Multi-objective optimization of hyperelastic material constants: a feasibility study

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ABSTRACT: A preliminary study of the use of multi-objective optimization methods in determining the material constants of a hyperelastic material model is presented. Classical experimental data is fitted to the third-order Ogden and the third-order Yeoh model hyperelastic material models through finite element analysis and multi-objective optimization, using the Simulia products Abaqus and Isight, respectively. The resulting hyperelastic material constants for both material models provide an improved curve-fit over the integrated fitting tool within Abaqus, in terms of average absolute difference (error). This indicates that this approach is suitable for development of an efficient and effective method for experimental characterization of rubber-like materials using both homogeneous and inhomogeneous experimental results.

1 INTRODUCTION

The ability to accurately simulate a component's behavior allows the engineer to gain a greater understanding of its expected service life. To do this successfully, the conditions the component is exposed to (structural and/or thermal) must be known, along with the corresponding material properties. In the case of rubber components, the description of a given material requires independent characterization due to the variation in properties resulting from chemical composition, processing and manufacturing methods. In addition, the stress-strain response of rubber is highly nonlinear and dependent on the applied mode of deformation, requiring measurement of its multi-axial response.

Hyperelastic material models are widely used in simulation or mathematical descriptions of the static response of rubber. These may have a micro-mechanical or phenomenological bases. Although the former is physical in its formulation, both approaches typically require the same degree of testing to calibrate the response. Compressibility effects may be taken into account by partitioning the deformation gradient into isochoric and deviatoric components, which requires an additional confined compression experiment. Models of more complex isothermal behaviors experienced by rubbers, such as the Mullins effect with permanent set and induced anisotropy (Diani, et al., 2009), hysteresis and visco-elasticity (Bergstrom & Boyce, 1999), are usually based on hyperelastic models. The remainder of this paper's content will study the isothermal and quasi-static material response of rubber and a novel method to obtain hyperelastic constants from experimental data.

The most common method of characterizing the static and incompressible response of rubber is through performing three homogeneous tests: uniaxial tension (UT), planar tension (PT) (equivalent to pure shear) and equibiaxial tension (ET). These tests were popularized by Treloar on 8% Sulphur rubber (Treloar, 1944) who collected the equibiaxial tension, or 2-dimensional extension, data with the use of an inflated rubber sheet. Uniaxial and planar tension data can both be collected using cut rubber sheets of simple geometry and a commonplace uniaxial testing machine. However, equibiaxial testing requires some form of bespoke testing equipment usually in the form of simultaneous loading of perpendicular axes within a 2D plane or perpendicularly applied inflation and punch tests.

Due to the difficulty of applying equibiaxial loading and gaining accurate results, several different methods have been developed to obtain this data. One method is to adapt a uniaxial testing machine with a device that translates a controlled ratio of the applied vertical force horizontally (Brieu, et al., 2006). The difficulty in this method is in achieving a homogeneous equibiaxial response, as the chosen geometry of the sample will affect the degree of biaxiality (Seibert, et al., 2014). Alternatively, a load can be applied radially to a circular sample using a complex pulley system. Other methods use inhomogeneous deformations and advanced imaging techniques to extract the required equibiaxial material response (Sasso, et al., 2008).

In this study, a multi-objective optimization procedure was developed to determine whether hyperelastic material constants can be obtained by an alternative method. Standard homogeneous tests are represented by simple finite element representations for use in the optimization process to obtain the optimal hyperelastic constants. If successful, this method will be extended to efficiently gain optimized material constants using simple inhomogeneous test data and equivalent finite element simulations.

2 OPTIMIZATION METHODOLOGY

2.1 *Material data and modelling*

Treloar's data for UT, PT & ET tests from (Steinmann, et al., 2012) were adopted for use in this optimization procedure. This data set is commonly used as a benchmark in the development and validation of hyperelastic material models.

