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Stability norms control using the virtual impedance concept for power frequency applications

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Abstract— In small-signal stability studies, various stability criteria have been proposed based on impedance or admittance norms to assess the stability of power systems. Controlling these norms can drag a system from a stable to an unstable operating condition. Therefore, finding a control variable that has a direct linear relationship with these norms will facilitate the utilisation of data-driven control principles to build a control system based on small-signal impedance. For a STATCOM connected to a power network, the control system parameters of the STATCOM are tested to identify their relationships with the stability norms. Different types of virtual impedance are tested, and the suitability of each virtual impedance type and connection is examined and presented.

Keywords— Statcom, control parameters, virtual impedance, small signal impedance

I. INTRODUCTION

Since the beginning of power network development, stability studies have become increasingly important in the research and industrial fields. Different stability assessment methods, such as eigenvalue, state-space and impedance analysis, have been developed to evaluate system stability. The small-signal impedance method is both sturdy and practical, especially in real-world applications when creating a fully detailed model of the power network becomes a challenging task [1]. It uses the concept of black box analysis by employing terminal measurements, instead of using a fully detailed model of the network as in the case of eigenvalue or state-space analysis, to assess stability.

The first implementation of the small-signal impedance method was in dc systems and was later adopted to determine ac system stability in $dq$ coordinates. $dq$ analysis transforms the power system variables to dc components under steady-state conditions, which made the adoption of dc stability tools easier in ac systems.

Criteria were developed to assess the small-signal stability of the power network based on generator and load admittance in $dq$ coordinates. The generalised Nyquist criterion (GNC) was one of the most important criteria. Developed firstly to assess dc system stability and modified later for ac systems, it examines whether or not the product of generator impedance and system admittance encircles the point (-1,0) in the complex plane.

From the small-signal impedance point of view, improving ac system stability at the interface between different systems or the interface between the system and the connected devices is a function of the impedances of the two parts of the system. The $dq$ impedance of an ac system has four parts which together define system stability based on the norm of the impedance matrix. Therefore, having a direct relationship between these norms and a control parameter can facilitate the use of stability norms in a control system.

Different techniques for improving system stability, including active and passive techniques, have been reported in the literature. Active techniques depend upon changing the control variable of the device to provide the required impedance, while passive techniques deploy a passive physical component such as a pure resistance or complex impedance to reshape the impedance.

Passive techniques reshape the output impedance by regulating the load voltage and/or current. They are not preferred in power system applications where they increase system losses and the cost of installed components [3]. This is reduced the use of passive components and, in the meantime, builds interest in using active techniques.

Virtual impedance [4] and synchronous virtual impedance [5] [6] are the main active methods utilised in the literature to reshape the impedance of different devices, such as flexible AC transmission system (FACTS) devices and HVDC systems, to achieve system requirements. These requirements vary from improving ride-through capability and reducing harmonics to improving system stability, in addition to power flow control and damping of sub-synchronous oscillations [7].

In power system applications, FACTS devices are becoming widely installed to compensate reactive power and to prevent system oscillations [1]. Amongst these devices, shunt-connected devices, such as static synchronous compensators (STATCOM), have a significant influence on system stability due to their connection to the network or interaction with other devices in the system, especially during system emergency events. Therefore, reshaping STATCOM impedance is an important issue in system stability.

Small-signal impedance performance studies and control methods have been reported in the literature. A synchronous machine has been used to reshape STATCOM impedances
The effects of STATCOM control parameters on the impedance matrix have also been examined. However, the impact of these parameters and impedances on the stability norm of the studied devices, which makes the use of these parameters in any practical control system difficult, has not been studied.

This paper reports a study of the effect of STATCOM control variables on stability norms. Also, virtual impedance parameters in any practical control system difficult, has not been studied. The reference current ($\mathbf{I}_{\text{ref}}$) can be derived according to (5).

$$\frac{\Delta \mathbf{I}_{\text{ref}}}{\Delta t} = G \begin{bmatrix} V_{dc}^* \\ V_{S_q} \end{bmatrix} - G \begin{bmatrix} \Delta V_{dc} \\ \Delta V_{S_q} \end{bmatrix}$$

(3)

The voltage controller is responsible for controlling dc and bus voltages using (4).

$$\frac{\Delta V_{dc}}{\Delta V_{S_q}} = H \begin{bmatrix} \Delta V_{dc} \\ \Delta V_{S_q} \end{bmatrix} + K \begin{bmatrix} \Delta \mathbf{I}_{d} \\ \Delta \mathbf{I}_{q} \end{bmatrix}$$

(4)

Definitions of the symbols used in (1) to (4) are presented in appendix A.

