A Decomposition Algorithm for Robust Lot Sizing Problem with Remanufacturing Option

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Abstract. In this paper, we propose a decomposition procedure for constructing robust optimal production plans for reverse inventory systems. Our method is motivated by the need of overcoming the excessive computational time requirements, as well as the inaccuracies caused by imprecise representations of problem parameters. The method is based on a min-max formulation that avoids the excessive conservatism of the dualization technique employed by Wei et al. (2011). We perform a computational study using our decomposition framework on several classes of computer generated test instances and we report our experience. Bienstock and Özbay (2008) computed optimal base stock levels for the traditional lot sizing problem when the production cost is linear and we extend this work here by considering return inventories and setup costs for production. We use the approach of Bertsimas and Sim (2004) to model the uncertainties in the input.

Keywords: robust lot sizing, remanufacturing, decomposition

1 Introduction

Traditional lot sizing problems mainly aim to construct production plans that minimize the total operational cost for a specific production system, while ensuring that demand in each time period is satisfied. For such models to remain applicable for present production systems, recent shifts in manufacturing practices have to be taken into consideration through revising model structure and assumptions. A practice that has been increasingly applied is the reuse of deformed items to manufacture as-good-as-new products, motivated by the increasing interest of implementing recycling activities. More specifically, such production systems with item recoveries are expected to have reduced overall production costs and waste through restoring deformed products to their usable state. Recovery of these items can be undertaken in several ways (see Thierry et al. 1995). In our work, we are interested in investigating the option of recovering these items through remanufacturing. Applications of remanufacturing are observed in the production of a wide range of products, such as electronic goods and industrial items (see, Thierry et al. 1995, Guide and Van Wassenhove 2009, Agrawal et al. 2015). Our main focus is to consider the additional decisions regarding remanufacturing while constructing an optimal production plan for a discrete and finite time horizon, where the exact values for demands and returned items are known to be uncertain.

Despite the wide range of research on lot sizing problems (see, e.g., Akartunalı et al. 2016), very few studies have focused on lot sizing problems with remanufacturing (LSR). Preliminary research on LSR problems includes the implementation of the Wagner-Whithin algorithm by Richter and Sombrutzki (2000), which was later extended to one with manufacturing and remanufacturing costs by Richter and Weber (2001). An economic LSR formulation (ELSR) with disposal costs was introduced by Golany et al. (2001), where the ELSR problem was shown to be NP-complete. A dynamic programming algorithm was presented by Teunter et al. (2006), which solves the ELSR problem in $O(T^4)$ time for a special case of the problem. In more recent work, Helmrich et al. (2014) have introduced alternative formulations for the ELSR problem, and have shown that the problem with joint or separate setups is NP-hard. The work of Akartunalı and Arulselvan (2016) has shown the tractability of a polynomial time special case and have introduced two classes of valid inequalities for the capacitated version of the problem. However, there is a lack of literature on the impact of uncertainty on these formulations, with the exception of Wei et al. (2011). The present study aims to contribute to the growing research on ELSR problems by studying the implications of parameter uncertainties within the framework of robust optimization.

Robust optimization was first introduced by Soyster (1973), where uncertain parameters are defined through uncertainty sets and a robust optimal solution is defined as one that remains optimal for every parameter representation in an uncertainty set. More recent studies relaxed this conservative assumption, with the seminal work of Ben-Tal and Nemirovski (1998, 1999) constructing uncertainty sets as ellipsoids. Later, Bertsimas and Sim (2004) defined the uncertainty sets as budgeted polytopes, where robust parameter representations are constrained by a specified value. Their approach was applied to traditional lot sizing problems by Bertsimas and Thiele (2006) and adapted to the robust ELSR in Wei et al. (2011). Bienstock and Özbay (2008) propose a decomposition approach for solving a min-max formulation of the special lot sizing problem consisting in the computation of basestock levels. Robust lot sizing problems have also been considered as particular cases of the general problems addressed in Agra et al. (2016), and Atamtürk and Zhang (2007). For comprehensive books on robust optimization we refer the reader to Bertsimas and Sim (2004) and Ben-Tal et al. (2009). For concise overviews on robust optimization methods see Bertsimas and Thiele (2011), Gabrel et al. (2014), Gorissen et al. (2015).

