between the efficiency of the DMU and the efficiency of each subunit, implications for the first steps in improving performance of the overall system can be drawn. In the situation that only one of DMU's subunits is inefficient, an

effective strategy is to improve the performance of the inefficient subunit. For example, DMU_{37} is efficient in its subunits of $1, 2, 3, 5,$ and 6, but is inefficient in subunit 4. In this case, DMU_{37} can improve its performance in subunit 4 to become efficient overall. That is, DMU_{37} as an acute general hospital should put more resources into providing the family medicine specialist clinic service. The other situation is that two or more subunits are inefficient and have similar efficiencies. For this situation, an effective strategy may be first to improve the performance of the subunit that has a relative higher correlation coefficient with the overall system's efficiency. For example, DMU_{18} is efficient in subunits $1, 4, 5,$ and 6, and is inefficient in subunits 1 and 3. Moreover, subunits 1 and 3 in DMU_{18} have similar efficiency (0.50 vs. 0.68). Then, DMU_{18} can firstly improve its performance in subunit 3 to improve its performance in the overall system since the correlation coefficient between efficiencies of the overall system and subunit 3 is higher than that between efficiencies of the overall system and subunit 1. In this sense, DMU_{18} can firstly divide more resources to supply the specialist outpatient service. Moreover, the scale of the number of beds in DMU_{18} is small. To improve the performance of subunit 1 may means to expand the scale on the number of beds.

3.4. Comparison of efficiencies of DMUs

To emphasize the distinctive features of our method, the CHIRZ efficiency of each DMU is calculated from CHIRZ models based on CRS version. The CHIRZ efficiency (eCHIRZ) is listed in the second column of Table 8 in this paper. The efficiency (e) of each DMU obtained from our proposed method is also presented in the third column. The efficiency in the third column of Table 8 and the result in the last column of Table 5 are the same. Note that results of efficiencies of Table 5 are calculated based on a set of limits (A) on input proportions α_{ikp}. To further test the sensitivity of the overall system's efficiency to limits on α_{ikp}, another two sets of limits are employed. One of the two sets of limits gives tighter ranges to input proportions α_{ikp} as compared to the limits A. The other gives wider ranges for α_{ikp}. Denote the tighter limits as A_{tight} and the wider limits as A_{loose}, both A_{tight} and A_{loose} can be found in Table 4. In the efficiency comparison, only the overall efficiency is considered. Thus, the obtained overall efficiency of each DMU based on A_{tight} is presented in the fourth column (e_{tight}) and the efficiency based on A_{loose} is presented in the fifth column (e_{loose}) of Table

8.

<Insert Table 8 About Here>

It can be seen from Table 8 that no DMUs are evaluated as efficient when employing CHIRZ models (model (7) and model (8)) to measure efficiencies. However, three DMUs are efficient when applying our proposed approach, namely DMU_4, DMU_{14}, and DMU_{28}. Moreover, the efficiencies of the majority of DMUs based on our proposed approach are higher than that based on CHIRZ models, such as DMU_1, DMU_2, DMU_5, and DMU_6. To visualize the comparison between efficiencies, the CHIRZ efficiency (eCHIRZ) versus the efficiency of our proposed approach (e) for each DMU is displayed in Fig. 3. The horizontal axis of Fig. 3 is the CHIRZ efficiency and the vertical axis is the efficiency of our proposed model. The diagonal represents the case of equality between the CHIRZ efficiency and the efficiency of our proposed model.

<**Insert Fig. 3 About Here**>

It can be seen from Fig. 3 that the majority of DMUs are distributed above the diagonal. This accords with the efficiency result in Table 8 that efficiencies calculated from our proposed approach is higher than that calculated from the CHIRZ approach for the majority of DMUs. The reason for this phenomenon is that our proposed approach is developed on the VRS framework and CHIRZ models are based on CRS model. In general DEA models, the VRS efficiency is expected to be not lower than the CRS efficiency since the envelopment of the data in the VRS framework is tighter. Three points that represent three DMUs, namely DMU_4, DMU_{14}, and DMU_{28}, are distributed at the top line, since their efficiencies from our proposed approach is 1. It is interesting to note that seven DMUs are distributed below the diagonal. This is because seven DMUs' CHIRZ efficiencies are higher than their efficiencies calculated from our proposed approach, namely DMU_3, DMU_{15}, DMU_{19}, DMU_{20}, DMU_{24}, DMU_{31}, and DMU_{35}. The reason for this result is due to the fact that the model for input allocation in CHIRZ is also on the basis of CRS framework and the input allocation model in our proposed approach is developed on VRS version, and thus inputs divided to subunits are different between CHIRZ approach and our approach. That is, the model for determining the split of inputs in CHIRZ is model (7) which is developed on the CRS framework. The model for input allocation in our

