Generalized spectroscopy; coherence, superposition, and loss

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We analyze single particle coherence and interference in the presence of particle loss and derive an inequality that relates the preservation of coherence, the creation of superposition with the vacuum, and the degree of particle loss. We find that loss channels constructed using linear optics form a special subclass. We suggests a generalized spectroscopy where, in analogy with the absorption spectrum, we measure a "coherence loss spectrum" and a "superposition creation spectrum". The theory is illustrated with examples.

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Interference phenomena are versatile tools for studying quantum systems. The coherence of a physical process, i.e., its ability to preserve the capacity for interference, can likewise provide valuable information on the dynamics, which we suggest could be utilized for a spectroscopic approach. The coherence properties of quantum operations have been considered in Refs. [1, 2, 3], for an alternative approach see [4], and used to define operational interferometric fidelity and coherence measures [5]. However, these investigations assume no particle loss, limiting their applicability to mainly idealized situations. In this Letter, we consider the effect of particle loss on coherence and suggest procedures to measure the quantum properties of loss processes. The relation between particle loss and coherence has been considered both theoretically and experimentally in the context of neutron interferometry [6] using complex phase shifts, and also in the context of geometric phases [7] using non-Hermitian Hamiltonians. Complex phase shifts and non-Hermitian Hamiltonians [8] are useful phenomenological approaches to particle loss, and the latter can be derived as an approximation describing no-jump trajectories in the quantum jump approach [9]. However, within the standard framework of quantum mechanics, we model loss using quantum channels (trace preserving completely positive maps) [10] on second quantized systems to allow for varying particle number.

Briefly reviewing the ideal case of no particle loss, consider a particle with an internal degree of freedom (e.g. spin or polarization) traversing a Mach-Zehnder interferometer with two paths A and B. In path A we insert the material or device we wish to probe. In path B we apply a variable unitary operator U and a phase shift $e^{i\chi}$. After re-interfering the two paths, the probability to find the particle in path A is $p_A = \frac{1}{2} + \frac{1}{2}|F(\rho, U)|\cos(\arg F(\rho, U) - \chi)$, where $F(\rho, U) = \mathrm{Tr}(U^\dagger V\rho)$, and where ρ is the initial internal state of the particle, assuming no particle loss. The operator V, which we refer to as the coherence operator [2], is not uniquely determined by the internal state evolution channel Φ of the material [1, 2, 3], and thus provides additional information. In a second quantized description, restricted to the vacuum and single particle subspace, the action of the device or material can be described as $\widetilde{\Phi}_A \otimes \widetilde{I}_B$ where $\widetilde{I}_B$ denotes the identity channel on path B, and where $\widetilde{\Phi}_A(\widetilde{\rho}) = \Phi(\widetilde{\rho}) + V\widetilde{\rho}|0_A\rangle\langle 0_A| + |0_A\rangle\langle 0_A|\widetilde{\rho}V^\dagger + |0_A\rangle\langle 0_A|\widetilde{\rho}|0_A\rangle\langle 0_A|$ [1, 3]. Hence, the coherence operator V simultaneously determines to what extent superposition between the vacuum and single particle states is preserved, and the capacity for interference.

The additional information in the coherence operator suggests a spectroscopic procedure, recording the coherence as a function of the wavelength of the probing particle, to obtain a "coherence loss spectrum", akin to an absorption spectrum. (Not to be confused with coherence spectroscopy [11].) Here, we generalize our previous approaches to include loss in order to characterize this type of spectroscopy. Clearly, loss of the interfering particles causes a reduction of interference, but how much? We find an expression that relates the interference capacity with particle loss, but also with a third quantity that describes the creation of superposition between the vacuum and single particle states. To achieve this we need to quantify superposition [12]. Given two projectors P_0 and $P^\perp$ onto orthogonal subspaces, the function $A(\rho) = ||P^\perp \rho P_0||$ [12], where $||\cdot||$ is the standard operator norm, quantifies the superposition in a state ρ with respect to the two subspaces. In the present case, where the vacuum state results in the one-dimensional projector $P_0 = |0\rangle\langle 0|$, we find $||P^\perp \rho P_0|| = ||P^\perp \rho|0\rangle||$, where on the right hand side we have the ordinary Hilbert space norm.