Here we consider a min-max formulation for the robust ELSR problem, since the approach followed in Wei et al. (2011) is known to be too conservative and uses many dual variables which restricts its applicability. For a detailed explanation of this conservativeness, see Bienstock and Özbay (2008). A common approach to handle min-max robust optimization problems is to use a variant of the Benders' decomposition, (see Thiele et al. 2010). Frequently, this decomposition results in the iterative inclusion of rows and columns (Agra et al. 2013; Zeng and Zhao 2013). Such approach is also known as the *Adversarial* approach (Gorissen et al. 2015). For solving robust inventory problems, the decomposition framework was introduced by Bienstock and Özbay (2008) and revisited later by Agra et al. (2016), where demand in each time period is assumed to be uncertain. Our approach for the robust ELSR problem is also motivated by these studies, where our objective is to generate optimal production plans when demands and returns are uncertain. We use the approach of Bertsimas and Sim (2004) to model the uncertainty set as budgeted polytopes, where the variation of the demands and returns in relation to their nominal values is constrained by a specified value. A robust model with recourse is considered where the inventory levels are allowed to adjust to the realization of the uncertain parameters. Our contribution is two-fold. Firstly, we model an extended version of the lot sizing problem, wherein we consider uncertainty in both the return and demand sets with set up costs for production. Our second contribution lies in reporting our computational experience with several input classes of costs and inventory levels.

The remainder of this paper is organized as follows: In Section 2, we introduce the deterministic and robust formulations for the ELSR problem. We introduce the robust decomposition algorithm in Section 3, and finally we conclude by presenting preliminary performance results in Section 4.

2 Problem Definition

The main problem addressed in this study is the economic lot sizing problem with remanufacturing and joint setups in a robust setting. Throughout the paper, the term "robust" refers to probability-free uncertainty. Problem assumptions and notations, the deterministic ELSR formulation, and a detailed description of parameter uncertainties are presented prior to the robust formulation. The decomposition algorithm introduced in Section 3 is based on the robust min-max formulation given in Section 2.2. Thus, the assumptions and notations given under this section remain valid for the decomposition algorithm. The objective of our problem is to produce a production plan detailing the amounts to be manufactured, remanufactured, kept in inventory, backlogged and disposed of, where total operational costs are minimized for the maximum value of return and demand deviations. Our assumptions are: a) remanufacturing is a single operation (has no accompanying inspection/disassembly) b) remanufactured items are as good as manufactured ones c) serviceable inventory (ready to serve demand) can either be positive (incurs holding cost) or negative (incurs a backlogging cost) d) return items can be disposed at a cost e) manufacturing and remanufacturing are not capacitated and incur a joint setup cost of K f) The production plan is generated for a finite and discrete time horizon, T.

In addition, we assume that all problem parameters are known. The values of demands and returns are inexact, however, the inputs used to construct the relevant uncertainty sets are known. All cost parameters are time-invariant, and serviceables have a greater holding cost than returned items. Similarly, manufacturing an item is more expensive than remanufacturing a returned item.

Manufacturing, remanufacturing, disposal and backlogging costs of a single item are represented as m, r, f and b, respectively. The unit holding cost of serviceable (returned) goods are shown as h^s (h^r). Let the demands (returns) for periods $t = 1, \dots, T$ be D_t (R_t). For modelling the set up decision, we introduce a binary variable y_t and a sufficiently big M_t for all t. Variable x_t^m (x_t^r) indicates the number of items manufactured (remanufactured) in time t. Let the the number of items disposed at the end of period t be d_t. Finally, Z_t^D (Z_t^R) models the scaled deviation of demands (returns) from the nominal value in period t. We might drop the time index t, to denote the corresponding vector. For instance, Γ^R will denote the vector in T dimensions with the t^{th} component being Γ_t^R.