Comparison studies investigating several material models in terms of the best fit to the entire data set (Marckmann & Verron, 2006) and the best fit to all data sets using data from a single test (Steinmann, et al., 2012, Hossain & Steinmann, 2013) indicate that the micro-mechanical extended-tube model (Kaliske & Heinrich, 1999) performed best overall. The best phenomenological model was the third-order Ogden model (Ogden, 1972) with six material constants, alhough this model is incapable of predicting other data sets when only one test's data was used.

Ideally, the extended-tube and third-order Ogden material models would therefore be used in this study, however, the extended-tube model was not available at the time of study and required a UHYPER or UMAT subroutine for its implementation. Therefore, the third-order Ogden model was used along side a third-order Yeoh model (Yeoh, 1993), which is a hybrid micro-mechanical model with a phenomenological adjustment. Since the number of coefficients is proportional to the time required for the optimization process, with only three material constants, the third-order Yeoh model provided an initial insight into the feasibility of the chosen optimization methods and a suitable comparison.

2.2 *Finite element modelling*

For each mode of deformation, an Abaqus input script was generated with only a single millimeter cubed element. Within this script, the hyperelastic constants were parameterized such that they could be easily identified and controlled by the optimizer. Displacement boundary conditions were applied for each variation to the appropriate nodes, shown for the case of equibiaxial tension in Figure 1. With displacements applied in millimeters, the nodal force reaction was extracted along with the nodal displacement to give stress and strain results in MPa and mm/mm respectively. Though the extracted force from a single node gives only a quarter of the actual stress value, the matched data was consequently quartered for comparison.

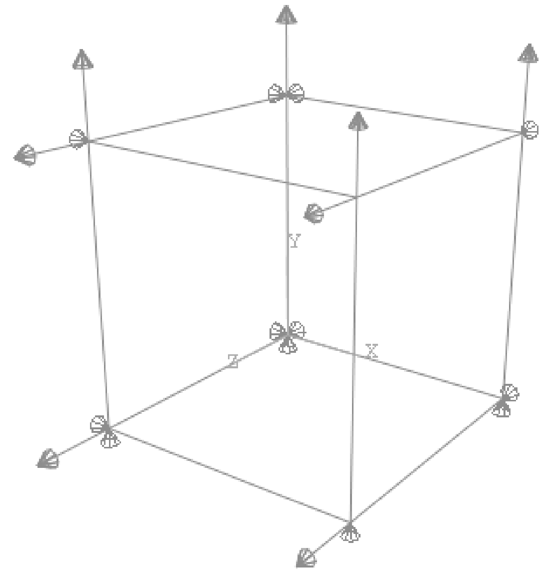

Figure 1. Equibiaxial Tension boundary conditions for the single element simulated model

2.3 *Optimization method*

The optimization procedure is performed automatically using a combination of two Simulia products: Abaqus and Isight. Abaqus is used for the simulation of the input set of hyperelastic constants using the simple finite element models, discussed in section 2.2. Isight is responsible for assessing and directing the input of the hyperelastic constants based on the results of the comparison of the error between the finite element results and the experimental data sets. As there are multiple data sets, one for each mode of deformation, the optimization process is therefore multi-objective and a weight function is used to ensure that the error of each data set's magnitude is normalized. For all optimizations, the absolute difference (error) function is used.

Prior to optimization, Abaqus' evaluation tool was used for comparison to gain material constants for the selected hyperelastic model. This tool requires only the input of the experimental data sets and the selection of the material models to be evaluated. Abaqus then generates the 'optimal' hyperelastic constants using linear least squares method or a Levenberg-Marquardt algorithm for the Yeoh and Ogden curve fits respectively. The stability is then checked for uniaxial, planar and equibiaxial deformation in tension and compression within the nominal strain range: $-0.9 \leq \varepsilon \leq 9.0$, using the Drucker stability criterion. (Abaqus, 2016)

In the selection of the optimization method, two distinct variations were used: the first used initial guesses based on the values obtained by the Abaqus evaluation and employed an optimization algorithm; the second method used trivial starting points and wide-ranging bounds with a hybrid optimization-exploratory algorithm. In the former, the Hooke-Jeeves algorithm (Hooke & Jeeves, 1961) was used due to its ability to find local minima. As for the latter, the parallel Pointer-2 algorithm (Van der Velden & Koch, 2010) was selected. Pointer-2 is an algorithm that controls four optimisation methods in serial or in parallel, ensuring that the design space is explored and most, if not all, minima are found within it. The Isight optimisation process for the Hooke-Jeeves method is shown in Figure 2.