Let the channel $\widetilde{\Phi}$ be "vacuum preserving", i.e., $\widetilde{\Phi}(|0\rangle\langle 0|) = |0\rangle\langle 0|$. Let $|\psi^\perp\rangle$ be a normalized element in the orthogonal complement of the vacuum state. We define three functions that characterize the action of the channel $\widetilde{\Phi}$. The first, $\mathcal{L}(\psi^\perp) = \langle 0|\widetilde{\Phi}(|\psi^\perp\rangle\langle\psi^\perp|)|0\rangle$, tells us to what extent the state $|\psi^\perp\rangle$ is mapped to the vacuum state, i.e., the degree of loss. The next function, $\mathcal{P}(\psi^\perp) = ||P^\perp\Phi(|\psi^\perp\rangle\langle 0|)P_0||$, describes how well $\widetilde{\Phi}$ preserves superposition between $|0\rangle$ and $|\psi^\perp\rangle$. Finally, $\mathcal{C}(\psi^\perp) = ||P^\perp\Phi(|\psi^\perp\rangle\langle\psi^\perp|)P_0||$ quantifies how much superposition the operation creates from the input $|\psi^{\perp}\rangle$. If $\widetilde{\Phi}$ is vacuum preserving, the following relation holds

$$\mathcal{L}(\psi^{\perp})\mathcal{P}^2(\psi^{\perp}) + \mathcal{C}^2(\psi^{\perp}) \leq \mathcal{L}(\psi^{\perp})[1-\mathcal{L}(\psi^{\perp})]. \tag{1}$$

Moreover, if $\mathcal{L}(\psi^{\perp}) = 0$, then $\mathcal{C}(\psi^{\perp}) = 0$.

To prove Eq. (1) we note that there always exists a Hilbert space $\mathcal{H}_a$ and a unitary operator $\mathbb{U}$ on $\widetilde{\mathcal{H}} \otimes \mathcal{H}_a$ such that $\widetilde{\Phi}(\rho) = \mathrm{Tr}_a(\mathbb{U}\rho\otimes|a\rangle\langle a|\mathbb{U}^{\dagger})$. By the requirement that $\widetilde{\Phi}$ should be vacuum preserving it follows that there exists a normalized $|a_0\rangle \in \mathcal{H}_a$, such that $\mathbb{U}|0,a\rangle = |0,a_0\rangle$ (where $|x,y\rangle = |x\rangle|y\rangle$). We define $|f\rangle = \mathbb{U}|\psi^{\perp},a\rangle$, and note that $\langle 0,a_0|f\rangle = 0$. We can make the following identifications: $\mathcal{L}(\psi^{\perp}) = ||\langle 0|f\rangle||^2$, $\mathcal{P}(\psi^{\perp}) = ||\langle a_0|f\rangle||$, and $\mathcal{C}(\psi^{\perp}) = ||P_0^{\perp}\mathrm{Tr}_a(|f\rangle\langle f|)P_0||$, where we keep in mind that $\langle 0|f\rangle \in \mathcal{H}_a$ and $\langle a_0|f\rangle \in \widetilde{\mathcal{H}}$. We let $P_{a_0}^{\perp}$ denote the projector onto the orthogonal complement of $|a_0\rangle$, and note that we can write $\mathcal{L}(\psi^{\perp}) = \langle f|P_0\otimes P_{a_0}^{\perp}|f\rangle$ and $\mathcal{P}^2(\psi^{\perp}) = \langle f|P_0^{\perp}\otimes P_{a_0}|f\rangle$. Finally, one can show that $\mathcal{C}^2(\psi^{\perp}) \leq \langle P_0^{\perp}\otimes P_{a_0}^{\perp}|f\rangle\langle f|P_0\otimes P_{a_0}^{\perp}|f\rangle$. We can now prove Eq. (1) by using the identity $\langle f|P_0^{\perp}\otimes P_{a_0}^{\perp}|f\rangle + \langle f|P_0^{\perp}\otimes P_{a_0}|f\rangle + \langle f|P_0\otimes P_{a_0}|f\rangle = 1$. We can interpret Eq. (1) as an exclusion principle satisfied by vacuum preserving channels; for a given level of loss the preservation of superposition and the creation of superposition are competing, one takes its maximum only if the other is zero.