2.1 Classical Deterministic Model

The ELSR problem can be written as:

$$\min_{(\mathbf{x},y)\in\mathcal{P}} \theta^{D,R}(\mathbf{x}, y) \tag{1}$$

where

$$\theta^{D,R}(\mathbf{x}, y) = \sum_{\mathrm{t=1}}^{\mathrm{T}} (K y_t + m x_t^m + r x_t^r + f d_t + H_t^s + H_t^r), \tag{2}$$

and $\mathbf{x} = (x^m, x^r)$ and y are vectors specifying a feasible production plan that belongs to the set

$$\begin{aligned} \mathcal{P} := \{(x_t^m, x_t^r, y_t) \in \mathbb{R}_+^{2T} \times \mathbb{Z}_+^T : I_0^r + \sum_{i=1}^{t} (R_i - x_i^r - d_i) \geq 0, \quad \forall t = 1, ..., T \\ M_t y_t \geq x_t^m + x_t^r, \quad \forall t = 1, ..., T\} \end{aligned} \tag{3}$$

As reverse flows do not exist for returns, their inventory levels are restricted to be nonnegative and we ensure that the setup cost K is incurred for the time period t when $y_t = 1$, where $M_t = \sum_{i=t}^{T} D_i$. Variables H_t^s (H_t^r) model the total cost of serviceable (return) inventory held in period t and is given by

$$H_t^s = \max\{h^s[I_0^s + \sum_{i=1}^{t} (x_i^m + x_i^r - D_i)], -b[I_0^s + \sum_{i=1}^{t} (x_i^m + x_i^r - D_i)]\} \tag{4}$$

$$H_t^r = h^r[I_0^r + \sum_{i=1}^{t} (R_i - x_i^r - d_i)] \tag{5}$$

2.2 Uncertainty

In practical cases some of the parameters may not be known in advance. Here we assume the demands D_t and the returns R_t are uncertain, and consider a twostage robust model. The number of items manufactured, remanufactured and disposals (and consequently the set-up decisions) are assumed to be first stage or "here-and-now" decisions. Thus, such decisions are taken before the value of the uncertain parameters is revealed. While the serviceable and return inventory levels are second-stage variables since they are allowed to adjust to the value of the parameters.

We apply the robust optimization approach of Bertsimas and Sim (2004) defining uncertainty sets as budgeted polytopes. The uncertainty on demand and return parameters is considered to be independent from each other. Therefore, an independent uncertainty set for demands (U^D), and returns (U^R) exist. For each time period $t = 1, \ldots, T$, parameters $\Gamma_t^D, \bar{D}_t, \hat{D}_t$ ($\Gamma_t^R, \bar{R}_t, \hat{R}_t$) are the budget of uncertainty for demands (returns), nominal demands (returns) and maximum deviation in demands (returns) respectively. The robust parameter D_t takes its value in the interval $[\bar{D}_t, \bar{D}_t + \hat{D}_t]$. Similarly, R_t takes its value in the interval $[\bar{R}_t, \bar{R}_t + \hat{R}_t]$. Hence, our uncertainty sets are defined as:

$$U^D(\Gamma^D) := \{D \in \mathbb{R}_+^T : \; D_t = \bar{D}_t + \hat{D}_t z_t^D, \quad \forall t = 1, ..., T, \; z_t^D \in Z_t^D(\Gamma_t^D)\} \tag{6}$$

$$U^R(\Gamma^R) := \{R \in \mathbb{R}_+^T : \; R_t = \bar{R}_t + \hat{R}_t z_t^R, \quad \forall t = 1, ..., T, \; z_t^R \in Z_t^R(\Gamma_t^R)\} \tag{7}$$

