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Control Performance Monitoring of State-Dependent Nonlinear Processes

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Abstract: This paper presents a novel approach to monitor control performance of nonlinear processes that can be described as state-dependent models (SDMs). A discrete Kalman filter (KF) is established to estimate the SDM parameters. A covariance control formulation is introduced to split the system closed-loop variance/covariance into two terms, one term to account for the minimum expected quadratic loss bound (equivalent to the minimum variance performance bound but in state space formulation), and another to account for performance deviations from the minimum variance bound. Simulation studies have been conducted on several nonlinear process systems including a cold rolling mill model with roll eccentricity and a steel making system with real time oxyfuel slab reheating furnace control data. The case study results demonstrate the computational efficiency of the proposed strategy in real time monitoring and control of systems with fast, nonlinear and time-varying dynamics.

Keywords: State-dependent model (SDM), parameter estimation, Kalman filter (KF), control performance monitoring, covariance control, steel industry

1. INTRODUCTION

Control performance assessment/monitoring (CPA/CPM) provides automated monitoring, evaluation and diagnosis of possible under-performing control systems. The assessment is normally conducted on models fitted to closed-loop operational data through the estimation of a minimum variance performance bound (MVPB) when Gaussian stochastic disturbance signals are considered. A comprehensive review of CPA technologies and their impacts in industry can be found in (Jelali, 2006).

Performance assessment of nonlinear systems remains an important and challenging topic which has not been fully investigated. One difficulty is that estimation of nonlinear model parameters from operating data is computationally demanding. Furthermore, to include statistical properties, nonlinear processes are estimated as Volterra time series. The use of Volterra time series, albeit general, can also be intractable, limiting the assessment to only special classes of nonlinear models. In the work of (Harris and Yu, 2007) and (Yu et al., 2010), for example, the performance assessment is restricted to nonlinear models with additive linear or partially nonlinear disturbances and with explicit input/output representations. A more general strategy is required for performance monitoring of nonlinear systems.

State-dependent models (SDM) are general nonlinear models with finite dimensionality, amenable to statistical analysis and relatively easy to be fitted to data with no prior assumptions about the process nonlinearity (Priestly, 1988). This motivates our use of SDM to allow the closed-loop performance information being included in the states covariance for general nonlinear systems. The extraction of MVPB from the resulting SDM model is therefore made by means of covariance control theory. The solution to the covariance control problem provides a controller which assigns a given state covariance to the closed-loop system (Skelton et al., 1998). More specifically, the covariance control algorithm can provide the MVPB within the actual closed-loop performance.

In this work, state-dependent modelling from operational data for CPM is carried out through a discrete KF since the SDM can be formulated as truncated Volterra series with state-dependent coefficients. This SDM representation is locally linear. The main advantage of using KF is that its innovations error covariance (the covariance of the difference between the available observations and their optimal estimates) contains the overall outputs variance. A performance index can therefore be formulated based on the difference between the outputs variance/covariance and the extracted MVPB. The developed index has two advantages against linear indexes, it can accommodate the time varying behaviour encountered in process industries, and can quantify the effects of nonlinearities on the systems performance. The proposed algorithm is examined with application studies to steel processing lines and other nonlinear processes. Steel processing lines are highly nonlinear, fast speed and setpoint-dependent processes where the final product depends on customer-order specifications.

The remaining paper is organised as follows. The SDM description and its parameter estimation through KF are presented in Section 2.1. Calculation of the MVPB using covariance control and the derivation of the performance

* Special Thanks to The European Commission, Research Fund for Coal and Steel, Project acronym: Cognitive Control, Proposal No: RFS-PR-09123.
index are presented in Section 3. In Section 4, simulations studies are conducted on several industrial examples. Conclusions and future work are discussed in Section 5.