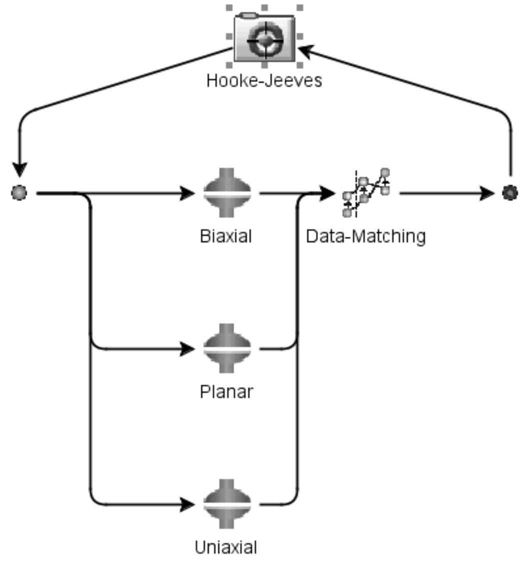

Figure 2. Hooke-Jeeves multi-objective optimization within Isight

3 OPTIMIZATION RESULTS

For both material models, an optimization using each method was attempted. In the case of the Hooke-Jeeves optimization, which is somewhat dependent on the chosen starting point, the initial material constants were chosen to within one decimal point of the evaluated constant from Abaqus. As for the upper and lower bounds, the nearest whole integer was used rounding upwards and downwards respectively. In the Pointer-2 optimization, the starting points were chosen arbitrarily and the constants were bound by a wide range.

The solutions found to be optimal for the fit of all data sets were compared in terms of the absolute difference (error) function used for the optimization. Stress-strain graphs were plotted to visually assess their fit to the material data. In the presentation of the results, the following abbreviations are used: Error: absolute difference (error), Abq: Abaqus evaluated results, H-J: Hooke-Jeeves and P-2: Pointer-2. Other symbols used are the common notations for the hyperelastic material constants or have been previously defined.

3.1 *3rd-order Yeoh*

Given that the third-order Yeoh model was not ranked as a highly accurate model in the referenced comparison studies, it was not expected to achieve as close a fit as the Ogden model. This was observed to be the case in the initial Abaqus evaluation of the material models. The Hooke-Jeeves model was capable of gaining a better fit to the data, in terms of average error and the Pointer-2 optimization gained solutions with a significantly smaller error value. Solutions for the Pointer-2 and Hooke-Jeeves optimizations are shown in Table 1, along with the Abaqus evaluated results. The Drucker stability check revealed that all sets were stable for the assessed nominal strain range.

Table 1. Hyperelastic constants and absolute difference (error) for third-order Yeoh optimizations

Yeoh:	Abq	H-J	P-2
c10	0.1852	0.1740	0.2032
c20 (E-3)	-1.449	-3.242E-3	-2.786
c30 (E-5)	3.973	2.460	6.847
Average Error	1.407	1.298	1.097
ET Error	2.367	0.8590	1.767
PT Error	0.4686	1.582	0.6573
UT Error	1.386	1.453	0.8663