approach is model (6) that is a VRS version model. The optimal input proportions $\alpha'^{*}_{ikp^d}$ from model (7) are different to the optimal input proportions α_{ikp^d} of model (6). In that case, for the same subunit, the inputs allocated to it between CHIRZ approach and our approach are different. For example, for subunit 2 and subunit 4 in DMU_3, the optimal input proportions based on CHIRZ approach are 0.25 and 0.1. However, the optimal input proportions for subunit 2 and subunit 4 of DMU_3 in our approach are 0.1 and 0.3. Thus, the property of the VRS efficiency is not lower than the CRS efficiency is not applicable to efficiencies of the subunit since allocated inputs to subunits are different between CHIRZ approach and our approach. Moreover, the efficiency of each DMU is a weight average of efficiencies of its subunits with the weight of each subunit is the share of the aggregate inputs assigned to it. Therefore, the aforementioned property of DMU's VRS efficiency is not lower than its CRS efficiency may be invalid in some cases of our approach.

In Table 8, efficiencies for the majority of DMUs are increased when a tighter set of limits on input proportions is used. However, when a wider set of constraint on the input proportions is used, efficiencies of the majority of DMUs are decreased. This phenomenon can also be seen from Fig. 4. As shown in Fig. 4, the horizontal axis is the overall efficiency difference between e_{tight} and e while the vertical axis is the efficiency difference between e_{loose} and e. For the efficiency difference between e_{tight} and e, the majority of DMUs are distributed to the right of the vertical axis, because e_{tight} is larger than e for the majority of DMUs. For the difference between e_{loose} and e, the majority of DMUs are distributed below the horizon axis since the efficiency value of e_{loose} is smaller than e for the majority of DMUs.

<Insert Fig. 4 About Here>

4. Conclusions

The health care sector is a fast growing sector in the world and expenditures on health care are increasing globally. Hospital expenditure represents an important part of this overall spend and the performance of hospitals has attracted increasingly intense attention. Consequently, hospitals are under pressure to improve their efficiencies. In recent years, many researchers have applied DEA to measuring hospitals' performance, but all of this literature using CRS or

VRS DEA models to evaluate performance is on the basis of the assumption that DMUs use a similar mix of inputs to produce a similar set of outputs. However, this is not the case. In fact, hospitals are nonhomogeneous reflecting their differing service models. That is, hospitals produce different kinds of outputs rather than just being distinguished in terms of the quality of their outputs. This paper takes hospitals in Hong Kong as an example and measures their efficiencies by taking their nonhomogeneity into account. The models of CHIRZ are extended from the CRS framework to the VRS framework, as this is more appropriate in this setting. The proposed approach is entirely general and can be used to measure other nonhomogeneous hospitals (or indeed other sorts of entity which exhibit variable returns to scale). The performance results for 37 hospitals in Hong Kong show that efficiencies obtained from our proposed approach can discriminate between efficiency and inefficient units. The correlation coefficient analysis results show how our analysis can be used to provide useful information to management about where to look for improvements in the system.

The issue of distinguishing different categories of inputs in the performance evaluation has been intensively studied. This paper considers joint inputs which simultaneously used by all subunits in the performance evaluation. The discretionary input, the output-specific input, and the sub-joint input are other common kinds of input [35]. How to consider the nonhomogeneity among hospitals including other categories of inputs in the performance evaluation is a possible research topic for future study. This paper analyzes the correlation among efficiencies of subunit and hospitals to make suggestions for performance improvement. The issue of whether hospitals' ranking change when additional subunits are presented can help managers to make decisions on additional health service provision. How to tackle this issue is a possible direction for future research. Moreover, the distance function has been a commonly used technique for the efficiency evaluation over the past two decades [38]. Whether it is possible to extend the distance function to deal with the nonhomogeneous problem in performance evaluation has not been studied, and it would be an interesting area to explore. We hope that this paper will stimulate others to continue to refine the methodological toolkit available to researchers and managers who wish to develop a deeper understanding of drivers on efficiency in the healthcare sector.