The variables z_t^D and z_t^R in (6) and (7) take their values in the interval $[0, 1]$ and are used to indicate a given proportion of the maximum deviations $\hat{D}_t$ and $\hat{R}_t$. In order to avoid overconservative parameter representations, the parameters Γ_t^D and Γ_t^R are introduced to constrain z_t^D and z_t^R. More specifically, the cumulative values of scaled deviation variables for demands and returns are required to be strictly less than or equal to Γ_t^D and Γ_t^R, hence we obtain:

$$Z_t^D(\Gamma_t^D) := \{z_t^D \in [0, 1]^t : \sum_{i=1}^{t} z_i^D \leq \Gamma_t^D, \forall t = 1, ..., T\} \tag{8}$$

$$Z_t^R(\Gamma_t^R) := \{z_t^R \in [0, 1]^t : \sum_{i=1}^{t} z_i^R \leq \Gamma_t^R, \forall t = 1, ..., T\} \tag{9}$$

As the inventory levels are allowed to adjust to the uncertain parameters, the variables H_t^s and H_t^r will depend on the demands and returns. So, for each $t = 1, \ldots, T$ and $D \in U^D$, we have $H_t^s(D)$ given from (4), and for each $t = 1, \ldots, T$ and $R \in U^R$, we have $H_t^r(R)$ given from (5).

We can now extend the deterministic ELSR problem to this uncertain case as a robust min-max formulation:

$$\min_{(\mathbf{x}, \mathbf{y}) \in \mathcal{P}} \max_{\substack{D \in U^D(\Gamma^D) \\ R \in U^R(\Gamma^R)}} \theta^{D,R}(\mathbf{x}, \mathbf{y}) \tag{10}$$

where $\theta^{D,R}(\mathbf{x}, y)$ is extended as follows:

$$\theta^{D,R}(\mathbf{x}, y) = \sum_{\mathrm{t=1}}^{\mathrm{T}} (Ky_t + mx_t^m + rx_t^r + fd_t + H_t^s(D) + H_t^r(R)) \tag{11}$$

3 Decomposition Approach

As the number of variables $H_t^s(D)$ and $H_t^r(R)$ is not finite, the inner maximization problem is not finite. However, practical experience based on decomposition algorithms for related problems (see, for instance, Agra et al. 2013 for the case of the robust vehicle routing problem with time windows, Agra et al. 2016 for a general class of problems including the robut lot-sizing problem, and Bienstock and Özbay 2008 for the problem of computing robust basestock levels) has shown that only a few of the values of the uncertainty sets $U^D(\Gamma^D)$ and $U^R(\Gamma^R)$ are necessary to solve the problem.

Here we present a decomposition algorithm that iteratively solves a restricted version of the robust min-max problem (10) with respect to a subset of points of $U^D(\Gamma^D)$ and of $U^R(\Gamma^R)$ which will be denoted by $\tilde{U}^D$ and $\tilde{U}^R$, respectively. We call this restricted version of (10) as "Decision Maker's" problem (DMP). Given an optimal solution $(\mathbf{x}^*, y^*) \in \mathcal{P}$ to the DMP, we solve a certain maximization problem, which seeks a demand $D \in U^D(\Gamma^D)$ and return $R \in U^R(\Gamma^R)$ that maximises the total inventory and backlogging costs for the production plan $(\mathbf{x}^*, y^*) \in \mathcal{P}$. We refer to this subproblem as the "Adversarial Problem" (AP). The extreme point D^*, R^* generated by AP is used to update $\tilde{U}^D$ and $\tilde{U}^R$ and the process is repeated. Convergence is guaranteed through the finiteness of the number of extreme points of the uncertainty sets $U^D(\Gamma^D)$ and $U^R(\Gamma^D)$. The formal description of this idea is given in Algorithm **??**.