The relation in Eq. (1) is valid for arbitrary vacuum preserving operations, no matter how they act on the orthogonal complement of the vacuum state (e.g., single particle states may be mapped to two-particle states). However, a clear relation between $\mathcal{P}(\psi^{\perp})$ and coherence, as measured with a single particle interferometer, requires the assumption that single particle states are not mapped outside the vacuum-single particle subspace, e.g., if $\widetilde{\Phi}$ has no gain. With this additional assumption, we can further understand Eq. (1) by considering a particle without internal degree of freedom. In this case we can explicitly construct the loss channels on the vacuum-single particle states as

$$\widetilde{\Phi}(\rho) = |0\rangle\langle 0|\rho_{00} + \sigma\rho_{11} + \gamma|0\rangle\langle 1|\rho_{01} + \gamma^*|1\rangle\langle 0|\rho_{10}, \tag{2}$$

where σ is a density operator on $\mathrm{Sp}\{|0\rangle,|1\rangle\}$, and γ is a complex number, such that $|\gamma| \leq 1$ and $\sigma_{00}|\gamma|^2 + |\sigma_{01}|^2 \leq \sigma_{00}(1-\sigma_{00})$. Hence, $\mathcal{L} = \sigma_{00}$, $\mathcal{P} = |\gamma|$, and $\mathcal{C} = |\sigma_{01}|$. One can show that the converse also holds; if σ is a density operator then $\widetilde{\Phi}$ defined by Eq. (2) is a vacuum preserving channel on the vacuum and single particle states.

How do we measure $\mathcal{L}(\psi^{\perp})$, $\mathcal{P}(\psi^{\perp})$, and $\mathcal{C}(\psi^{\perp})$? We first note that $\mathcal{L}(\psi^{\perp})$ can be measured directly from the probability of finding the vacuum in $\widetilde{\Phi}(|\psi^{\perp}\rangle\langle\psi^{\perp}|)$. If $\widetilde{\Phi}(|\psi^{\perp}\rangle\langle\psi^{\perp}|)$ stays within the vacuum and single particle subspace, then $\mathcal{P}(\psi^{\perp})$, with $|\psi^{\perp}\rangle$ a single-particle state, can be measured using a Mach-Zehnder setup, where the internal input state of the particle is $|\psi^{\perp}\rangle$. We find that the final probability to detect the particle in path A is $p_A(\chi) = \frac{1}{2} - \frac{1}{4}\mathcal{L}(\psi^{\perp}) + \frac{1}{2}|F|\cos(\arg F - \chi)$, where $F = \langle\psi^{\perp}|U^{\dagger}\widetilde{\Phi}(|\psi^{\perp}\rangle\langle 0|)|0\rangle$. Thus, the maximal visibility $\frac{1}{2}\mathcal{P}(\psi^{\perp})$ is obtained when $U|\psi^{\perp}\rangle$ is parallel to $P^{\perp}\widetilde{\Phi}(|\psi^{\perp}\rangle\langle 0|)|0\rangle$. It is difficult to see how $\mathcal{C}(\psi^{\perp})$ could be measured using an interferometric setup. However, there are other means to determine this quantity, at least if the particle is a photon. There are experimental techniques [13] to determine the probability $p_{\chi} = \langle\chi|\sigma|\chi\rangle$, where $|\chi\rangle = (|0\rangle + e^{-i\chi}|1\rangle)/\sqrt{2}$ for $\chi \in [0,2\pi)$, and where σ is a density operator on the vacuum and single particle states. It is thus possible to determine σ_{01} by varying χ. We can apply $\widetilde{\Phi}$ on a single particle state $|\psi^{\perp}\rangle$ and use the above technique to estimate p_{χ}, and thus determine $|\sigma_{01}|$. This approach can in principle be extended to take into account an internal degree of freedom such that we can measure $\mathcal{C}(\psi^{\perp})$.

