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Characterising plastic collapse of pipe bend structures

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Abstract

Two recently proposed design by analysis criteria of plastic collapse based on plastic work concepts, the plastic work (PW) criterion and the plastic work curvature (PWC) criterion, are applied to a strain hardening pipe bend arrangement subject to combined pressure and in-plane moment loading. Calculated plastic pressure–moment interaction surfaces are compared with limit surfaces, large deformation analysis instability surfaces and plastic load surfaces given by the ASME Twice Elastic Slope criterion and the tangent intersection criterion. The results show that both large deformation theory and material strain hardening have a significant effect on the elastic–plastic response and calculated static strength of the component. The PW criterion is relatively simple to apply in practice and gives plastic load values similar to the tangent intersection criterion. The PWC criterion is more subjective to apply in practice but it allows the designer to follow the development of the gross plastic deformation mechanism in more detail. The PWC criterion indicates a more significant strain hardening strength enhancement effect than the other criteria considered, leading to a higher calculated plastic load.

Keywords: Pipe bends; Gross plastic deformation; Plastic load; Criterion of plastic collapse

1. Introduction

Pipe bends are flexible components of piping systems, often absorbing large loads and thermal expansions during service under normal operating conditions. The structural response of a bend depends on the type of load or loads applied. Internal pressure loading tends to expand the cross-section of the bend. In-plane closing moments flatten the cross-section (ovalising with major axis out of the plane of the bend) so as to reduce its second moment of area and introduce geometric weakening. In-plane opening moments increase the depth of the cross-section (ovalising in-plane) and hence, increase the second moment of area and introduce geometric strengthening. Out of plane bending moments ovalise the cross-section at approximately 45° to the plane of the bend and lead to slight geometric weakening. The response of a pipe bend under combined pressure and bending loads is further complicated by the fact that the pressure and bending responses are known to be highly coupled [1]. These large deformation and coupling effects are not modelled by conventional limit analysis, which assumes small deformation theory, and can only be determined by large-deformation elastic–plastic analysis or experiments.

Depending on the specific configuration considered, large deformation elastic–perfectly plastic analysis may result in a conventional limit load failure (violation of force equilibrium) or failure by structural instability due to large deformations. When a strain hardening material model is used, the response becomes even more complicated. Lack of convergence may occur due to structural instability but otherwise convergence may occur for unreasonably high loads, depending on the strain hardening material model used. In such cases, the plastic collapse load is considered to occur before convergence of the solution fails and is determined in practice by applying a *criterion of plastic collapse*. The ASME Boiler and Pressure Vessel Code Section VIII Division 2 [2] design by analysis procedures specifies use of the twice elastic slope (TES) plastic collapse criterion in Appendix 4-136.5 *plastic analysis* (as defined in Appendix 6-153, *criterion of collapse load*). This is one of several similar criteria that have been proposed for determining the plastic collapse load of pressurised components [3]. The criterion does not define when actual physical plastic collapse of the structure occurs but rather is intended to indicate when gross plastic deformation, GPD, of the vessel occurs. Gerdeen [3] therefore, recommends that the load determined by application of the criterion is referred to as the 'plastic load' rather than 'plastic collapse load'.

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Nomenclature

GPD	gross plastic deformation
h	dimensionless bend factor, Rt/r_m^2
M	applied moment
$\bar{M}$	normalized plastic moment, $M/4r_m^2\sigma_y t$
$\bar{P}$	normalized pressure $Pr_m/\sigma_y t$
PW	plastic work criterion
PWC	plastic work curvature criterion
R	the bend radius
r_m	mean radius of elbow cross-section
t	elbow wall thickness
TES	twice elastic slope criterion
TI	tangent intersection criterion
W^e	elastic work
W^p	plastic work
λ	global load parameter
σ_{ij}	stress component
ε_{ij}	strain component
$d\varepsilon_{ij}^e$	elastic strain increments in a infinitesimally small element of material
$d\varepsilon_{ij}^p$	plastic strain increments in a infinitesimally small element of material

The elastic–plastic response of pipe bends under pressure and bending loads has received considerable attention in the literature. Mello and Griffin [4] carried out elastic–plastic analysis of an elbow using a non-linear finite element computer program, including material strain hardening, stress redistribution, and ovalisation effects. The plastic loads calculated by various criteria of plastic collapse were compared to collapse loads determined by limit analysis. Sobel and Newman [5] investigated the instability of elbows in the plastic range using the finite element method and special pipe bend elements (and concluded that geometric non-linearity effects should be included in the analysis). They later compared the predictions from MARC finite element analysis with experimental results for the elastic–plastic behaviour of a stainless steel piping structure with two straight tangent pipes subjected to an in-plane closing bending moment [6]. They found that simplified analysis, omitting non-linear geometry, overestimated the strength of the bend. Dhalla [7] also used MARC, to investigate geometric and material non-linearity interaction effects, showing that the geometric non-linear effects became significant at load levels above 80% of the collapse load. In a later experimental-finite element investigation of two 0.41 m (16′) mean pipe diameter elbows with attached straight pipes [8], he found that at higher loads the analysis predicted an increasingly stiffer response, overestimating the collapse load by 15%.