Initialize $UB = +\infty$, $LB = 0, \tilde{U}^D = \{\bar{D}\}, \tilde{U}^D = \{\bar{R}\}$
while $(UB - LB)/LB \geq \epsilon$ **do**
  1. Solve DMP
  a. $(\mathbf{x}^*, y^*)$ be the solution of $\min_{(\mathbf{x},y)\in\mathcal{P}} \max_{D,R\in\tilde{U}^D\times\tilde{U}^R} \theta^{D,R}(\mathbf{x}, y)$
  b. Set $LB = \max_{D,R\in\tilde{U}^D\times\tilde{U}^R} \theta^{D,R}(\mathbf{x}^*, y^*)$
  2. Solve AP
  a. $(D^*, R^*) = \arg\max_{D,R\in U^D\times U^R} \theta^{D,R}(\mathbf{x}^*, y^*)$
  b. $\tilde{U}^D = \tilde{U}^D \cup \{D^*\}$, $\tilde{U}^R = \tilde{U}^R \cup \{R^*\}$
  c. $UB = \min\{UB, \theta^{D^*,R^*}(\mathbf{x}^*, y^*)\}$
end

Algorithm 1: Robust decomposition algorithm

For the sake of completeness, we give the DMP and the AP. In order to model the DMP, notice that the inner maximization problem in (10) defined for the restricted set $\tilde{U}^D \times \tilde{U}^R$, $\max_{D,R\in\tilde{U}^D\times\tilde{U}^R} \theta^{D,R}(\mathbf{x},\mathbf{y})$, can be written as:

$$\sum_{\mathrm{t=1}}^{\mathrm{T}}(Ky_t + mx_t^m + rx_t^r + fd_t) + \max_{D,R\in\tilde{U}^D\times\tilde{U}^R} \sum_{\mathrm{t=1}}^{\mathrm{T}}(H_t^s(D) + H_t^r(R))). \tag{12}$$

Introducing variable π to indicate the maximum value of the total inventory and backlogging costs over all possible realizations of demands and returns, the DMP can be written as follows:

$$\begin{aligned}
&\min \sum_{\mathrm{t=1}}^{\mathrm{T}}(Ky_t + mx_t^m + rx_t^r + fd_t) + \pi && && (13)\\
&\text{s.t. } \pi \geq \sum_{\mathrm{t=1}}^{\mathrm{T}}(H_t^s(D) + H_t^r(R)) && \begin{matrix}\forall D \in \tilde{U}^D\\ \forall R \in \tilde{U}^R\end{matrix} && (14)\\
&\quad H_t^s(D) \geq h^s\Big(I_0^s + \sum_{\mathrm{i=1}}^{\mathrm{t}}(x_i^m + x_i^r - D_i)\Big) && \begin{matrix}\forall t = 1,...,T\\ \forall D \in \tilde{U}^D\end{matrix} && (15)\\
&\quad H_t^s(D) \geq -b\Big(I_0^s + \sum_{\mathrm{i=1}}^{\mathrm{t}}(x_i^m + x_i^r - D_i))\Big) && \begin{matrix}\forall t = 1,...,T\\ \forall D \in \tilde{U}^D\end{matrix} && (16)\\
&\quad H_t^r(R) = h^r\Big(I_0^r + \sum_{i=1}^{t}(R_i - x_i^r - d_i)\Big), && \begin{matrix}\forall t = 1,...,T\\ \forall R \in \tilde{U}^R\end{matrix} && (17)\\
&\quad \sum_{\mathrm{i=1}}^{\mathrm{t}}(R_i - d_i - x_i^r) \geq 0 && \begin{matrix}\forall t = 1,...,T\\ \forall R \in \tilde{U}^R\end{matrix} && (18)\\
&\quad M_t y_t \geq x_t^m + x_t^r && \forall t = 1,...,T && (19)\\
&\quad (x,y) \in P
\end{aligned}$$