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Figures

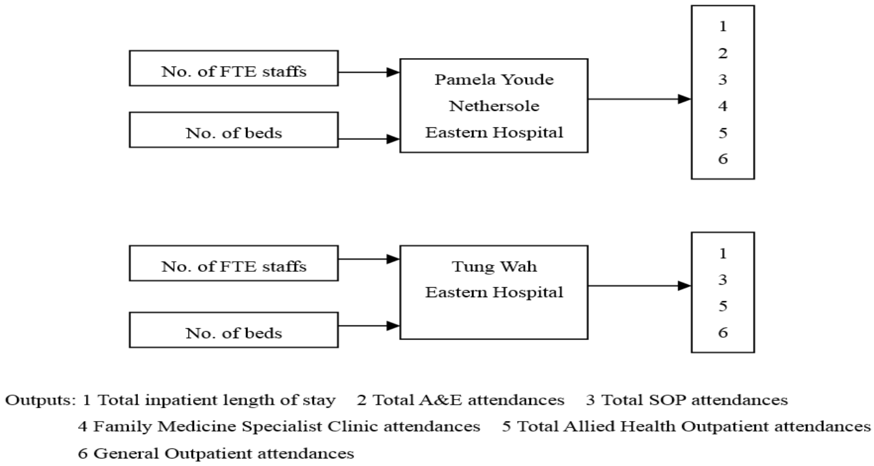

Fig. 1 Inputs and outputs of Hospitals

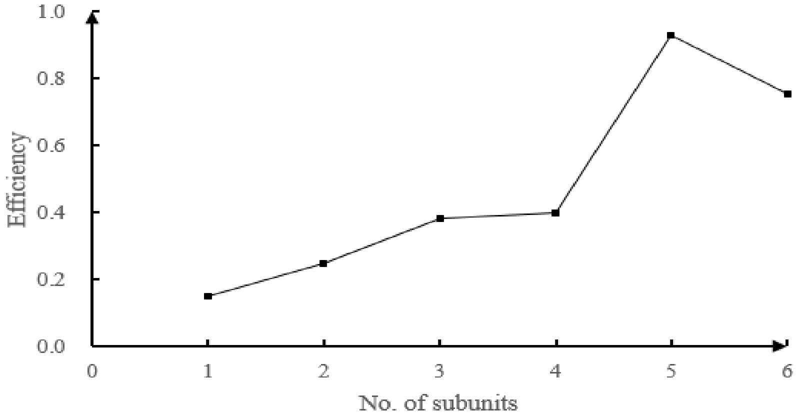

Fig. 2 The average efficiency of DMUs vs. the number of subunits

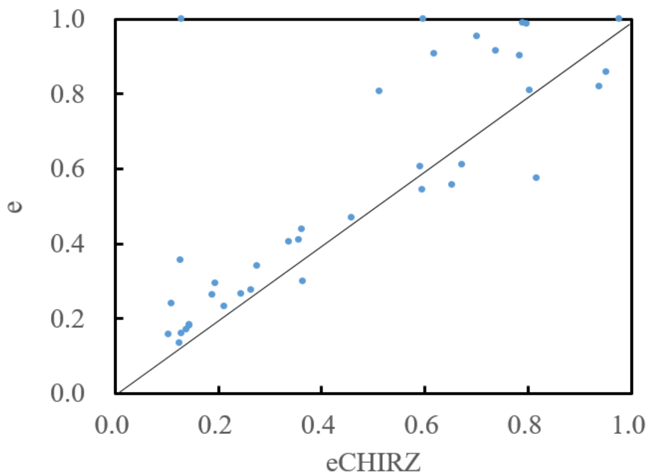

Fig. 3 The CHIRZ efficiency vs. our proposed efficiency

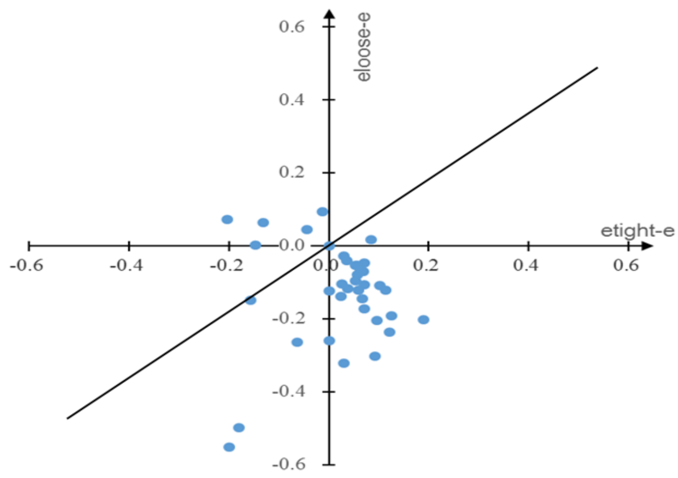

Fig. 4 The efficiency difference between e_{tight} and e v.s. that between e_{loose} and e

Tables

Table 1 Input and output variables of hospitals and descriptive statistics of raw data

	Stat.				
	Max	Min	Average	Range	Std.Dev
Inputs					
No. of FTE staff	5,870.16	57.02	1,634.87	5,813.14	1,718.55
No. of beds	1,843.00	26.00	733.78	1,817.00	563.06
Outputs					
Total inpatient length of stay (Output 1)	1,381,178.00	8,855.60	289,544.00	1,372,322.00	290,188.70
Total Accident & Emergency attendances (Output 2)	228,871.00	10,975.00	140,831.90	217,896.00	48,783.93
Total Specialist Outpatient attendances (Output 3)	690,407.00	21.00	214,501.72	690,386.00	231,603.08
Family medicine specialist clinic attendances (Output 4)	58,190.00	251.00	17,369.00	57,939.00	19,888.69
Total allied health outpatient attendances (Output 5)	223,020.00	125.00	64,007.00	222,895.00	70,461.73
General outpatient attendances (Output 6)	766,062.00	26,117.00	296,495.00	739,945.00	199,749.10

Table 2 Outputs of DMU groups

Group \ Output	Output 1	Output 2	Output 3	Output 4	Output 5	Output 6
1	√				√	
2	√	√	√	√	√	√
3	√	√	√		√	√
4	√		√		√	√
5	√					
6	√		√		√	
7	√		√	√	√	√
8	√	√	√	√	√	