Let us examine two examples. First, consider a beam-splitter with transmissivity $\cos^2\theta$, coupling the mode of interest with an ancillary mode initially in the vacuum state. The resulting channel $\widetilde{\Phi}_{\mathrm{bs}}$ on vacuum-single particle subspace of the mode of interest is as in Eq. (2) with $\sigma = \sin^2(\theta)|0\rangle\langle 0| + \cos^2(\theta)|1\rangle\langle 1|$ and $\gamma = \cos(\theta)$. We have no superposition creation, $\mathcal{C} \equiv |\sigma_{01}| = 0$, and $|\gamma|$ takes its maximal value for a given degree of loss. As a second example we let the beam-splitter be either totally transparent ($\theta = 0$) with probability p or totally reflective ($\theta = \pi/2$) with probability $1-p$. On average we obtain a channel $\widetilde{\Phi}_{\mathrm{rand}}$ as in Eq. (2), with $\sigma = p|1\rangle\langle 1| + (1-p)|0\rangle\langle 0|$ and $\gamma = p$. For comparison we let $p = \cos^2(\theta)$, yielding the same σ as previously, but $\gamma = \cos^2(\theta)$. Thus, for the same level of particle loss, $\widetilde{\Phi}_{\mathrm{rand}}$ preserves less coherence than $\widetilde{\Phi}_{\mathrm{bs}}$. In Ref. [6] (with a potentially confusing terminology mismatch with this Letter), "deterministic" absorption using a beam chopper is equivalent to $\widetilde{\Phi}_{\mathrm{rand}}$, and "stochastic" absorption by the foil absorber we associate with $\widetilde{\Phi}_{bs}$.

Beam splitters are examples of linear optics. Here we consider the more general question of which vacuum preserving channels can be obtained using linear optics only. We consider K bosonic system modes $\mathcal{A}_1,\ldots,\mathcal{A}_K$ (corresponding, e.g., to an internal degree of freedom of the particle) with annihilation operators $\boldsymbol{a} = (a_1,\ldots,a_K)$, and J ancillary modes $\mathcal{B}_1,\ldots,\mathcal{B}_J$ with annihilation operators $\boldsymbol{b} = (b_1,\ldots,b_J)$. On the total system we assume linear optics [14], which we describe with a unitary operator $\mathbb{U}$ such that

$$\mathbb{U}^{\dagger}\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}\mathbb{U} = \boldsymbol{S}\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix}\boldsymbol{S}^{(11)} & \boldsymbol{S}^{(12)}\\ \boldsymbol{S}^{(21)} & \boldsymbol{S}^{(22)}\end{bmatrix}, \tag{3}$$

where $\boldsymbol{S}$ is a unitary matrix. We wish to find all vacuum preserving channels $\widetilde{\Phi}(\rho) = \mathrm{Tr}_e(\mathbb{U}\rho\otimes\eta\mathbb{U}^{\dagger})$, where η is an arbitrary but fixed density operator on the ancillary modes. Let $\boldsymbol{S}^{(12)} = \boldsymbol{V}\boldsymbol{D}\boldsymbol{W}^{\dagger}$ be a singular value decomposition, i.e., $\boldsymbol{V}$ and $\boldsymbol{W}$ are unitary matrices, and $\boldsymbol{D}$ is such that $\boldsymbol{D}_{ll'} = d_l\delta_{ll'}$ for $1 \leq l,l' \leq \min(K,J)$ and zero