Note that variables $H_t^r(R)$ can be eliminated using equations (17). Given a solution for variables x_i^m, x_i^r, d_i, the AP is formulated as follows:

$$\max \pi \tag{20}$$

$$\text{s.t. } \pi \leq \sum_{\text{t=1}}^{\text{T}} (H_t^s + h^r \sum_{\text{i=1}}^{\text{t}} (\bar{R}_i + \hat{R}_i z_i^R - d_i - x_i^r)) \tag{21}$$

$$H_t^s = \max\Big\{ h^s \Big(I_0^s + \sum_{\text{i=1}}^{\text{t}} (x_i^m + x_i^r - (\bar{D}_i + \hat{D}_i z_i^D) \Big),$$

$$- b\Big(I_0^s + \sum_{\text{i=1}}^{\text{t}} (x_i^m + x_i^r - (\bar{D}_i + \hat{D}_i z_i^D) \Big) \Big\} \qquad \forall t = 1, ..., T \tag{22}$$

$$I_0^r + \sum_{i=1}^{t} (\bar{R}_i + \hat{R}_i z_i^R - x_i^r - d_i) \geq 0 \qquad \forall t = 1, ..., T \tag{23}$$

$$\sum_{\text{i=1}}^{\text{t}} z_i^D \leq \Gamma_t^D, \quad \sum_{\text{i=1}}^{\text{t}} z_i^R \leq \Gamma_t^R \qquad \forall t = 1, ..., T \tag{24}$$

$$0 \leq z_t^{Dj} \leq 1, \quad 0 \leq z_t^{Rj} \leq 1 \qquad \forall t = 1, ..., T \tag{25}$$

In order to linearize (22), we introduce binary variable s_t indicating whether inventory is kept or demand is backlogged, and rewrite it as:

$$H_t^s \leq h^s \Big(I_0^s + \sum_{\text{i=1}}^{\text{t}} (x_i^m + x_i^r - (\bar{D}_i + \hat{D}_i z_i^D) \Big) + M_{1t}(1 - s_t) \quad \forall t = 1, ..., T \tag{26}$$

$$H_t^s \leq -b\Big(I_0^s + \sum_{\text{i=1}}^{\text{t}} (x_i^m + x_i^r - (\bar{D}_i + \hat{D}_i z_i^D) \Big) + M_{2t} s_t \quad \forall t = 1, ..., T \tag{27}$$

4 Experiments

The proposed decomposition algorithm was implemented in Java using Eclipse Mars. Our formulations were implemented and solved as MIPs using Java API for CPLEX 12.6 on an Intel Core i5, 3.30GHz CPU, 3.29GHz, 8 GB RAM machine.

Additionally, each run has been restricted to a total running time of 10,000 seconds. The terminating condition for instances with a smaller running time is set as $\epsilon = 0.01$, where $\epsilon = \frac{UB-LB}{LB}$.

4.1 Data Generation

Data sets have been generated for different levels of four parameter types: number of returns, probability of constraint violation caused by Γ_t^D and Γ_t^R, the setup cost and the disposal cost. We consider three different levels for each group, except for disposal costs: low, medium and high. For disposal costs, we are interested in observing two different cases, namely when the disposal cost is greater or less than the remanufacturing cost. Throughout this section, the data sets are abbreviated as "ABCD_T", where each letter indicates the levels of the aforementioned parameters in their given order, with T time periods.

For all data sets, nominal demand is generated randomly in the interval $[50, 100]$. Likewise, returns are generated randomly in intervals $[15, 30]$, $[25, 50]$ and $[35, 70]$, for low, medium and high levels, respectively. Maximum demand and return deviations are calculated as $\hat{D}_t = 0.1\bar{D}_t$ and $\hat{R}_t = 0.1\bar{R}_t$. In order to determine Γ_t^D and Γ_t^R, we use the probabilistic bounds given by Bertsimas and Sim (2004). We set the probability of constraint violation as 0.01, 0.05 and 0.10, for low, medium and high levels, respectively. To determine the setup cost, we use the following equations: $K = 0.1\bar{D}_{min}h^s$, $K = 2\bar{D}_{med}h^s$ and $K = 5\bar{D}_{max}h^s$, where $\bar{D}_{min} = 50, \bar{D}_{med} = 75$ and $\bar{D}_{max} = 100$, for low, medium and high levels. Finally, the disposal cost is set as $d = 0.5r$ when it is less than the remanufacturing cost, and as $d = 2r$ otherwise.

