

Hall effect and geometric phases in Josephson junction arrays

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Abstract

Since effectively the local contact vortex velocity dependent part of the Magnus force in a Josephson junction array is zero in the classical limit, we predict zero classical Hall effect. In the quantum limit because of the geometric phases due to the finite superfluid density at superconductor grains, rich and complex Hall effect is found in this quantum regime due to the Thouless-Kohmoto-Nightingale-den-Nijs effect.¹

Since vortex dynamics is identical to that of an electron in the presence of a magnetic field, numerous models for the quantum Hall effect in both homogeneous and inhomogeneous superconductor films have been proposed.¹

² Those models have fully explored the analogy to the quantum Hall effect in semiconductor heterojunctions by treating vortices moving in a uniform magnetic field with a homogeneous background. While the treatment can be justified in a homogeneous superconductor film, it may not be so in inhomogeneous cases such as in Josephson junction arrays, where both the (fraction part) number of magnetic flux, the frustration n , and the (fraction part) of the fictitious magnetic flux ϕ_0 per plaquette are usually large, and the periodic potential for a vortex is strong. Therefore the simple continuous limit without

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the accounting of the potential is not appropriate for a Josephson junction array.

In a Josephson junction array, because of the huge core energy, vortices cannot move into the superconducting grains. They are confined to move along the junctions and the voids (nonsuperconducting areas), an example of the guided vortex motion. Since the vortex velocity part of the Magnus force is proportional to the local superfluid density, derivable from the nonlinear Schrödinger Lagrangian formulation², this force is zero for a vortex at a void, and, exponentially small at a junction. Furthermore, because of the guided motion, even the small transverse force at a junction does not produce a sideways motion. This implies that this local contact transverse force does not play a role in vortex dynamics in a Josephson junction array. Hence there is no Hall effect in the classical limit. This absence of the *en route* transverse force is in agreement with experimental observations³. The condition for the classical limit will be given at the end of the paper.

In the quantum regime, however, vortices experience geometric phases similar to the Aharonov-Bohm effect, due to the finite superfluid density at superconductor grains.⁴ To be commensurate with the existence of the vortex inaccessible regions and the geometric phases, we consider the tight-binding limit of vortex motion. The corresponding Hamiltonian may be written as

$$H = t \sum_{(l,j)} a_l^\dagger a_j e^{iA_{lj}} + \sum_{l,j} a_l^\dagger a_l V_{lj} a_j^\dagger a_j, \quad (1)$$

where a_l is the boson annihilation operator for a vortex at l -th void, and $\sum_{(l,j)}$ stands for the summation over nearest neighbors. The phase A_{lj} is defined on the links connected the nearest neighbors, and its sum around a plaquette is equal to the geometric phase $2\pi\phi_0$: $\sum_{\text{plaquette}} A_{lj} = 2\pi\phi_0$. A uniform geometric phase in a square lattice will be assumed, where the number of

‘fluxes’ ϕ_0 is the number of Cooper pairs on a superconductor grain, which may be controlled by a gate voltage. The interaction between vortices is described by V_{lj} , which is long range and repulsive. We will treat it as a short range repulsive interaction for a first approximation, further approximated by the hard-core conditions. The tunneling matrix element t is,

$$t \simeq \sqrt{E_J E_C} \exp\{-O(1)\sqrt{E_J/E_C}\}, \quad (2)$$

where E_J is the Josephson junction energy and E_C the junction charging energy⁵.

To discuss the Hall effect of the idealized vortex problem in the quantum regime, we map the hard-core boson problem onto a fermion problem by attaching odd number of ‘fluxes’ on each vortex. The resulting Hamiltonian for the fermion problem is

$$H = t \sum_{(l,j)} c_l^\dagger c_j e^{i[A_{lj} + \mathcal{A}_{lj}]}, \quad (3)$$

where c_j is the corresponding the fermion annihilation operator at the j -th void. The number of statistical fluxes ϕ_s at the j -th void satisfies the constrain $\phi_s = -(2m+1) \langle c_j^\dagger c_j \rangle$, with $\sum_{plaque\text{tte}} \mathcal{A}_{lj} = 2\pi\phi_s$, which means that $2m+1$ fluxes have been attached to each vortex.⁶ If this mapping gives a mean field solution with an energy gap separated from its excitations, the statistical fluxes can be adiabatically smeared over the lattice and effectively detached from vortices. In this case $\phi_s = -(2m+1)n$, with n is the magnetic flux frustration, the number of vortices per plaquette. Then the resulting mean field problem is exactly the Harper-Azbel-Wannier-Hofstadter problem, where energy gaps do exist. The quantum Hall behaviors of such a problem have been studied in detail by Thouless, Kohmoto, Nightingale, and den Nijs⁷. For such a system the quantum Hall conductance σ_H^f is $\sigma_H^f = t_r$, with the integer t_r the solution of the Diophantine equation $r = s_r q + t_r p$. Here the number of

fluxes per plaquette $\phi = \phi_0 - \phi_s = p/q$, with p and q coprime, $n = r/q$, and r , s_r , t_r integers with $|t_r| \leq q/2$. Counting the mapping generated Chern-Simons contribution to the Hall conductance, $\sigma_H^s = \frac{1}{2m+1}$, the Hall conductance of the original vortex system is then^{6,8} $1/\sigma_H^v = 1/\sigma_H^f + 1/\sigma_H^s$. Converting back into the electric Hall conductance and putting back the unit, we find the electric quantum conductance of the Josephson junction array is

$$\sigma_H = \frac{4e^2}{h} \frac{\sigma_H^f + \sigma_H^s}{\sigma_H^f \sigma_H^s}. \quad (4)$$

As known in the previous study of quantum Hall effect^{8,6} for a given set of the ‘flux’ ϕ_0 and the frustration n , there may exist several values of m , that is, several mappings, with their mean-field solutions all corresponding to filled bands which are separated from excitations by energy gaps. If such a case occurs, detailed calculation is needed to find the m with the largest energy gap, which is the most stable one.

One can check that following symmetries hold for the quantum Hall conductance σ_H : the periodicity, $\sigma_H(\phi_0, n) = \sigma_H(\phi_0 + 1, n)$; the odd symmetry, $\sigma_H(\phi_0, n) = -\sigma_H(-\phi_0, n)$; the particle-hole symmetry, $\sigma_H(\phi_0, n) = -\sigma_H(\phi_0, 1 - n)$. We note that both positive and negative Hall conductance may be easily reached, contrast to the previous proposal of the quantum Hall effect in a Josephson junction array¹. For the special mapping $2m n = \phi_0$ the mean-field solution is automatically within a gap, and the Hall conductance is $\sigma_H = 2m \frac{4e^2}{h}$. With these specific sets of ϕ_0 and n and in the zero limit of their fraction parts one can take the continuous limit of the tight-binding model.

We conclude by discussing of a criterion for the classical limit, in which there is no Hall effect. The relevant energy scale is the tunneling matrix element t . When the temperature is higher than t , thermal fluctuation will destroy the quantum coherence and the vortices move classically. The quantum

regime is realized for temperatures lower than t where the phase coherence is preserved. For a Josephson junction energy $E_J \sim 1$ K and the junction charging energy $E_C \sim E_J$, $t \sim 100$ mK. We point out that the Hall effect in the quantum regime may have been realized experimentally⁹.

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