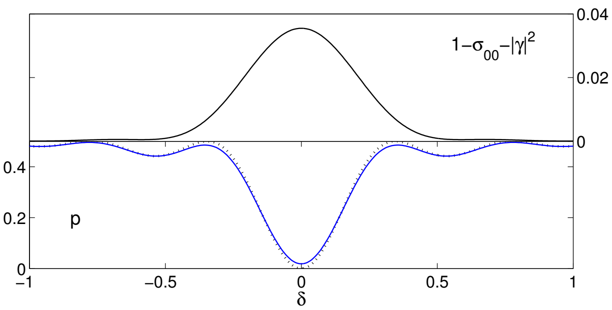

FIG. 1: (Color online) A photon interacting with a dephasing two-level atom via the Jaynes-Cummings model. The solid (dotted) line in the lower panel depicts the probability that a photon is found in the field, as a function of the detuning $\delta = \omega - \omega_a$, at time $t = \pi/(2g)$ with (without) dephasing. The upper panel shows the excess coherence loss $1 - \sigma_{00} - |\gamma|^2$ as a function of detuning δ in the case with dephasing. Neither the pure Jaynes-Cummings model, nor a simple relaxation model for the atom, gives any excess coherence loss.

otherwise, and $d_l \geq 0$. We transform to new modes $\overline{\mathcal{A}}$ and $\overline{\mathcal{B}}$, with annihilation operators $\overline{\boldsymbol{a}} = \boldsymbol{V}^\dagger \boldsymbol{a}$ and $\overline{\boldsymbol{b}} = \boldsymbol{W}^\dagger \boldsymbol{b}$, respectively. We find that $\mathbb{U}^\dagger \overline{\boldsymbol{a}} \mathbb{U} = \boldsymbol{V}^\dagger \boldsymbol{S}^{(11)} \boldsymbol{a} + \boldsymbol{D}\overline{\boldsymbol{b}}$. (Note $\boldsymbol{a}$, not $\overline{\boldsymbol{a}}$, on the right hand side.) Since the channel $\widetilde{\Phi}$ is supposed to be vacuum preserving it follows that $\mathrm{Tr}[\overline{n}_k \widetilde{\Phi}(|0\rangle\langle 0|)] = 0$, where $\overline{n}_k = \overline{a}_k^\dagger \overline{a}_k$. Without loss of generality we may assume $\eta = |\psi\rangle\langle\psi|$, yielding $\langle 0, \psi | \mathbb{U}^\dagger \overline{n}_k \mathbb{U} | 0, \psi\rangle = d_k ||\overline{b}_k|\psi\rangle||^2$, and hence $d_k \overline{b}_k |\psi\rangle = 0$. Now we consider the action of the channel $\widetilde{\Phi}$ on the vacuum and single particle states and use the above results to find:

$$\begin{aligned} \widetilde{\Phi}(\rho) &= |0\rangle\langle 0| \mathrm{Tr}[(\hat{1} - S^\dagger S)\rho] + S\rho S^\dagger \\ &\quad + S\rho|0\rangle\langle 0| + |0\rangle\langle 0|\rho S^\dagger, \end{aligned} \tag{4}$$

where $S = \sum_{l,l'=1}^{J} |\mathcal{A}_l\rangle \boldsymbol{S}_{ll'}^{(11)} \langle \mathcal{A}_{l'}|$, and where $|\mathcal{A}_l\rangle$ denotes the single particle excitation in mode $\mathcal{A}_l$. A necessary and sufficient condition for the channel in Eq. (4) to be obtainable via linear optics is that $SS^\dagger \leq P_1$ and $S^\dagger S \leq P_1$, with P_1 the projector onto the single-particle subspace. One can see that $\mathcal{P}^2(\psi^\perp) = 1 - \mathcal{L}(\psi^\perp)$ and $\mathcal{C}(\psi^\perp) = 0$. Hence, the coherence preservation is maximal relative to the degree of particle loss, and there is no superposition creation. Note that excessive coherence loss is obtained if we form convex combinations of linear optics channels, i.e. by random selection of "pure" linear optics channels. We can also conclude that, even at the level of vacuum and single particle states, the linear optics channels form only a subset of all possible channels.

