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Introduction of fractional elements for improvising the performance of PID controller for heating furnace using AMIGO tuning technique☆

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Summary The paper demonstrates about melioration of integer order and fractional order model of heating furnace. Both models are being placed in closed loop along with the proportional integral derivative (PID) controller and fractional order proportional integral derivative (FOPID) controller so that the various time domain performance characteristics of the heating furnace can be meliorated. The tuning parameters (K_p, K_i and K_d) of the controllers has been found using the Astrom-Hagglund tuning technique and the differ-integrals (λ and μ) are found using the Nelder-Mead optimisation technique.

Introduction

In mathematical modelling, an understanding is made of those feelings into the vernacular of science (Lawson and Marion, 2008). Heating furnace is a mechanical gadget that is used to warm distinctive substances at the required temperature. There are bunches of parameters that are not up to the imprint in the warming heater like overshoot, steady state error and settling time. The controllers are designed and tuned utilising diverse tuning methods and streamlining systems. Tuning methods are the techniques for achieving the different tuning parameters of the controllers. Optimising procedures are additionally the same yet they likewise deliver the estimations of two additional parameters called the differ-integrals which assume an essential part in the designing of the controller of fractional order controller (Figs. 1 and 2).

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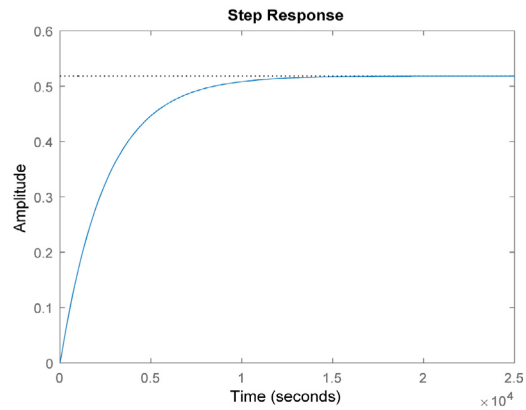

Figure 1 Step response of the heating furnace Eq. (11).

IOPID and FOPID controller

IOPID controller is mathematically defined as (Shahri and Balochian, 2012),

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau + K_d \frac{de}{dt} \quad (1)$$

where, K_p is the gain of proportionality, K_i is the gain of Integral, K_d is the gain of Derivative, e is the Error, t signifies the instantaneous time and τ is the variable of integration. On performing the Laplace transform of the Eq. (1) which is the PID controller equation is,

$$L_I(s) = K_p + \frac{K_i}{s} + K_d s \quad (2)$$

Numerically, the FOPID controller can be defined as (Rastogi and Tiwari, 2013),

$$u(t) = K_P e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^{\mu} e(t) \quad (3)$$

On performing the Laplace transform of the Eq. (3) we get (Shahri and Balochian, 2012),

$$L_f(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \quad (4)$$

Where, K_p is the gain of proportionality, K_i is the gain of Integral, K_d is the gain of Derivative and λ and μ are the differential-integral's order for FOPID controller.

Astrom-Hagglund or AMIGO tuning technique

The other name for this tuning method is AMIGO which stands for approximate M-constrained integral gain optimisation method for tuning. The tuning procedure of the AMIGO is as follows (Astrom and Hagglund, 1995),