2. DEVELOPMENT OF STATE-DEPENDENT MODELS

2.1 Model Description

In a discrete time setting, fitting a time series into a nonlinear model in the form of SDM can be described by the relationship between the observation, $y_t$, and the past history of the series as follows:

$$ y_t = f (y_{t-1}, \ldots, y_{t-m}, \epsilon_{t-1}, \ldots, \epsilon_{t-n}) + \epsilon_t $$

(1)

The nonlinear function $f$ describes the information of $y_t$ contained within its past history $\{y_{t-1}\}_{i=1}^m$ of order $m$, and the innovations process $\{\epsilon_{t-j}\}_{j=1}^n$ of order $n$. Any past observation, $y_{t-i}$, is, by itself a function of the innovation sequence $\{\epsilon_{t-i-j}\}_{j=1}^n$, thus giving the notion of the state vector, i.e. at time $t-1$:

$$ x_{t-1} = [\epsilon_{t-n}, \ldots, \epsilon_{t-1}, y_{t-m}, \ldots, y_{t-1}]^T $$

(2)

Here the state vector is not in the context of standard linear state space formulation, since it does not provide a minimal realisation, but rather being used as a set of quantities that contain all the necessary information about the process (Priestley, 1988). If $f$ is analytical, it can be expressed as a Volterra first-order expansion as follows:

$$ y_t = - \sum_{i=1}^{m} a_i (x_{t-1}) \epsilon_{t-i} + \epsilon_t + \sum_{j=1}^{n} g_j (x_{t-1}) \epsilon_{t-j} $$$$ + \mu (x_{t-1}) $$

(3)

where $\mu (x_{t-1})$ is the mean value of the state vector $x_{t-1}$. The coefficients $a_i$ and $g_j$ are first order derivatives of $f (x_{t-1})$ with respect to $x_{t-1}$.

To estimate the SDM parameters through a sequential algorithm such as KF, the output observation $y_t$ can be re-written as follows:

$$ y_t = x_{t-1} \Gamma_t + \epsilon_t $$

(4)

where:

$$ x_{t-1} = [1, x_{t-1}^T] $$

$$ \Gamma_t = [\mu, g_1, \ldots, g_n, -a_m, \ldots, -a_1]^T $$

For simplicity, the state vector notation $x_{t-1}$ has been omitted in vector $\Gamma_t$ and the vector $X_{t-1}$ is an extended state vector that includes an element for the mean value $\mu$. The innovations term, $\epsilon_t$, is expected to be zero mean white noise, with innovations variance $\Sigma$. such that the KF provides the optimal estimates. With this formulation, the entries of the past observations and the innovations appear as locally linear. In the presence of delays, the innovations should be regarded as independent from the feedback effects. This is a necessary condition for the formulation of the feedback invariance for systems with delays (Harris and Yu, 2007).

To obtain the MVPB, a state space approach is used. In state space form, the minimum expected quadratic loss is equal to the minimum variance from its equivalent polynomial form. Under the minimum expected quadratic loss, the expectation of output, $E \{y_t^2\}$, is equal to the expectation of the innovation term, $E \{\epsilon_t^2\}$, or to the sequence, $E \{\sum_{k=1}^{d-1} \epsilon_k^2\}$, when a controllable input is delayed with $d$ time instances. From (4), we have:

$$ E \{y_t^2\} = E \{X_{t-1} \Gamma_t \Gamma_t^T X_{t-1}^T\} + E \{\epsilon_t^2\} $$

(5)

The presence of the expectation term in (5) from the past observation data suggests that a non-minimum expected quadratic loss feedback controller has been used, which is actually more realistic from real application point of view. In practice, controllers achieving the minimum variance or the minimum quadratic loss may be difficult to implement due to physical and energy efficiency constrains. Furthermore, a well-tuned controller may be under-performing in real processes, thus leading to non-minimum design performance.