Table 3 Definitions of variables

Variable	Definition	Variable	Definition
i	index of inputs	j	index of DMUs
P	the number of DMUs groups	p	the p^{th} DMU group
d	index of DMU under evaluation	K	the set of subunit types
k	the k^{th} subunit and output	p^d	the DMU group that contains DMU_d
k^d	the k^{th} subunit of DMU_d	L_p	the set of subunits consisting of p
x_{ij}^k	the i^{th} input of subunit k of DMU_j	L_{p^d}	the set of subunits consisting of p^d
x_{ij}	the i^{th} input of DMU_j	w_i	the multiplier assigned to the i^{th} input
y_{kj}	the k^{th} output of DMU_j	u_k	the multiplier assigned to the k^{th} output
ω_{kj}	the weight of subunit k to DMU_j	ω_{kd}	the weight of subunit k to DMU_d
v_i	the multiplier assigned to the i^{th} input after Charnes-Cooper (C-C) transformation	μ_k	the multiplier assigned to the k^{th} output after C-C transformation
α_{ikp}	the proportion of the i^{th} input assigned to subunit k of the DMU group p	α_{ikp^d}	the proportion of the i^{th} input assigned to subunit k of the DMU group p^d
u_k^0	the unrestricted variable of subunit k	μ_k^0	the unrestricted variable after C-C transformation
z_{ikp}	a converted variable and $z_{ikp} = w_i * \alpha_{ikp}$	γ_{ikp}	the variable of z_{ikp} after C-C transformation
M_k	the set of DMU groups of subunit k	M_{k^d}	the set of DMU groups of subunit k^d
x_{ij}^{k*}	obtained i^{th} input of subunit k of DMU_j	$x_{id}^{k^d*}$	obtained i^{th} input of subunit k to DMU_d
e_d^*	the overall efficiency of DMU_d	$e_{k^d d}^*$	the efficiency of subunit k^d of DMU_d