Using Mason’s gain formula or any block reduction method, (1) to (4) are used to derive the total transfer function of the small-signal STATCOM impedance as follows:

$$\frac{\Delta V_{S_q}}{\Delta V_{S_d}} = Z_{\text{STATCOM}} \begin{bmatrix} \Delta \mathbf{I}_{d} \\ \Delta \mathbf{I}_{q} \end{bmatrix}$$

(5)

$$Z_{\text{STATCOM}} = \frac{A + C K + B (E + F G K)}{D - B (D + F G H) - C H}$$

(6)

The equation (6) is transformed to a block diagram to depict this relation as shown in block diagram presented in Fig.3. The STATCOM operation is considered as an ideal operation; the pulse width modulation delay (PWM) delay and the measurement delay (md) is ignored in this paper and can be added the model as shown in block diagram of impedance model presented in Fig.3. The main effect of ignoring the PWM delay is the off diagonal impedance will be very small, while the effect of sensing delay will present for big system mainly.

### III. Basic Principle of Stability Criteria Based Impedance Norms

For stability studies based on small signal impedance, the impedance is measured at the generator-network connection point to assess the stability at this system interface. In the synchronous rotating $dq$ frame the generator impedance for the system shown in Fig.4 is given by (9).

$$\mathbf{Z}_g = \begin{bmatrix} Z_{S_{dd}} & Z_{S_{dq}} \\ Z_{S_{qd}} & Z_{S_{qq}} \end{bmatrix}$$

(7)

The small-signal impedance of the network as seen by the generator is of the form:

$$\mathbf{Z}_{\text{NT}} = \begin{bmatrix} Z_{NT_{dd}} & Z_{NT_{dq}} \\ Z_{NT_{qd}} & Z_{NT_{qq}} \end{bmatrix}$$

(8)
For ac networks the generalised Nyquist criterion (GNC) has the following form [15] [16]:

- The infinite one norm

\[ Z_{\infty} = \| z_{\infty}^{dq} \|_{\infty} \| y_{\infty}^{dq} \|_{1} < 0.5 \]  

(9)

In this paper, the infinite norm of the STATCOM will be investigated for both control parameters and the virtual impedance topologies.

- The G-norm criterion, which results in a product of the G-norms on both sides of the interfacing point:

\[ Z_{\infty} = \| z_{\infty}^{dq} \|_{G} \| y_{\infty}^{dq} \|_{G} < 0.25 \]  

(10)

\[ \| X \|_{G} = \max \left( |X_{\alpha\beta}|, |X_{\alpha\gamma}|, |X_{\beta\gamma}| \right) \]

\( X \) here can represent impedance or admittance.

- A third criterion which examines the stability of the system based on the maximum singular value of both sides of the interfacing point of the system:

\[ Z_{\sigma} = \delta(z_{\sigma}^{dq})\delta(y_{\sigma}^{dq}) < 1 \]  

(11)

It is worth noting that these three criteria, along with some others in this field, are sufficient but not necessary to predict the system’s small-signal stability. In this study, the relationship between the infinite stability norm and the change of control parameters, for a group of virtual impedances, is examined.