In addition, the holding cost of serviceables is generated in the interval $[5, 10]$, through which the remaining cost parameters are defined. We set the holding cost for returns as $h^r = 0.1h^s$, the backlogging cost as $b = 4h^s$, the manufacturing cost as $m = 2h^r$, and the remanufacturing cost as $r = 2h^r$.

4.2 Preliminary Results

To observe the performance of our decomposition algorithm, we analyse the total time requirements for obtaining the smallest possible $UB - LB$ gap. The following performance measures are preliminary results that are obtained from 16 different data sets for $T = 10$, and 5 different data sets for $T = 50$. A total of 10 instances were solved for each data set.

When $T = 10$, all instances can be solved to $\epsilon = 0.07$ or better. It is possible to achieve $\epsilon = 0.001$ very quickly for the vast majority of the data sets. However, a few instances were aborted due to the time limit, for which the gap remains relatively large. For each data set, the total running time, final gap and total iterations for $T = 10$ are given in Table 1. The overall performance of cases when the gap could not be reduced down to $\epsilon = 0.001$ under 10,000 seconds are given in Figures 1 and 2.

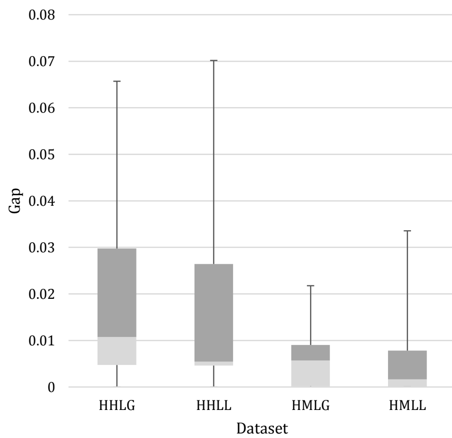

Fig. 1: Total UB - LB gap for T=10 (when gap is greater than 0.01).

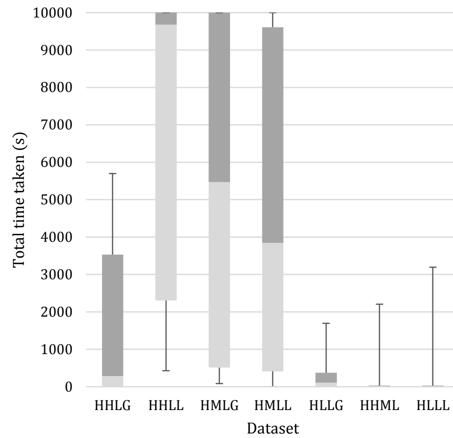

Fig. 2: Total running time for T=10 (when $\epsilon > 0.01$ could not be achieved under 10,000 seconds).

Table 1: Gap, time, iteration performance for T=10.

	Gap (ϵ)		Time Performance (s)		Number of Iterations	
Dataset	Avg.	Std.Dev.	Avg.	Std.Dev.	Avg.	Std.Dev.
HHHG	0.004	0.002	0.4	0.1	4.2	0.9
HHHL	0.003	0.003	0.7	0.3	4.9	1.1
HHMG	0.005	0.003	27.8	40.5	5.9	1.2
HHML	0.003	0.002	236.2	692.5	5.3	0.9
HHLG	0.020	0.021	1706.0	2314.5	4.2	1.5
HHLL	0.018	0.022	6783.9	4337.9	4.2	1.2
HMHG	0.004	0.004	0.4	0.2	3.7	0.9
HMHL	0.005	0.004	0.3	0.2	3.9	0.3
HMLG	0.007	0.008	5247.2	4542.0	4.6	1.0
HMLL	0.008	0.013	4713.2	4512.3	4.5	1.2
HLHG	0.007	0.004	0.3	0.1	2.8	0.4
HLHL	0.005	0.004	0.4	0.2	3.0	0.5
HLMG	0.005	0.003	32.7	54.7	3.8	0.4
HLML	0.004	0.005	19.8	34.1	3.9	0.6
HLLG	0.002	0.004	401.4	617.4	4.2	0.6
HLLL	0.002	0.003	433.5	1024.5	4.2	0.8

As the detailed results in Table 1 indicate, the gaps achieved and number of iterations needed are in general consistent across different data sets, in addition to being in general very small (e.g., the highest maximum gap is still under 0.1, and no more than 8 iterations were necessary for any instance). On the other hand, as it can be also observed from the Figures 1 and 2, the time performance can vary significantly not only among different datasets but also among different instances of most datasets.

