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Mathematical Modelling the Spread of Zika and Microcephaly in Brazil.

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Abstract

In this paper we look at a non-age-structured model for the spread of the Zika Virus and Microcephaly in Brazil. We first outline the non-seasonal differential equation model, and discuss parameter values and their estimation. Then we talk about the basic reproduction number and details of the calculation of the number of Microcephaly cases. Next we estimate how the model can be made more realistic by introducing seasonality into the mosquito population. Finally we consider sensitivity of the results to the mosquito biting rate.

Key words: Zika, Brazil, Microcephaly, Aedes Aegypti mosquito, basic reproduction number, differential equation model, seasonality.
MSC 2000: AMS codes 92B15, 92C60, 92D30.

1 Extended Abstract

1.1 Non-seasonal Model

The Zika virus is spread by the same species of mosquito, namely the *Aedes Aegypti* (*A. Aegypti*), as Dengue. Zika is a member of the virus family *Flaviviridae*. The first discovery of the Zika virus was in 1947, however despite being around for a while, Zika has not received much attention until recently when it has been discovered that it is associated with Microcephaly which is a serious birth defect in newborns, caused if women are infected with Zika during pregnancy. Most importantly there is still no vaccine to prevent the Zika virus. Apart from causing severe birth defects to newborn babies, infected individuals can also experience fever, rash and joint pain. As a result, in this paper we will use an existing

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Parameter values	Biological meanings	Values
a	*A. Aegypti* biting rate	0.5×7/week [1, 5]
b	Probability of transmission of Zika when an infectious mosquito bites a susceptible human	$0.10 - 0.75$ [1]
c	Probability of transmission of Zika when a susceptible mosquito bites an infectious human	$0.30 - 0.75$ [3]
N_H	Human population in Brazil in 2015	$207,848,000$ [12]
μ_H	Per capita human mortality rate in Brazil	$1/(75 \times 52)$/week [12]
γ	Per capita human recovery rate	7/6/week [5]
μ_v	Per capita mortality rate for *A. Aegypti*	0.025×7/week [9]
N_v	*A. Aegypti* population	$1.5 \times N_H$ [4, 6]
τ	Zika extrinsic incubation period	8.2/7 weeks [5]

Table 1: Parameter values given in Equation (1).

time-delayed mathematical model for Dengue mentioned in [9] to analyse the dynamical behaviour for the Zika virus, in Brazil, as well as estimating the future expected number of cases of Microcephaly due to Zika.

The definitions of the parameter values used in the differential equation model and their corresponding values are given in Table 1. The model that we are working with is given as follows:

$$\begin{aligned}
\frac{dS_H(t)}{dt} &= -abI_v(t)\frac{S_H(t)}{N_H} - \mu_H S_H(t) + \mu_H N_H, \\
\frac{dI_H(t)}{dt} &= abI_v(t)\frac{S_H(t)}{N_H} - (\mu_H + \gamma)I_H(t), \quad \frac{dR_H(t)}{dt} = \gamma I_H(t) - \mu_H R_H(t), \qquad (1) \\
\frac{dS_v(t)}{dt} &= -acS_v(t)\frac{I_H(t)}{N_H} - \mu_v S_v(t) + \mu_v N_v, \\
\frac{dL_v(t)}{dt} &= acS_v(t)\frac{I_H(t)}{N_H} - \mu_v L_v(t) - acS_v(t-\tau)\frac{I_H(t-\tau)}{N_H}e^{-\mu_v \tau}, \\
\frac{dI_v(t)}{dt} &= acS_v(t-\tau)\frac{I_H(t-\tau)}{N_H}e^{-\mu_v \tau} - \mu_v I_v(t),
\end{aligned}$$

with initial conditions $S_H(0), I_H(0), R_H(0), S_v(0), L_v(0)$ and $I_v(0)$, where $S_H(t), I_H(t)$ and $R_H(t)$ respectively represent the susceptible, infected and recovered individuals for humans, while $S_v(t), L_v(t)$ and $I_v(t)$ respectively represent the susceptible, latent and infected mosquitoes. Note that $N_H = S_H + I_H + R_H$ denotes the total human population size and $N_v = S_v + L_v + I_v$ represents the total *A. Aegypti* population size where both populations

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are constant. Although the Zika virus and Dengue are spread by the same transmission route and thus some parameter values would remain the same, parameters such as the transmission probabilities between humans and *A. Aegypti* mosquitoes which are defined as b and c in Equation (1) may vary. Therefore one of the aims in this project is to use the least squares estimation technique and the real Zika virus data from Brazil given in [5] to estimate these two values.

We assume that a single Zika infected human enters the disease free population at some time t_0, where $t_0 < t_1$ and t_1 is the first time when we have available Zika data values in Brazil obtained from [5] as the first week in 2015. We have estimated t_0 and hence $S_H(t_1), I_H(t_1), R_H(t_1), S_v(t_1), L_v(t_1)$ and $I_v(t_1)$, used as simulation starting values, by least squares. The basic reproduction number for our delayed Zika model given in [10] is defined as

$$R_0 = \frac{ma^2bce^{-\mu_v\tau}}{\mu_v(\mu_H + \gamma)}, \tag{2}$$

where all the parameter values are defined as in Table 1. For a in the range 0.7 – 3.5/week we get R_0 in the range 1.27 – 11.01/week.

1.2 Numerical Solutions

Once all the parameter values are obtained and estimated, we use R to solve the differential equations given in Equation (1) and produced simulations which illustrate the number of susceptible, infected and recovered individuals over both a short time period, to represent the immediate future, and over a long time period, to represent what happens when the endemic equilibrium has been reached.

We focus on analysing the effect of pregnant women infected with Zika virus during their first trimester as various reports (e.g. [2, 8]) suggest that pregnant women who are infected with the Zika virus during the first trimester have a much higher risk of their babies developing Microcephaly as opposed to those who are infected with Zika in their second or third trimesters. We have obtained an estimated expected future number of cases of Microcephaly due to pregnant women infected with Zika during their first trimester both in the short and in the long term.

1.3 Model With Seasonality

It is well-known that the life cycle of *A. Aegypti* is influenced by many environmental factors such as rainfall and temperature (e.g. [7, 11, 13]). As a result, in order to fully capture the behaviour of the *A. Aegypti* mosquitoes under the influence of environmental factors and its effect on the number of Microcephaly cases, later on we decided to improve on our model by adding seasonality into the birth function of *A. Aegypti* mosquitoes. Similarly, with this seasonality model, we use the least squares estimation technique to parameter

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estimate new values of b and c, and thus calculate the future expected number of cases of Microcephaly both in the short and long term due to pregnant women being infected in their first trimester.

1.4 Results

For both models, numerical simulations are produced to illustrate the spread of the Zika virus over a period of time and the future expected number of cases of Microcephaly as a result of the Zika virus are calculated. The suggested value of $a = 3.5$/week for the parameter a, the *A. Aegypti* biting rate, is high compared to the other values in the literature. So we discuss the sensitivity of our results to different values of this parameter. We will later extend the results to an age-structured model.

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