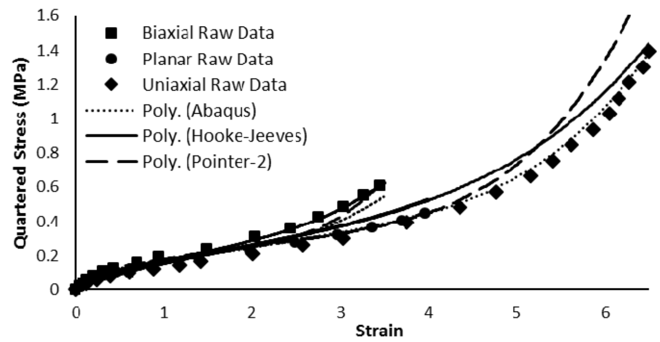

Figure 3. Third-order Yeoh quartered stress-strain plot

The test data alongside the results of the fitted curves are shown in Figure 3. Sixth-order polynomial functions have been used to plot the Abaqus and optimization results for clarity. It can be seen that the Abaqus evaluated Yeoh constants seem to produce a visually better fit than the Pointer-2 optimization, regarding their uniaxial behavior. However, the Pointer-2 data was found to be mathematically the better fit in terms of the chosen error function for uniaxial tension and the overall average error. Different error functions will be investigated in future studies.

3.2 *Third-order Ogden*

The third-Ogden model was expected to capably achieve a solution for the simpler Hooke-Jeeves method. However, the Pointer-2 method was less likely to achieve the desired result due to the vast number of numerical combinations with six coefficients. After running the Pointer-2 optimization for a 24 hour period, the program was stopped and the best result was taken. As can be seen in Table 2, the result is significantly worse than the Abaqus and Hooke-Jeeves results, in terms of absolute difference (error), for all data sets. This set was also found to be unstable in uniaxial tension greater than $\varepsilon = 3.08$ and, the equivalent deformation mode, in biaxial compression less than $\varepsilon = -0.505$. The other sets were completely stable over the assessed range.

Table 2. Hyperelastic constants and absolute difference (error) for third-order Ogden optimizations

Ogden:	Abq	H-J	P-2
μ1	0.3829	0.3875	0.9232
α1	1.452	1.463	1.382
μ2 (E-3)	1.309	1.006	25.92
α2	5.489	5.587	1.490
μ3 (E-2)	1.545	1.258	-54.33
α3	-1.875	-1.963	0.591
Average Error	0.1954	0.1239	2.042
ET Error	0.1583	0.1205	3.174
PT Error	0.1213	0.1531	1.466
UT Error	0.3066	0.0982	1.488

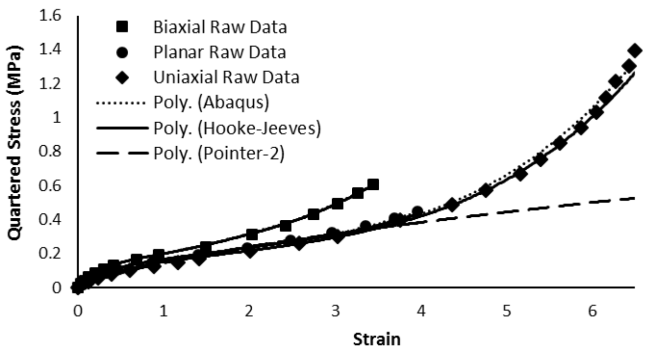

Figure 4. Third-order Ogden quartered stress-strain plot

The stress-strain results of the third-Ogden optimization are shown in Figure 4. As previously, the optimization results are plotted using sixth-order polynomials. Both the Abaqus and Hooke-Jeeves results are a good fit of the entire data set. However, as suggested by the numerical error, the Pointer-2 set did not manage to gain a suitable set of coefficients within the prescribed time. If run for long enough, a comparable or better result would be obtained. However, the required time for a solution would not be a feasible alternative means for material characterization.

3.3 *Observations*

The third-order Ogden model produced a significantly better fit of the data than the third-order Yeoh model. However, with six coefficients, the Ogden model did not gain a suitable solution with the exploratory method. When viewing the sets of coefficients produced within the Pointer-2 optimization, the constants of the Ogden model were found to fluctuate significantly for solutions with similar magnitudes of error. This demonstrates that a unique set of constants may not exist for the third-order Ogden model for this data set due to it being phenomenological in nature, which is in agreement with Ogden et al (Ogden, et al., 2004). This is shown in Figure 5, where the magnitude of the third-order Ogden coefficients are plotted for five solutions with similar error. This suggests that, where an approximate initial guess is not known, mechanically based models with fewer coefficients are preferable.