2.2 Parameter Estimation of SDM

The model parameters can be identified by fitting the closed-loop operating data to the SDM. In (Priestley, 1988), the evolution of the SDM parameters is approximated to linear functions of the difference between the elements of the extended state vector $X_t$ and the elements of its predictor $\hat{X}_t$:

$$ a_{i,t} = a_{i,t-1} + \Delta \hat{X}_{t+i+n}^T e_{i,t} $$

$$ g_{j,t} = g_{j,t-1} + \Delta \hat{X}_{t+j+1}^T \xi_{j,t} $$

$$ \mu_t = \mu_{t-1} + \Delta \hat{X}_t^T \lambda_t $$

(6)

with $\Delta \hat{X}_t = X_t - \hat{X}_t$ and $\Delta \hat{X}_t = 1$ so that the mean value can also be updated. $\epsilon, \xi$ and $\lambda$ can be random locally to preserve nonlinearity and add non-stationarity to the model, that is:

$$ \Gamma_t = \Gamma_{t-1} + \Delta \hat{X}_t^T \beta_t $$

$$ \beta_t = \beta_{t-1} + V_t $$

(7)

where $\beta_t = [\lambda_t, \epsilon_{1,t}, \ldots, \epsilon_{m,t}, \xi_{1,t}, \ldots, \xi_{n,t}]$ and $V_t$ is a sequence of compatible dimensions that contains independent random-valued elements with multivariate normal distribution. For simplicity, it is assumed that $V_t \sim N(0, \Sigma_V)$ with noise covariance $\Sigma_V$.

Combining both equations in (7), the estimation of the SDM parameters can be developed based on the following model:

$$ \Phi_t = F_t \Phi_{t-1} + W_t $$

$$ y_t = H_t \Phi_t + e_t $$

(8)

where:
For an under-performing system, the control objective now became under-performing over time, thus an unreachable system is a more general assumption for CPA. It can now be stated that poor control performance from a state space approach occurs mainly when the system is not fully reachable in a finite period of time. It could be argued that the system is not fully controllable but it could also be the case that the system was originally controllable and became under-performing over time, thus an unreachable system is a more general assumption for CPA.

3. MINIMUM PERFORMANCE BOUND

Consider that the output observation $y_t$ from (4) has a non-minimum open-loop state space realisation given by:

$$X_{t+1} = A_tX_t + B_tu_t + e_{t+1}$$

$$y_t = CX_t$$

with:

$$A_t = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \mu & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g^{op}_{m+n-1} & g^{op}_{m+n-2} & \cdots & g^{op}_{n-1} & -g^{op}_{n-2} & \cdots & -g^{op}_0 \end{bmatrix}$$

$$B_t = [b_0 \cdots b_{m+n-1} \cdots b_0]^T$$

$$C = [0 \cdots 0 \cdots 0 \cdots 1]^T$$

$$e_{t+1} = [0 \cdots e_{t+1}]^T$$

$u_t$ is the controllable input, $A_t$ and $B_t$ are matrix containing coefficients of $X_t$ and $u_t$, respectively. The superscript $op$ stands for open loop. The minimum expected quadratic loss is identical to the minimum variance only if an optimal estimator is applied over a state space model with noise-free output measurement signal as in (10) (Wark, 1990). This assumption is valid for controller design, however, the problem under study is model parameters identification. The lack of an optimal state estimator is circumvented by the fact that the optimality of the KF is added to the model state parameters and all the state vector elements are observable. Under these conditions, the closed-loop system should be able to achieve the minimum variance through the following controller:

$$u_t = -J^{MV}_t X_t$$

(11)

where $J^{MV}_t$ is the static state minimum variance feedback gain (the superscript $MV$ stands for minimum variance):

$$J^{MV}_t = (\hat{B}_t^T C^T \hat{C}_t)^{-1} \hat{B}_t^T C^T \hat{A}_t$$

$\hat{A}_t$ and $\hat{B}_t$ are optimal parameter estimates calculated by the KF. The controller in (11) also compensates the past innovations since $X_t$ is used for feedback.

It can now be stated that poor control performance from a state space approach occurs mainly when the system is not fully reachable in a finite period of time. It could be argued that the system is not fully controllable but it could also be the case that the system was originally controllable and became under-performing over time, thus an unreachable system is a more general assumption for CPA.