Table 4 Limits on input proportions for each DMU group

Limits / Group	A	A_{tight}	A_{loose}
1	[0.15, 0.80]	[0.20, 0.70]	[0.10, 0.90]
2	[0.10, 0.60]	[0.15, 0.50]	[0.05, 0.70]
3	[0.10, 0.50]	[0.15, 0.40]	[0.05, 0.60]
4	[0.20, 0.60]	[0.20, 0.50]	[0.10, 0.70]
5	[0.00, 1.00]	[0.00, 1.00]	[0.00, 1.00]
6	[0.20, 0.70]	[0.25, 0.60]	[0.10, 0.80]
7	[0.10, 0.50]	[0.15, 0.40]	[0.05, 0.60]
8	[0.10, 0.50]	[0.15, 0.40]	[0.05, 0.60]

Table 5 Efficiency scores of subunits and DMUs

	DMU	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6	e
p = 1	1	0.35				0.20		0.30
	25	0.27				0.05		0.18
	29	0.33				0.16		0.26
p = 2	2	1.00	0.80	1.00	1.00	0.58	0.63	0.90
	3	0.79	1.00	0.69	0.55	0.40	0.77	0.61
	11	0.75	0.58	1.00	0.59	0.72	0.43	0.81
	16	1.00	1.00	1.00	0.40	1.00	0.67	0.91
	18	0.93	1.00	0.96	1.00	1.00	1.00	0.99
	19	0.77	1.00	0.99	1.00	0.63	1.00	0.82
	20	0.70	0.32	0.92	0.68	0.50	0.81	0.56
	22	0.79	0.97	0.84	0.55	0.22	0.46	0.47
	24	0.86	0.27	0.72	0.53	0.49	0.53	0.54
	26	0.79	0.87	0.80	0.85	0.79	0.65	0.81
	27	0.90	1.00	1.00	1.00	1.00	1.00	0.99
	30	0.79	0.37	0.72	0.59	0.91	1.00	0.61
	31	1.00	0.81	0.26	1.00	0.94	0.71	0.57
	37	1.00	1.00	1.00	0.54	1.00	1.00	0.95
p = 3	4	1.00	1.00	1.00		1.00	1.00	1.00
p = 4	5	0.31		0.66		0.46	0.47	0.44
	12	0.56		0.25		0.10	0.24	0.34
	13	0.46		0.26		0.27	0.58	0.41
p = 5	6	0.17						0.17
	36	0.13						0.13
p = 6	7	0.49		0.54		0.34		0.41
	8	0.45		0.12		0.25		0.24
	9	0.47		0.14		0.13		0.23
	10	0.46		0.43		0.27		0.36
	14	1.00		1.00		1.00		1.00
	15	0.63		0.17		0.24		0.30
	17	0.36		0.13		0.11		0.18
	21	0.41		0.32		0.14		0.28
	28	1.00		1.00		1.00		1.00
	32	0.36		0.08		0.07		0.16
	33	0.46		0.04		0.08		0.16
	34	0.50		0.16		0.14		0.27
p = 7	23	0.94		0.75	1.00	0.95	0.90	0.92
p = 8	35	1.00	1.00	0.59	0.83	0.81		0.86

Table 6 Statistics of efficiencies on subunits

Stat.	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6
No. of DMU	37	16	32	16	35	19
No. of efficient DMU	8	8	8	6	7	5
Max	1.00	1.00	1.00	1.00	1.00	1.00
Min	0.13	0.27	0.04	0.40	0.05	0.24
Range	0.87	0.73	0.96	0.60	0.95	0.76
Std.Dev	0.28	0.27	0.35	0.22	0.36	0.24

Table 7 Correlation coefficients among efficiencies

	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6
$e(p = 1)$	1.00				1.00	
$e(p = 2)$	0.50	0.68	0.68	0.29	0.76	0.46
$e(p = 4)$	-1.00		1.00		1.00	0.50
$e(p = 5)$	1.00					
$e(p = 6)$	0.77		0.96		0.95	