When we look into the datasets with $T = 50$, a greater number of instances naturally run until the maximum time limit is reached as a consequence of the increased number of periods. However, the algorithm is still able to close the gap up to $\epsilon = 0.003$ for some instances.

In comparison to $T = 10$, the total number of iterations reduce as the time limit is reached in earlier iterations. We also observe a greater variety in terms of the total solution time for several data sets (see Figure 4). For groups where the total running time is invariant, the final gap remains larger compared to others, in which case ϵ is less than 0.05. The gap, time and iteration performances for $T = 50$ are presented in Table 2. Although many instances exhausted the time limit, it is encouraging to see that the maximum gaps still remain very small.

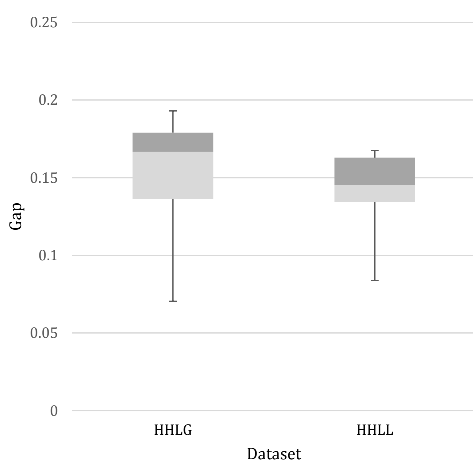

Fig. 3: Total UB - LB gap for T=50 (when gap is greater than 0.05).

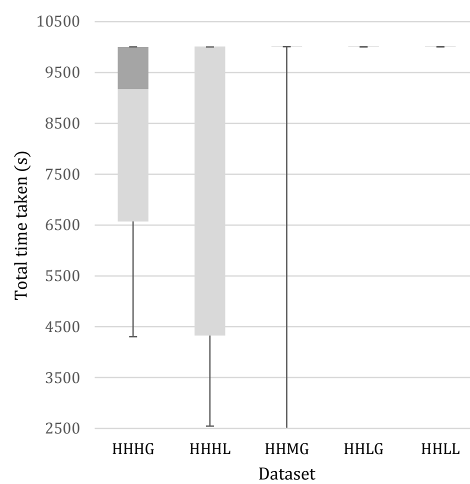

Fig. 4: Total running time for T=50.

5 Conclusions

In this paper, we studied the robust lot sizing problem with remanufacturing option, where, to the best of our knowledge, the literature is at best scarce.

Table 2: Gap, time, iteration performance for T=50.

	Gap (ϵ)		Time Performance (s)		Number of Iterations	
Data Set	Avg.	Std.Dev.	Avg.	Std.Dev.	Avg.	Std.Dev.
HHHG	0.010	0.006	8203.7	2092.2	3.5	0.7
HHHL	0.012	0.006	7595.5	3305.4	2.8	0.8
HHLG	0.159	0.043	10002.9	1.1	2.1	0.3
HHLL	0.141	0.028	10004.3	1.1	2.0	0.0
HHMG	0.029	0.008	9044.4	3033.3	2.7	0.5

We proposed a simple but effective decomposition procedure for constructing robust optimal production plans, where the approach of Bertsimas and Sim (2004) was used to model the uncertainties in input. Preliminary computational results on various datasets indicate that this procedure can work effectively, in particular to address current issues caused by imprecise representations of problem parameters.

Future work includes further development and improvement of the current computational framework in order to achieve more effective results, in particular of the computational times. It is also important to perform extensive computational testing to allow a thorough statistical analysis of the performance.

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