Now we turn to the question of coherence loss spectra, and as a model system we consider a photonic mode and a two-level atom interacting via a Jaynes-Cummings Hamiltonian [15]. On the relevant subspace we can write the effective total Hamiltonian as

$$H = \frac{\omega}{2}\sigma_z \otimes \hat{1}_a + \frac{\omega_a}{2}\hat{1} \otimes \sigma_z^a + g(|01\rangle\langle 10| + |10\rangle\langle 01|), \tag{5}$$

where ω is the photon energy, ω_a the excitation energy of the atom, and g the coupling constant, all in units of some suitable reference energy $\mathcal{E}$, where we assume $\hbar = 1$. If the atom initially is in the ground state then the resulting channel on the vacuum and single particle subspace of the field is such that $|\sigma_{01}| = 0$ and $|\gamma|^2 = 1 - \sigma_{00}$. Hence, there is neither generation of superposition nor excessive coherence loss. We now assume the atom is affected by an environment, modeled with the master equation $\frac{d}{dt}\rho = -i[H,\rho] + q\mathcal{Q}(\rho)$, where $q \geq 0$, and where the time-parameter t is in units of $\mathcal{E}^{-1}$. We consider two cases

$$\begin{aligned} \mathcal{Q}_r(\rho) &= |0\rangle_a\langle 1|\rho|1\rangle_a\langle 0| - \frac{1}{2}|1\rangle_a\langle 1|\rho - \frac{1}{2}|1\rangle_a\langle 1|\rho, \\ \mathcal{Q}_d(\rho) &= -\frac{1}{4}[\sigma_z^a, [\sigma_z^a, \rho]], \end{aligned} \tag{6}$$

where $\mathcal{Q}_r$ gives relaxation to the ground state of the atom, and $\mathcal{Q}_d$ gives dephasing in its eigenbasis. For both these cases $\sigma_{01} = 0$. Hence, we can take $1 - \sigma_{00} - |\gamma|^2$ as a measure of excess coherence loss. One can show that for the relaxation model there is no excessive coherence loss, but there is for dephasing. We let $g = 0.1$, $\omega_a = 1$, and $q = 0.01$. The upper panel of Fig. 1 shows the excessive coherence loss $1 - \sigma_{00} - |\gamma|^2$ as a function of the detuning $\delta = \omega - \omega_a$, at $t = \pi/(2g)$. In the lower panel the solid (dotted) line depicts the probability p to find a photon in the field as a function of the detuning delta δ, at the same t with (without) dephasing. The excessive coherence loss spectrum distinguishes dephasing from the pure Jaynes-Cummings interaction and relaxation, and hence may discriminate atom-environment couplings.

So far, all examples have resulted in $\mathcal{C} = 0$. Here we show how $\mathcal{C} \neq 0$ can occur. If we can create the superposition $|\chi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, then the channel $\widetilde{\Phi}(\rho) = |0\rangle\langle 0|\rho|0\rangle\langle 0| + |\chi\rangle\langle\chi|\langle 1|\rho|1\rangle$ can be generated. We measure the particle number in the mode: if we measure vacuum we do nothing, otherwise we prepare $|\chi\rangle$. The measurement destroys any initial superposition, but as Eq. (1) shows, this is the price we have to pay for maximal superposition creation. Whether it is possible to generate the superposition $|\chi\rangle$ depends on the nature of the particle. If the particle is a photon then the superposition can be generated using, e.g., photon blockade [16]. More generally, if the particle is related to a super selection rule (SSR) [17], a reference frame (if such is available) can be used to locally break the SSR [18], which enables generation of the superposition.