Table 8 Efficiency scores of DMUs

DMU	eCHIRZ	e	e_{tight}	e_{loose}
1	0.19	0.30	0.41	0.17
2	0.78	0.90	0.75	0.90
3	0.67	0.61	0.73	0.37
4	0.60	1.00	1.00	1.00
5	0.36	0.44	0.46	0.30
6	0.14	0.17	0.24	0.10
7	0.36	0.41	0.54	0.22
8	0.11	0.24	0.31	0.13
9	0.21	0.23	0.28	0.18
10	0.13	0.36	0.43	0.18
11	0.51	0.81	0.60	0.88
12	0.27	0.34	0.40	0.22
13	0.34	0.41	0.49	0.42
14	0.98	1.00	1.00	0.88
15	0.36	0.30	0.37	0.15
16	0.62	0.91	0.78	0.97
17	0.14	0.18	0.21	0.15
18	0.80	0.99	0.81	0.49
19	0.94	0.82	0.91	0.52
20	0.65	0.56	0.63	0.51
21	0.26	0.28	0.30	0.17
22	0.46	0.47	0.57	0.27
23	0.74	0.92	0.85	0.65
24	0.59	0.54	0.53	0.64
25	0.14	0.18	0.24	0.10
26	0.80	0.81	0.65	0.66
27	0.79	0.99	0.79	0.44
28	0.13	1.00	1.00	0.74
29	0.19	0.26	0.37	0.16
30	0.59	0.61	0.66	0.51
31	0.82	0.57	0.76	0.37
32	0.10	0.16	0.19	0.12
33	0.13	0.16	0.22	0.10
34	0.24	0.27	0.30	0.15
35	0.95	0.86	0.89	0.54
36	0.12	0.13	0.19	0.08
37	0.70	0.95	0.91	1.00

Electronic Supplemental Material

Appendix 1: Generating subunits of a general case

The algorithm for generating subunits of a general case can be described as follows:

Step 1: Define an empty set as E.

Step 2: According to each output r, define a subunit $k_r(I, r)$, and I is inputs used to produce the output r.

Step 3: Define $\mathrm{p}(k_r)$ as the set of all DMU groups p that contains $k_r(I, r)$. Then, adding $\mathrm{p}(k_r)$ into the empty set E.

Step 4: In E, compare each $\mathrm{p}(k_r)$ to other $\mathrm{p}(k_{r\prime})$, and find out all $\mathrm{p}(k_{r\prime})$ that have the same DMU groups of $\mathrm{p}(k_r)$. If there has no such $\mathrm{p}(k_{r\prime})$, set a subunit as $k_{k=1}^{K} = (I, r)$ and remove $\mathrm{p}(k_r)$ from E. Otherwise, set a subunit as $k_{k=1}^{K} = (I, r\ and\ r')$ and remove $\mathrm{p}(k_r)$ and all $\mathrm{p}(k_{r\prime})$ from E.

Step 5: Repeat step 4 until E become an empty set.