3.1 Covariance Control Formulation

For an under-performing system, the control objective now becomes finding a controller based on the actual closed-loop variance that contains a minimum variance bound given by the controller in (11). This control objective is feasible by means of covariance control that uses feedback control to achieve the desired system performance (Skelton et al., 1998). Assigning an exact performance is theoretically possible but rather unrealistic for CPA, alternatively the control problem is oriented towards assigning a desired performance bound. Suppose that the state covariance that characterises the actual closed-loop performance is specified in terms of a performance bound, that is:

$$\lim_{t \to \infty} E \{ y_t^2 \} < C \Pi_X C^T$$

(12)

where $\Pi_X$ is a given positive-definite matrix that stabilises the closed-loop system under controllability assumptions. The closed-loop system covariance is bounded by:

$$\Pi_X > (\hat{A} - \hat{B}J) \Pi_X (\hat{A} - \hat{B}J)^T + \Sigma_e$$

(13)

and $J$ is the state feedback gain that achieves the actual covariance. From (13), $J$ can be developed to be:

$$J = (\hat{B}_t^T Q \hat{B})^{-\frac{1}{2}} \hat{B}_t^T Q \hat{A} + (\hat{B}_t^T Q \hat{B})^{-\frac{1}{2}} L S \hat{A}$$

(14)

where $L$ is an arbitrary matrix such that $\Pi_X > (\hat{A} - \hat{B}J) \Pi_X (\hat{A} - \hat{B}J)^T + \Sigma_e$ and $J$ is the state feedback gain that achieves the actual covariance. From (13), $J$ can be developed to be:

$$J = (\hat{B}_t^T Q \hat{B})^{-\frac{1}{2}} \hat{B}_t^T Q \hat{A} + (\hat{B}_t^T Q \hat{B})^{-\frac{1}{2}} L S \hat{A}$$

(14)

where $L$ is an arbitrary matrix such that $\Pi_X > (\hat{A} - \hat{B}J) \Pi_X (\hat{A} - \hat{B}J)^T + \Sigma_e$ and $J$ is the state feedback gain that achieves the actual covariance.
Q and S are positive-definite matrices, J is a general solution to (12) and can generate required covariance bound (Skelton et al., 1998).

When \( \lim_{t \to \infty} E \{ \gamma_t^2 \} = C \Sigma_x C^T \), the second term in (14) becomes zero and \( J = J^{MV} \). Both terms from the state feedback gain can thus be re-written as:

\[
J = J^{MV} + \Delta J
\]

For CPM, \( \Delta J \) represents the change in the system’s closed-loop variance/covariance over time. Using the state feedback controller from (15) into (10), the following closed-loop state space realisation is obtained:

\[
X_{t+1} = -B \Delta J X_t + e_t
\]

\[
y_t = C X_t
\]

since \( A_t = BJ_t^{MV} \). Writing \( \alpha_t = -B \Delta J \), the model in (16) is the non-minimum closed-loop state space realisation of (4) and can be estimated from closed-loop operating data.

A by-product of using covariance control for CPM is that for structured controllers the derivation of the feedback gain can provide a control re-tuning methodology. That is, the closed-loop state covariance can be partitioned as follows:

\[
\Pi_x = \begin{bmatrix} \Pi_p^b & \Pi_p c \\ \Pi_p c & \Pi_c \end{bmatrix}
\]

where \( \Pi_p, \Pi_c \) and \( \Pi_p c \) are the plant state covariance, dynamic controller state covariance, and the covariance between plant and controller states, respectively. This approach requires measurements of the system controllable inputs and outputs. The details of this methodology and the results on PI/PID control re-tuning will be presented in a future paper.

3.2 Performance Index

A straightforward use of the Harris index can be considered for the closed-loop system performance as follows:

\[
\eta_y = \frac{\Pi^{MV}}{\Pi x}\]

where \( |\Pi^{MV}| \) is the determinant of the system minimum covariance matrix \( \Pi^{MV} \). The index lies within \([0, 1]\). In practice, the control performance can be classified as \( \eta_y < 0.5 \) (poor) \( \eta_y \in [0.5, 0.8] \) (good) \( \eta_y > 0.8 \) (optimal), and will depend on the applications at hand (Jelali, 2007).