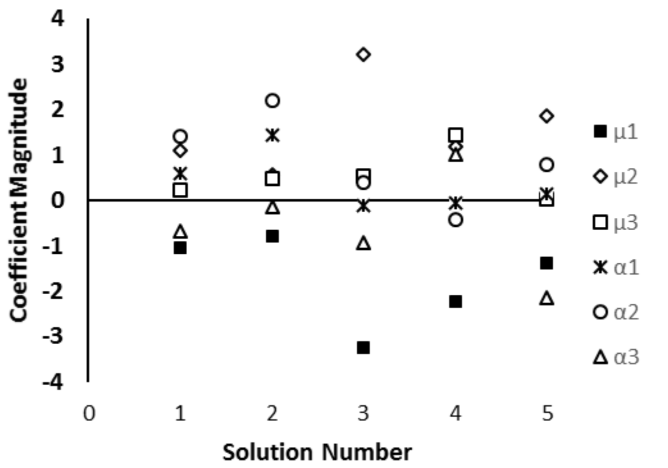

Figure 5. Third-order Ogden coefficients plotted for five optimization results with similar error

4 NOVEL MATERIAL CHARACTERISATION METHOD

Using the results of this feasibility study, it is proposed that this method could be utilized in an alternative means of characterizing rubber-like materials. This alternative method would take the results of homogeneous and inhomogeneous experiments and, using multi-objective optimization, find the optimal set of material constants to fit all experimental data sets. Simple uniaxial compression tests can achieve equivalent results to equibiaxial tension, with the assumption that rubber is incompressible, but these tests require negligible friction. By using the results from a bonded compression test and an equivalent simulated experiment within a multi-objective optimization, it is hypothesized that the equivalent material constants would be discovered.

This method, if successful, could provide an alternative to bespoke equibiaxial testing equipment when gathering the multi-axial response for rubber-like materials. The data to capably characterize the incompressible, static response is hypothesized to use three tests: uniaxial tension, planar tension and bonded uniaxial compression tests. The benefit of these tests is that they can all be performed using the same uniaxial testing machine, provided it is capable of producing the required loads in both tension and compression. Additionally, the required specimens are of simple geometry and easy to manufacture.

The accuracy of the method is largely dependent on both the experimental results and finite element results. The finite element results will be of approximately comparable accuracy to the material model's limitations, provided that finite element phenomena are appropriately considered, notably mesh convergence and volumetric-locking in this instance. However, the experimental error may be increased due to the inclusion of inhomogeneous tests. These may require significantly more cycling before a consistent response is attained, due to the propagation of stress-softening through the specimen, owing to the Mullin's effect. Also, the strain-rate will be somewhat variant throughout the material and will require consideration for the different tests to be consistent in this regard.

In order to validate this method, it will be important to use material models to generate simulated equibiaxial data for comparison to equibiaxial data gathered in a more conventional form. Also, it will be necessary to use compression specimens of different diameter to provide further validation and demonstrate repeatability.

5 CONCLUSIONS

Using Treloar's data and two multi-objective optimization techniques, the feasibility of integrating these techniques in an alternative method for material characterization has been investigated. The results of this study have found that a suitable initial value and approximate bounds for the hyperelastic coefficients is of significant importance. Also, higher-order phenomenological models are expected to be less appropriate unless an optimization process that exploits local minima is used.

In gaining optimized constants, the Hooke-Jeeves method was more efficient than the Pointer-2 method, as would be expected. However, the time taken to gain a solution using either multi-objective optimization method is substantially longer than the Abaqus evaluation tool. A significant portion of the time required is spent in 'housekeeping' tasks within Abaqus, some examples are: accessing the license server, writing the results files and reading output database files. For this type of optimization, the simple homogeneous deformations of the unit-cube models could be more efficiently simulated in a purely mathematical form.

6 ACKNOWLEDGEMENTS

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