Appendix 2: Tables

Table 1 Data of inputs and outputs

DMU	Inputs		Outputs					
	No. of FTE staffs	No. of beds	1	2	3	4	5	6
1	180.00	240.00	76,235.90	—	—	—	125.00	—
2	4,283.48	1,633.00	683,670.00	155,156.00	547,471.00	58,190.00	118,121.00	380,248.00
3	1,294.20	633.00	169,758.60	82,799.00	125,015.00	10,948.00	92,573.00	138,740.00
4	113.29	87.00	14,292.20	10,975.00	67.00	—	6,579.00	33,056.00
5	579.53	278.00	112,531.30	—	103,228.00	—	27,353.00	26,117.00
6	169.26	160.00	51,262.00	—	—	—	—	—
7	225.35	130.00	24,280.00	—	19,187.00	—	27,325.00	—
8	263.54	272.00	60,776.20	—	595.00	—	375.00	—
9	544.47	372.00	149,206.40	—	36,224.00	—	2,970.00	—
10	141.06	110.00	26,455.20	—	338.00	—	2,695.00	—
11	5,370.74	1,698.00	607,081.50	132,564.00	690,407.00	21,105.00	149,081.00	335,258.00
12	802.73	550.00	342,704.50	—	45,005.00	—	5,591.00	32,735.00
13	370.40	324.00	138,959.10	—	11,464.00	—	10,976.00	45,432.00
14	277.86	45.00	34,550.70	—	224,919.00	—	18,660.00	—
15	1,906.87	1,335.00	422,763.20	—	84,137.00	—	132,863.00	—
16	5,870.16	1,843.00	865,225.20	206,214.00	688,884.00	6,497.00	223,020.00	516,017.00
17	655.59	425.00	136,124.80	—	9,470.00	—	4,629.00	—
18	1,488.56	543.00	211,873.20	132,059.00	209,326.00	251.00	97,510.00	297,828.00
19	4,334.31	1,403.00	508,131.00	183,774.00	527,135.00	53,003.00	214,659.00	569,520.00
20	2,227.96	1,183.00	393,429.60	139,820.00	351,927.00	732.00	69,628.00	268,555.00
21	1,282.16	920.00	293,999.10	—	211,209.00	—	31,077.00	—
22	3,149.80	1,206.00	399,617.40	142,120.00	356,509.00	2,489.00	157,438.00	209,726.00
23	721.33	236.00	81,291.00	—	66,648.00	702.00	27,868.00	415,159.00
24	4,004.68	1,731.00	719,737.20	155,381.00	417,914.00	10,332.00	107,362.00	423,410.00
25	596.60	511.00	183,379.00	—	—	—	853.00	—
26	1,762.02	800.00	230,678.80	142,805.00	207,623.00	2,040.00	78,758.00	265,345.00
27	1,529.84	563.00	212,020.00	136,101.00	226,647.00	5,035.00	96,628.00	225,056.00
28	57.02	26.00	8,855.60	—	21.00	—	923.00	—
29	217.32	304.00	85,589.90	—	—	—	840.00	—
30	1,672.56	599.00	198,759.60	115,764.00	173,500.00	6,083.00	68,640.00	248,659.00
31	4,470.95	1,478.00	703,444.80	157,719.00	664,458.00	48,182.00	169,758.00	436,484.00
32	810.14	553.00	174,938.40	—	500.00	—	1,185.00	—
33	886.08	992.00	245,569.60	—	379.00	—	353.00	—
34	1,324.79	1,156.00	397,230.50	—	137,414.00	—	21,442.00	—
35	1,133.05	527.00	255,367.20	131,188.00	88,261.00	32,752.00	55,996.00	—
36	411.37	500.00	112,161.00	—	—	—	—	—
37	5,361.19	1,784.00	1381,178.00	228,871.00	638,173.00	19,556.00	216,388.00	766,062.00

Table 2 Input proportion of the No. of FTE staff to each subunit

	DMU	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6
p = 1	1	0.63				0.37	
	25	0.59				0.41	
	29	0.60				0.40	
p = 2	2	0.10	0.10	0.10	0.50	0.10	0.10
	3	0.10	0.10	0.10	0.30	0.30	0.10
	11	0.10	0.10	0.50	0.10	0.10	0.10
	16	0.10	0.10	0.10	0.10	0.50	0.10
	18	0.10	0.45	0.10	0.10	0.12	0.13
	19	0.10	0.10	0.10	0.40	0.20	0.10
	20	0.16	0.42	0.13	0.10	0.10	0.10
	22	0.10	0.10	0.10	0.10	0.50	0.10
	24	0.24	0.31	0.11	0.10	0.10	0.15
	26	0.10	0.44	0.10	0.10	0.10	0.16
	27	0.10	0.50	0.10	0.10	0.10	0.10
	30	0.10	0.41	0.10	0.19	0.10	0.10
	31	0.10	0.10	0.50	0.10	0.10	0.10
	37	0.23	0.23	0.10	0.10	0.10	0.23
p = 3	4	0.12	0.34	0.11		0.12	0.31
p = 4	5	0.40		0.20		0.20	0.20
	12	0.40		0.20		0.20	0.20
	13	0.40		0.20		0.20	0.20
p = 5	6	1.00					
	36	1.00					
p = 6	7	0.20		0.20		0.60	
	8	0.28		0.52		0.20	
	9	0.28		0.52		0.20	
	10	0.28		0.21		0.51	
	14	0.22		0.55		0.23	
	15	0.20		0.20		0.60	
	17	0.28		0.21		0.51	
	21	0.28		0.35		0.37	
	28	0.31		0.41		0.28	
	32	0.28		0.21		0.51	
	33	0.28		0.52		0.20	
	34	0.34		0.46		0.20	
p = 7	23	0.10		0.10	0.21	0.10	0.49
p = 8	35	0.10	0.21	0.10	0.49	0.10	