For CPM, the variance of the innovations may deviate from the white noise variance. It is therefore practical to take the determinant of the innovations as the metric and to make the change in variance/covariance over time independent of the innovations during the estimation of the SDM. That is:

\[
y_t = \alpha_{1,t} \begin{bmatrix} X_t^1 \\ \vdots \\ X_t^{n+1} \end{bmatrix} + \alpha_{2,t} (\Delta J_t) \begin{bmatrix} X_t^{n+2} \\ \vdots \\ X_t^{m+n+1} \end{bmatrix}
\]

where \( \alpha_{1,t} = [\mu, g_1, \ldots, g_1] \), \( \alpha_{2,t} (\Delta J_t) = [-a_0, \ldots, -a_l] \) and only \( \alpha_{2,t} (\Delta J) \) is dependent on \( (\Delta J) \). This approach is cumbersome for feedback/feedforward controllers since some terms of the innovations can include the change in variance from the feedforward action, but in no sense restrictive for feedback controllers since the innovation terms only include the minimum achievable variance. When time delays are present, the feedback invariance can be extracted by setting \( n > d \).

The selection of \( m \) and \( n \) values is related to the smoothness of the system nonlinearity as mentioned in Section 2.2. In practice, these values can be set to be \( m = n \) for feedback control, or \( m < n \) for feedback/feedforward control.

4. SIMULATION STUDIES

The developed CPM algorithm is applied to several process scenarios including a simple model with nonlinear actuator, a cold rolling mill model with roll eccentricity, and a steel making system with real time oxyfuel furnace control data. The method described in (Harris and Yu, 2007) is used for performance comparison.

4.1 Actuator Nonlinearity

A typical source of nonlinear oscillations in process control is valve stiction, which is the friction a valve needs to overcome before the actuator changes its position. A simple feedback/feedforward control system with He’s model (Jelali and Huang, 2010) of valve stiction is simulated in Matlab/Simulink. The control system is given by (Desborough and Harris, 1993):

\[
y_t = u_{t-3} + \frac{1}{1 - q^{-1}} \omega_{0,t} + \frac{q^{-5}}{1 - q^{-1}} \omega_{1,t}
\]

where \( y_t \) is the process output, \( \omega_{0,t} \) and \( \omega_{1,t} \) are the unknown and known zero mean, Gaussian disturbance terms, respectively. \( q^{-1} \) is the backward shift operator. Time delays of 3s and 5s were used for the controllable input and the known disturbance, respectively. The feedback/feedforward controllers are given as:

\[
u_t = -\frac{K_{ff}}{1 - q^{-1}} \omega_{f,t} + \frac{K_{fb}}{1 - q^{-1}} (y_{sp} - y_t)
\]

with feedback gain \( K_{fb} = 0.2690 \) and feedforward gain \( K_{ff} = 0.4055 \). \( y_{sp} \) is the setpoint signal for the output. The flow chart for valve stiction in He’s model is given in Fig. 1. Values of 0.35 and 0.25 were chosen for \( f_d \) and \( f_s \), respectively, to generate a strong stiction.

The controllers are properly tuned for setpoint tracking but not for disturbance rejection, causing the system output with and without valve stiction to be very noisy, see Fig 2. The performance of the system without valve stiction is very close to the lower bound for good performance, see Fig. 3. Valve stiction makes the index oscillatory at time intervals when the He’s model is active. Such oscillations would have not been picked up by a linear, non-iterative method even though they make the index value go below 0.5. The degradation in performance index is not significant but infers an oscillatory disturbance. The mean values of the calculated performance indexes are comparable to the values obtained of \( \eta_y = 0.5026 \) and 0.4617, using the comparison method for situations without and with valve stiction, respectively. The value of \( m \) was set to be 8 whereas the value of \( n \) was set to be 10 (\( m < n \)) to make the KF more sensitive to stiction oscillations.
Controller output

\[ u(t) \]

\[ cum_u = u + [u(t) u(t-1)] \]

\[ \text{yes} \]

\[ cum_u > f_i \]

\[ u(t) = u(t-1) \text{ sign } \text{cum}_u - f_i \]

\[ u = \text{cum}_u \]

\[ \text{no} \]