IV. EFFECT ON STABILITY NORMS OF CHANGING STATCOM CONTROL PARAMETERS

The test system introduced in Fig-1 is used to present the effect of control system of the STATCOM on the infinite norm. The test system and the STATCOM have the following parameters as shown in the table: 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{d} )</td>
<td>410</td>
<td>( K_{i,\alpha} )</td>
<td>8000</td>
</tr>
<tr>
<td>( V_{q} )</td>
<td>0</td>
<td>( K_{P_{\alpha}} )</td>
<td>10</td>
</tr>
<tr>
<td>( f_{s} )</td>
<td>60 Hz</td>
<td>( K_{i,\beta} )</td>
<td>0.001</td>
</tr>
<tr>
<td>( R_{f} )</td>
<td>0.5 ( \Omega )</td>
<td>( K_{P_{\beta}} )</td>
<td>800</td>
</tr>
<tr>
<td>( L_{f} )</td>
<td>5e-3 H</td>
<td>( K_{i,\gamma} )</td>
<td>8000</td>
</tr>
<tr>
<td>( C_{dc} )</td>
<td>400e-6 F</td>
<td>( K_{P_{\gamma}} )</td>
<td>0.01</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>1000V</td>
<td>( K_{i,\gamma} )</td>
<td>2</td>
</tr>
<tr>
<td>( K_{P_{\gamma}} )</td>
<td>800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As presented in (1) to (4), STATCOM control parameters have some effect on the STATCOM’s total impedance. Even though this relation for STATCOM impedance may be clear from the equations, this relation is not obvious for stability norms of impedance matrices, which is a disadvantage when tuning these parameters for stability improvement purposes.

Fig-5 shows how STATCOM controller gains affect the infinite stability norm. In general, increasing the voltage control loop gains has no effect on the infinite norm except for the proportional voltage gain (\( k_{P_{\alpha\beta}} \)). The increase of the gain tends to reduce the infinite norm for some range before it gets increased again at different values of (\( k_{P_{\alpha\beta}} \)) for the plotted frequencies.

As presented in (1) to (4), STATCOM control parameters have some effect on the STATCOM’s total impedance. Even though this relation for STATCOM impedance may be clear from the equations, this relation is not obvious for stability norms of impedance matrices, which is a disadvantage when tuning these parameters for stability improvement purposes.

In the meantime, changing the integral part of the current controller gains (\( k_{i,\beta} \)) have the no influence on the infinite norms as the shown in Fig 5.b. Even though the proportional gains (\( k_{P_{\beta}} \)) manage to reduce the infinite norm of the STATCOM over the range of gain change. The effect of direct current controller (\( k_{P_{\gamma}} \)) tends to become negligible for higher values of the gain while the quadrature gain (\( k_{P_{\gamma}} \)) can reduce the infinite norm for a wider range.

The values of control gain chosen impact significantly on their ability to adjust the infinite norm and might be restricted by the steady state and transient requirement of the connected network. Another restriction could be the device setting point where the infinite nor cannot be reduced. This conclusion
leads to the need to look for another control variable that can control the stability norms over a wide range and which can be represented by a simple mathematical relationship.

V. IMPLEMENTING VIRTUAL IMPEDANCE TO CONTROL STABILITY NORMS

As stated in the previous section, STATCOM control parameters are not suited for use in the implementation of stability norms tuning processes. This section presents the application of virtual impedance to change a device’s behaviour and to reshape its impedances. The basic idea of virtual impedance is to add the effect and behaviour of physical series or parallel impedance (passive impedance) to the control system (active impedance). The benefit of using virtual impedance along with other active techniques is that the active techniques regulate the STATCOM impedance magnitude and phase margins within a specific range without affecting the output voltage and currents.

In this study, the aim is to examine a simple virtual impedance implementation in a control system. Only virtual pure and complex impedances will be tested, and other techniques, such as the virtual synchronous machine [5], are beyond the scope of this study.

The aims of using virtual impedance here can be summarised as:

- Find a suitable control variable that has a direct relation with stability norms.
- Find a control variable that can increase or reduce the magnitude of total one norm or infinite norm of STATCOM impedance matrix.
- Test different virtual impedance topologies, such as shunt, series, resistive, RL and RC impedances, to assess the suitability of each type.

A. Series virtual impedance (SEVI)

The basic implementation of series impedance is to connect the virtual impedance between the STATCOM output current and the input voltage. However, the actual implementation is achieved by connecting the STATCOM output current to the reference voltage, as shown in Fig.6. For simplicity, the effect of PLL is ignored. The proposed SEVI has the form:

\[ Z_{se_{vir}} = \begin{cases} 
\frac{2v_{ir}}{g_{ir}} & f \in [f_1, f_2] \\
0 & f \notin [f_1, f_2] 
\end{cases} \]  

(12)

Where: \( f_1 \), \( f_2 \) are the boundaries of the interested frequency range of virtual impedance.