$$K_p = \frac{1}{K}\left(0.2 + 0.45\frac{T}{L}\right) \quad (5)$$

$$K_i = \left(\frac{0.4L + 0.8T}{L + 0.1T}\right) L \quad (6)$$

$$K_d = \frac{0.5LT}{0.3L + T} \quad (7)$$

Nelder-Mead optimisation technique

The different operations in Nelder-Mead optimisation method are (Wright, 2012), Taking a function $f(x)$, $x \in R^n$ which is to be minimised in which the current points are $x_1, x_2 \ldots\ldots x_{n+1}$. (i). Order: On the basis of values at the vertices, $f(x_1) \le f(x_2) \le \ldots\ldots\ldots\ldots \le f(x_{n+1})$. (ii). Calculate the centroid of all points (x_0) except x_{n+1}. (iii). Reflection: Calculate $x_r = x_0 + \alpha\,(x_0 - x_{n+1})$. If the reflected point is not better than the best and is better than the second worst, that is, $f(x_1) \le f(x_r) < f(x_n)$. After this by putting back the worst point x_{n+1} with reflected point x_r to get a new simplex and go to the first step. (iv). Expansion: If we have the best reflected part then $f(x_r) < f(x_1)$, then solve the expanded point $x_e = x_0 + \gamma(x_0 - x_{n+1})$. If the reflected point is not better than expanded point, that is, $[f(x_e) < f(x_r)]$ then either by replacing the most awful point x_{n+1} by expanded point x_e to get new simplex and then go to the first step or by replacing the most awful point x_{n+1} by reflected point x_r to acquire or get a new simplex and then go back to the first step. Else if the reflected point is not well again than subsequent worst then move to the fifth step. (v). Contraction: Here we know that $f(x_r) \ge f(x_n)$, contracted point is to be calculated $x_c = x_0 + \rho(x_0 - x_{n+1})$, if $f(x_c) < f(x_{n+1})$ that is the contracted point is better than the most awful point then by replacing the most awful point x_{n+1} with contracted point x_c to achieve a new simplex and then go to first step or proceed to sixth step. (vi). Reduction: reinstate the point with $x_i = x_1 + \sigma(x_i - x_1)$ for all i $\in \{2,\ldots\ldots,n+1\}$, then go to the first step.

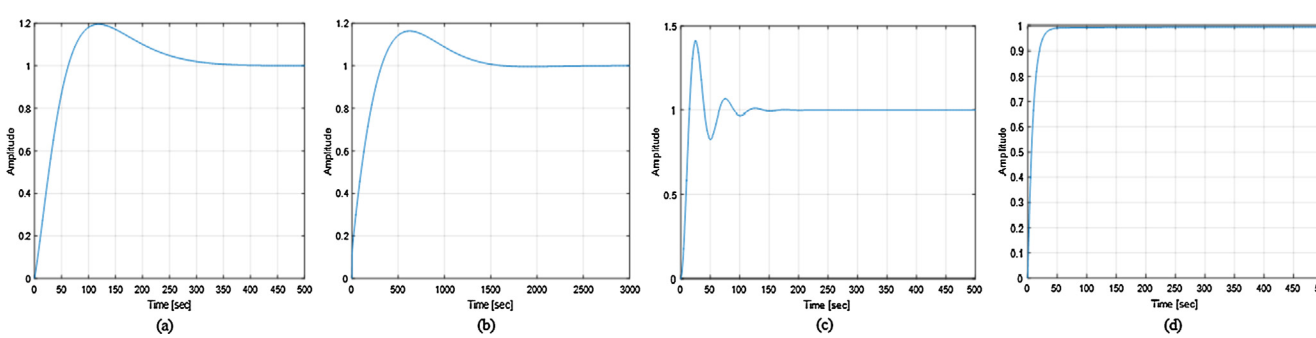

Figure 2 Step Responses of (a) Eqs. (20) (b) (21) (c) (22) (d) (23).

Table 1 Output parameter values.

	PID + IOM	FOPID + IOM	PID + FOM	FOPID + FOM
Overshoot (%)	19.5	41	16.3	0
Settling time (sec.)	435	171.7	2345	133

Results

For any physical system, the total force is equal to the summation of individual forces exerted by mass (m), damping (b) and spring (k) element. Mathematically, we can state the same as,

$$F = ma + bv + kx \tag{8}$$

In the Eq. (8) acceleration is signified as a, velocity is signified as v and displacement is signified as x. Therefore, the differential equation of Eq. (8) is,

$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \tag{9}$$

Note: For designing a network-based PID, the above equation or model is a rough process behaviour description. Therefore, the differential equation of the heating furnace using the above equation becomes (Zhao et al., 2005,Vic Dannon, 2009),

$$F = 73043\frac{d^2x}{dt^2} + 4893\frac{dx}{dt} + 1.93x \tag{10}$$

The Laplace transfer function of Eq. (10) is given as (Maiti and Konar, 2008),

$$G_I(s) = \frac{1}{73043s^2 + 4893s + 1.93} \tag{11}$$

In Eq. (20) 's' is the Laplace operator. The FOPDT model of Eq. (12) is,

$$G_{\text{IOM-FOPDT}}(s) = \frac{0.518133}{1 + 2520.04s}e^{-15.2189} \tag{12}$$

The FOM of the heating furnace is obtained using the Grunwald-Letnikov equation for fractional calculus which is given in Eq. (13),

$${}_aD_t^{\alpha} f(t) = \lim \frac{1}{(\alpha)\,h^{\alpha}}\sum_{k=0}^{\frac{(t-a)}{h}} \left\{\frac{(\alpha+k)}{(k+1)}\right\} f(t-kh) \tag{13}$$