Fig. 1. He’s valve stiction model flow chart

Fig. 2. Feedback/feedforward control system with and without valve stiction

Fig. 3. CPM for feedback/feedforward control system with and without valve stiction

4.2 Cold Rolling Mill with Roll Eccentricity

A cold rolling mill system is simulated in Matlab using the model from (Kodati, 2010). The model consists of 3 stand rolling mills. A multi-loop architecture is implemented to control the process using a combination of PID and high order controllers. Strip thickness, mass flow, looper angle (inter-stand) and speed control are implemented in every stage. Strip thickness at the exit of every stand is assessed. The time delay to the thickness sensor is varying but measurable. An initial calibration of setpoints is used to represent a new slab entering the rolling mill. Controllers are properly tuned but not set to compensate roll eccentricity which is added to each stand with frequencies of 4.679Hz and 4.997Hz for lower and upper rolls, respectively. Roll eccentricity refers to any conditions caused by axial deviations between the roll barrel and the roll necks that results in irregularities in the mill rolls.

The estimated strip thickness \( h_{\text{est}} \), measured strip thickness \( h_{\text{m}} \) and thickness setpoints \( h_{\text{sp}} \) of the 3 stands are compared (Fig. 4). The estimation process accurately accommodates roll eccentricity variations in all the stands with values of \( m \) and \( n \) both set to be 7 since the delay is varying but always smaller than 3. The performance index from each stand deteriorates when the strip moves forward along the processing line due to the accumulated roll eccentricity from the previous stand (Fig. 5). As expected, changes in setpoints are picked as peaks in the corresponding stand (due to controller transient), and as drops in other stands since the accumulated eccentricity is passed. The comparison method calculates performance values of \( \eta_y \) to be 0.7019, 0.6704 and 0.6545, respectively, for the 3 stands. The process is performing reasonably well since all performance values are within \([0.5, 0.8]\).

4.3 CPM with Real Oxyfuel Furnace Data

Real oxyfuel furnace data provided by Swerea MEFOS (One of the partners of the Cognitive Control Project completed in our previous work (Recalde, Katebi, and Tauro, 2013; Recalde, Katebi, and Yue, 2014) was employed in this CPM study. The furnace is used to reheat the slabs coming from the rolling mill and consists of 2 parts being
The temperature setpoints ($T_{sp}$) for these zones are also added for illustration and comparison. There are sudden drops in temperature in both zones when the slabs leave the zones and do not affect performance index. The performance values are very close to the poor performance bound, see Fig. 7. The index in zone 3 is worse than the one in zone 4 but does not present any oscillatory behaviour that may come from a nonlinearity. Without more information about the process, poorly performing controller/s can be the source of the degraded performance. Further assessment carried out by MEFOS revealed that the controller in zone 3 was indeed badly tuned and its effects were passed on to zone 4. The comparison method calculates the performance values of $\eta_y$ to be 0.5018 and 0.5037 for zones 3 and 4, respectively.

5. CONCLUSION

The CPM algorithm proposed in this work is capable of assessing nonlinear systems that can be fitted to a SDM and accommodating process variations in the state vector. It can track control performance changes over time and is computationally efficient. The identification of the SDM parameters can be carried out using KF or any other sequential identification algorithms. The KF is used in this work to assure the optimal parameter estimation in the SDM.

The flexibility of using covariance control to achieve the desired performance specifications is the basis of the proposed CPM algorithm. A by-product of this covariance control application is the possibility to develop a control re-tuning mechanism for structured controllers such as PI/PID control, widely used in industries. The successful implementation of this strategy supports the proposed CPM approach to be applied in assessment and diagnosis for general nonlinear dynamic systems.

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