The introduced virtual impedance is proposed to have the same transfer function on diagonal STATCOM impedances and consequently has the same influence on these impedances. Also, the effect of implemented virtual impedance should be defined to be effective within the interested frequency range and to be otherwise equal to zero, as presented in the equation(12). Limiting of the functionality of the virtual impedance mentioned earlier can be achieved using low and high pass filters or second-order band-pass [4].

The transfer function \( (G_{se}) \) of the SEVI denominator is derived from Fig.6, it is equal to:

\[ G_{se} = md \cdot F \cdot PWM \cdot B \]  

(13)

In this study, the power frequency range is defined to be between 60Hz and 420Hz, and results are plotted at the fundamental frequency and its harmonics (2\(^{nd}\), 3\(^{rd}\), 5\(^{th}\), 7\(^{th}\)). The fundamental impedance at other mid-frequency can be found by interpolating any bounded frequencies. The magnitude of both inductance (\( L \)) and capacitive (\( C \)) is equal to the resistance magnitude (\( R \)) when plotting the relationships between the infinite norm and virtual impedances presented in Fig.7.

![Fig.6 Implementation of series virtual impedance in STATCOM model](image)

(a) Series resistive  
(b) Series resistive-capacitive  
(c) Series resistive-inductive  
(d) Shunt resistive  
(e) Shunt resistive-capacitive  
(f) Shunt resistive-inductive

Fig.7 Effect of virtual impedance on stability norm at different perturbation frequencies
The three types of virtual impedances ($Z_{\text{vir}}$) are tested here to identify their relation to the infinite norm of STATCOM are the pure resistance virtual impedance (SRI), resistive-inductive virtual impedance (RLI) and resistive-capacitive virtual impedance (RCI) which on the form:

$$Z_{\text{vir}} = \begin{cases} R & \\ R + SL & \\ R + \frac{1}{SC} \end{cases}$$ (14)

Fig.7 (a) and (b) present the effects of changing series resistive virtual impedance (SRI) and series resistive-capacitive virtual impedance (RCI) on the STATCOM impedance infinite norm for a range of perturbation frequencies. Both series impedances increase the infinite norm for a range from 0 to about 400, and the infinite norm starts decreasing the infinite form for higher values of virtual impedance. Also, it is shown that the effect of the series connection of SRI and RCI is limited on the stability norm at the first three frequencies (fundamental, 2nd and 3rd). This restricts the suitability of using these types of connections on reducing the STATCOM infinite norm. It has some negative effect at lower values of the impedance and a great effect beyond that. In the meantime, the increase of series resistive-inductive impedance tends to reduce the infinite norm for all perturbation higher frequencies as shown in Fig.7(c).

B. Shunt virtual impedance (SHVI)

Shunt virtual impedance is connected between the input voltage and the current reference to reducing the amount of current flow (according to the direction of current flow, $i_{sa}$, $i_{sb}$ and $i_{sc}$, assumed in Fig.1) to the STATCOM and, consequently, to reduce the STATCOM impedance, as depicted in Fig.8. SHVI is proposed to affect the same impedances as (12). The same frequency boundaries can be applied here to the functionality of the shunt virtual impedance as shown in (17).

$$Z_{\text{shVir}} = \begin{cases} \frac{Z_{\text{vir}}}{Gsh} & \text{if } f \in [f_1, f_2] \\ 0 & \text{if } f \notin [f_1, f_2] \end{cases}$$ (15)

The transfer function ($Gsh$) presented in equation (15) is equal to:

$$Gsh = m.d. \text{PWM}. B$$ (16)

Fig.7 (d), (e) and (f) present the effect of shunt virtual impedances (SHVI) at different perturbation frequencies. Shunt resistive and shunt resistive-capacitive has almost no effect at all perturbation frequencies over a range of change, as shown in Fig.7 (d,e) except a minor decrease at 420Hz. Shunt resistive-inductive (SRL) virtual impedance has a negative impact on the STATCOM infinite norm as presented in Fig.7 (f). It increases the total stability norm of the STATCOM considerably. As the perturbation frequency is decreased, the effect becomes limited. In general reducing the current using the shunt connections is not sufficient to reduce the STATCOM impedance. This is referred to the nature of the STATCOM controller.