Table 3 Input proportion of the No. of beds to each subunit

	DMU	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6
p = 1	1	0.50				0.50	
	25	0.49				0.51	
	29	0.50				0.50	
p = 2	2	0.10	0.10	0.10	0.50	0.10	0.10
	3	0.10	0.10	0.10	0.30	0.30	0.10
	11	0.10	0.10	0.50	0.10	0.10	0.10
	16	0.10	0.10	0.10	0.10	0.50	0.10
	18	0.10	0.10	0.10	0.46	0.14	0.10
	19	0.10	0.10	0.10	0.18	0.42	0.10
	20	0.12	0.31	0.14	0.20	0.12	0.11
	22	0.10	0.10	0.10	0.10	0.50	0.10
	24	0.10	0.48	0.11	0.10	0.10	0.10
	26	0.10	0.10	0.10	0.50	0.10	0.10
	27	0.10	0.10	0.10	0.46	0.14	0.10
	30	0.10	0.39	0.10	0.21	0.10	0.10
	31	0.10	0.10	0.50	0.10	0.10	0.10
	37	0.20	0.25	0.10	0.10	0.10	0.25
p = 3	4	0.13	0.32	0.12		0.13	0.30
p = 4	5	0.40		0.20		0.20	0.20
	12	0.40		0.20		0.20	0.20
	13	0.39		0.20		0.20	0.20
p = 5	6	1.00					
	36	1.00					
p = 6	7	0.20		0.20		0.60	
	8	0.42		0.29		0.29	
	9	0.58		0.22		0.21	
	10	0.35		0.35		0.31	
	14	0.23		0.47		0.30	
	15	0.21		0.21		0.57	
	17	0.58		0.21		0.21	
	21	0.58		0.20		0.21	
	28	0.32		0.36		0.33	
	32	0.58		0.22		0.21	
	33	0.45		0.28		0.28	
	34	0.55		0.21		0.24	
p = 7	23	0.10		0.10	0.27	0.10	0.43
p = 8	35	0.19	0.32	0.10	0.28	0.11	

Table 4 Weights of the performance of subunits to the performance of DMUs

	DMU	Subunit 1	Subunit 2	Subunit 3	Subunit 4	Subunit 5	Subunit 6
p = 1	1	0.63				0.37	
	25	0.59				0.41	
	29	0.60				0.40	
p = 2	2	0.10	0.10	0.10	0.50	0.10	0.10
	3	0.10	0.10	0.10	0.30	0.30	0.10
	11	0.10	0.10	0.50	0.10	0.10	0.10
	16	0.10	0.10	0.10	0.10	0.50	0.10
	18	0.10	0.40	0.10	0.15	0.12	0.13
	19	0.10	0.10	0.10	0.18	0.42	0.10
	20	0.16	0.42	0.13	0.10	0.10	0.10
	22	0.10	0.10	0.10	0.10	0.50	0.10
	24	0.24	0.31	0.11	0.10	0.10	0.15
	26	0.10	0.44	0.10	0.10	0.10	0.16
	27	0.10	0.44	0.10	0.16	0.11	0.10
	30	0.10	0.40	0.10	0.20	0.10	0.10
	31	0.10	0.10	0.50	0.10	0.10	0.10
	37	0.20	0.25	0.10	0.10	0.10	0.25
p = 3	4	0.12	0.34	0.11		0.12	0.31
p = 4	5	0.40		0.20		0.20	0.20
	12	0.40		0.20		0.20	0.20
	13	0.40		0.20		0.20	0.20
p = 5	6	1.00					
	36	1.00					
p = 6	7	0.20		0.20		0.60	
	8	0.28		0.52		0.20	
	9	0.28		0.52		0.20	
	10	0.28		0.21		0.51	
	14	0.23		0.49		0.28	
	15	0.20		0.20		0.60	
	17	0.28		0.21		0.51	
	21	0.28		0.35		0.37	
	28	0.31		0.38		0.31	
	32	0.28		0.21		0.51	
	33	0.28		0.52		0.20	
	34	0.34		0.46		0.20	
p = 7	23	0.10		0.10	0.22	0.10	0.48
p = 8	35	0.10	0.21	0.10	0.49	0.10	