The following Hamiltonian of a photon and three level atom interaction results in a nontrivial superposition creation spectrum:

$$\begin{aligned} H &= \frac{\omega}{2}\sigma_z \otimes \hat{1}_a + \hat{1} \otimes (\omega_0|0\rangle\langle 0| + \omega_1|1\rangle\langle 1| + \omega_2|2\rangle\langle 2|) \\ &\quad + \sum_{k,k':k>k'} g_{k,k'}(\sigma_- \otimes |k\rangle\langle k'| + \sigma_+ \otimes |k'\rangle\langle k|), \end{aligned} \tag{7}$$

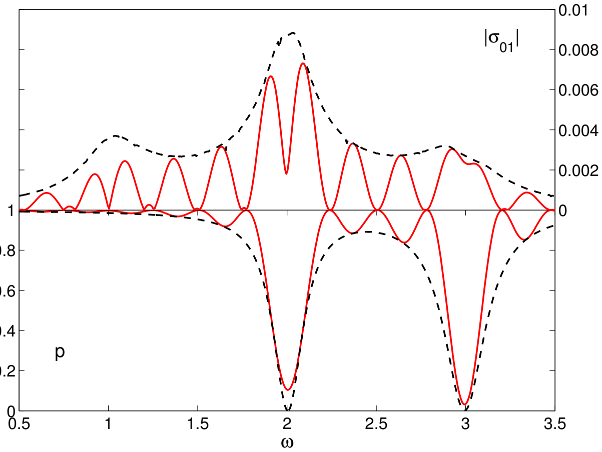

FIG. 2: (Color online) A photon interacting with a three-level atom, according to the Hamiltonian in Eq. (7). The solid line in the lower panel shows the probability p to detect the photon at time $t = 25$ as a function of the photon energy ω measured in units of a reference energy $\mathcal{E}$. The solid line in the upper panel shows the amount of superposition $|\sigma_{01}|$ between the vacuum and a single photon at $t = 25$ (in units of $\mathcal{E}^{-1}$) as a function of ω. The dashed line in the upper (lower) panel is obtained by at each ω take the maximum (minimum) of $|\sigma_{01}|$ (p) over a long evolution time.

where $\sigma_- = |0\rangle\langle 1|$ and $\sigma_+ = |1\rangle\langle 0|$, and where ω, ω_k, and $g_{k,k'}$ are in units of $\mathcal{E}$. Initially we have a single photon and the atom in its ground state. To illustrate the effect we choose $\omega_0 = 5$, $\omega_1 = 7$, $\omega_2 = 8$, $g_{01} = 0.05$, $g_{02} = 0.07$, and $g_{12} = 0.08$. In Fig. 2 the solid lines give the probability p to find a photon (lower panel) and the superposition creation $|\sigma_{01}|$ (upper panel) as functions of ω at $t = 25$. In the upper panel the dashed line is the maximum of $|\sigma_{01}|$ at each ω, taken over a long evolution time, thus approximating the envelope of the evolution of the $|\sigma_{01}|$. In the lower panel the dashed line similarly depicts the minimum of p. As expected, there are two lines in the absorption spectrum corresponding to the two transitions from the ground state. The superposition creation spectrum, however, shows a peak also at the transition between the two excited states. Further investigations are needed to understand the significance of this type of spectrum.

In conclusion we consider the coherence of single particles under particle loss. Channels with no gain are not only characterized by the loss they cause, but also by how well they preserve coherence, and their tendency to create superposition between vacuum and single particle states. We find an inequality relating these quantities. This characterization of loss processes suggests a generalized spectroscopic approach where we record a "coherence loss spectrum" and a "superposition creation spectrum", akin to absorption spectra. We illustrate these concepts with examples. Although the vacuum preservation condition simplifies the analysis, the notions of coherence loss spectra and superposition creation spectra are not limited to this setting. However, the general case requires a more extensive analysis, and will most likely lead to richer phenomena. A comparison with coherence spectroscopy in dissipative media [11, 19], and coherent control techniques in general [20], may also be fruitful.

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