Therefore, the FOM of heating furnace which comes out to be (Tepljakov et al., 2011),

$$G_F(s) = \frac{1}{14494s^{1.31} + 6009.5s^{0.97} + 1.69} \tag{14}$$

Therefore, the FOPDT model for the Eq. (14) is,

$$G_{\text{FOM-FOPDT}}(s) = \frac{0.404257}{1 + 3440.71s}e^{-66.9314} \tag{15}$$

The PID controller equation deduced using the FOPDT model of the IOM is,

$$L_{I_1}(s) = 144.198 + \frac{1.25211}{s} + 1095.28s \tag{16}$$

The PID controller equation deduced using the FOPDT model of FOM is,

$$L_{f_1}(s) = 57.7181 + \frac{0.127522}{s} + 1920.37s \tag{17}$$

The FOPID controller equation deduced for the IOM of heating furnace is,

$$L_{I_2}(s) = 144.198 + \frac{1.25211}{s^{0.4876}} + 1095.28s^{0.01011} \tag{18}$$

The FOPID controller equation deduced for the FOM of heating furnace is,

$$L_{f_2}(s) = 57.7181 + \frac{0.127522}{s^{0.3636}} + 1920.37s^{0.12483} \tag{19}$$

When the Eqs. (16) and (17) are, respectively, put in the closed loop system along with Eq. (11) the outputs obtained are

$$H_1(s) = \frac{1095.3s^2 + 144.2s + 1.2521}{73043s^3 + 5988.3s^2 + 146.13s + 1.2521} \tag{20}$$

$$H_2(s) = \frac{1920.4s^2 + 57.718s + 0.12752}{14994s^{2.31} + 1920.4s^2 + 6009.5s^{1.97} + 59.408s + 0.12752} \tag{21}$$

When the Eqs. (18) and (19) are, respectively, put in the closed loop system along with Eq. (14) the outputs obtained are,

$$H_3(s) = \frac{1095.3s^{0.49771} + 144.2s^{0.4876} + 1.2521}{73043s^{2.4876} + 4893s^{1.4876} + 1095.3s^{0.49771} + 146.13s^{0.4876} + 1.2521} \tag{22}$$

$$H_4(s) = \frac{1920.4s^{0.48843} + 57.718s^{0.3636} + 0.12752}{14994s^{1.6736} + 6009.5s^{1.3336} + 1920.4s^{0.48843} + 59.408s^{0.3636} + 0.12752} \tag{23}$$

Discussion

It is clear that the IOM transfer function of heating furnace exhibits very poor response with a steady-state error of more than 50%. So a PID controller is designed using AMIGO tuning technique. But the overshoot of the system then became 19.5% where as the settling became 435 s. Therefore, Nelder-Mead Optimisation algorithm was used to this PID to find the fractional elements λ & μ, so that FOPID can be designed for the System. But it also exhibited an overshoot and settling time of 41% & 170 s, respectively. Therefore, PID is designed based on the Fractional Order Model of Heating Furnace Transfer function. When AMIGO method was applied to FOM for the tuning parameters, the final system became stable with an exhibited overshoot of 16%, where as the settling time increased drastically up to 2400 s. Therefore to improvise the response Nelder-Mead optimisation algorithms was used to tune the already tuned tuning parameters using AMIGO method and also to optimise the differ-integral parameters. It is clear from the step response that the system overshoot decreased to a zero value where as the settling time was around 130 s (Table 1).

Conclusion

The plots of time response characteristics cleared that the PID & Fractional order PID designed for the IOM gave a very disturbed response. FOM of furnace gave comparatively good response while used with AMIGO tuning method, but it exhibited a high overshoot and also a sluggish response. As the overshoot in furnace generates sudden high pressure which may endanger the life of workers and properties, this method was avoided. But when fractional elements of PID were optimised using Nelder-Mead optimisation, the system exhibited almost negligible range of overshoot and also a comparatively low settling time. Therefore, it can be concluded that more the fractional elements are introduced more the result will be smooth and swift.

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