VI. PROPOSED CONTROL SYSTEM BASED ON STABILITY NORMS AND VIRTUAL IMPEDANCE

The general construction of control system based norms is presented in Fig.9. The control system transforms measured voltage and current using Park's transformation and analysed them using Fast Fourier Transforms (FFT). The outputs of the FFT 'voltages and currents' are used to calculate the small-signal impedance of the network and to calculate network norms accordingly. Calculating the required STATCOM norm is achieved using one of (9), (10) or (11), with a small stability margin ($\alpha, \beta$) introduced here to prevent the appearance of oscillation in the system, and to convert from an inequality to an equality relationship as:

$$\|Z_{\text{STATCOM}}\|_{\infty} = \begin{cases} \alpha \frac{0.5}{\|\text{FFT}_{\text{dq}}\|_1} & \|\text{FFT}_{\text{dq}}\|_1 > 0 \\ \beta \frac{+0.5}{\|\text{FFT}_{\text{dq}}\|_1} & \|\text{FFT}_{\text{dq}}\|_1 < 0 \end{cases}$$ (17)

where: $\alpha < 1$ and $\beta > 1$
The same process applied to (17) can also be applied to the other stability criteria presented in equations (10) and (11). The criteria for selecting parameters ($\alpha, \beta$) are based on the number of devices to be connected to the system and range of stability required for the device. The measurement circuit included in the control system extracts the injected frequencies using a fast Fourier transform (FFT), and calculates the network norm which will be used to find the required STATCOM norm using one of the equations (17). The last block in the controller defines the relationship between the stability norms and the required value for virtual impedance, based upon whether series or shunt virtual impedances are used. Any curve fitting technique can be used to simplify the relation between the virtual impedance and the stability norms; to simplify the tuning process. The system proposed in Fig.9 can provide auto-tuning control for a STATCOM within the range of frequencies of interest. It can calculate the system impedance within a few cycles to identify the required value for the virtual impedance. The time response of this supplementary control is to low due the measurement time required for the impedance; therefor the main application of this controller can be for the new devices.

VII. CONCLUSION

This paper discusses the possibility of using a control variable to directly control the stability norm of the STATCOM impedance matrix which may be implemented in control systems. Even though some of STATCOM parameters can provide the necessary decrease on the infinite norm, the change of those parameters could be restricted by connected system required. So, different types of virtual impedances have been tested to find a suitable control parameter for STATCOM.

Generally, the shunt virtual impedances have adverse effect on the infinite norm of the STATCOM impedance matrix. Alternatively, a better behaviour can be seen in the series connection of the virtual impedance. The change of resistive and the resistive-capacitive can increase or decrease the infinite norms of the STATCOM; therefore the range of those impedances should be limited to the required behaviour. The increase of resistive-inductive impedance steadily decreases the infinite norm and this effect is almost not restricted for the range of changing the impedance.

This relation can be utilised in the control system proposed in this paper using the data-driven control concept to improve the stability at the connection point of the STATCOM. Even though the stability norms are not a necessary condition of the stable systems but sufficient condition; the proper design of such supplementary control system could lead to auto-tuning devices for the power electronics and help to improve the stability of the systems.

APPENDIX A

\[ B = \begin{bmatrix} V_{dc} & 0 \\ 0 & V_{dc} \end{bmatrix} \]

\[ E = \begin{bmatrix} K_p i_d + K_i i_d / S & 0 \\ -\omega L_f & K_p i_d + K_i i_d / S \end{bmatrix} \]

\[ F = \begin{bmatrix} K_p i_d + K_i i_d / S & 0 \\ 0 & K_p i_d + K_i i_d / S \end{bmatrix} \]

\[ G = \begin{bmatrix} K_p i_d + K_i i_d / S & 0 \\ 0 & K_p v_q + K_i v_q / S \end{bmatrix} \]

\[ K = \begin{bmatrix} G_{dc} (v_s - 2i_{sd} r_f) / C_{dc} + (3/2) G_{dc} V_s / C_{dc} & 0 \\ 0 & 0 \end{bmatrix} \]

REFERENCES