Suzuki and Nasu [9] performed non-linear finite element analysis of butt welded elbows subjected to in-plane bending, obtaining a good correlation with experimental results in the linear range and a maximum difference of 10% in the non-linear analysis. Kussmaul et al. [10] conducted six pipe bend tests to determine the local and global failure behaviour as a function of load history and showed that the location of initial yielding depends on geometry, with the point of maximum strain on the inner surface at the bend crown. Shalaby and Younan [11] applied ABAQUS non-linear finite element analysis to a range of pipe bends (with no attached runs) under combined pressure and in-plane closing bending moment, using an elbow element and an elastic–perfectly plastic material model. They showed that both the plastic instability load and collapse load increase with increasing internal pressure up to a certain value and then decrease with increasing pressure. It was also shown that the plastic instability load increased with increasing pipe bend factor and that the effects of internal pressure depend on the diameter-to-thickness ratio (D/t) of the elbow. They subsequently investigated the strain and stress distribution of pipe bends under in-plane bending [12]. It was found that the distributions are equivalent but with opposite signs in the elastic range under in-plane closing and opening moments. In the plastic range, the axial and hoop strain distributions at instability are similar to those at yielding for a closing moment but different for an opening moment. Mourad and Younan [13] performed similar analysis to evaluate the behaviour of a pipe bend subjected to out-of-plane bending and internal pressure. They showed that the loaded end of the bend was the most severely strained cross-section and found that considerable plastic deformation occurred before instability was reached, especially in the presence of applied pressure.

Chattopadhyay et al. [14] carried out NISA elastic–plastic finite element analysis to evaluate plastic loads for six elbows of different geometry under in-plane bending moments and internal pressure. The true stress–strain response of SA350 Gr LF2 was used in a multi-linear hardening analysis. The elbow was attached to straight pipe runs of length equal to the six times the mean cross-sectional radius of the bend. The plastic moment was obtained by applying the twice elastic slope criterion to moment–rotation characteristic curves. They found that the application of internal pressure enhanced the plastic moment of an elbow up to a certain limit, beyond which it decreased with further increase in internal pressure. Two closed-form equations based on these results were proposed to evaluate the plastic moments of elbows under combined internal pressure and in-plane closing and opening bending moments

$$\bar{M} = 1.122h^{2/3} + 0.175\bar{P}/h - 0.598\bar{P}^2 \text{ (for closing case)} \quad (1)$$

$$\bar{M} = 1.047h^{1/3} + 0.124\bar{P}/h - 0.568\bar{P}^2 \text{ (for opening case)} \quad (2)$$

where $\bar{M}$ is the normalized plastic moment

$$\bar{M} = \frac{M}{4r_m^2\sigma_y t} \tag{3}$$

and $\bar{P}$ the normalized pressure

$$\bar{P} = \frac{Pr_m}{\sigma_y t} \tag{4}$$

and h is the dimensionless bend factor

$$h = \frac{Rt}{r_m^2} \tag{5}$$

where R is the bend radius, r_m mean radius of elbow cross-section and t the wall thickness.

Karamanos et al. [15] investigated the finite element and experimental response of elbows under in-plane bending and pressure, taking account of geometric and material non-linearity, using a non-linear three-node tube-element. ABAQUS shell and elbow elements were also used in some cases. The analysis indicated that internal pressure had an important effect on both ultimate moment and cross-sectional ovalisation and a significant difference between closing and opening moment response was observed. It was concluded that the adjacent straight parts of the pipe had a considerable influence on the response and the ultimate moment of the elbow. They also found that different failure modes of thin-wall elbows were observed for closing and opening moments. Elbows under closing bending moments were found to fail due to significant cross-sectional deformation, whereas elbow under opening bending moments exhibited local buckles. Robertson and the present writers [16] investigated the plastic collapse behaviour of pipe bends under combined pressure and closing in-plane moment using two conventional plastic collapse criteria, the twice elastic slope criterion and the tangent intersection criterion. Several practical problems were encountered when applying these criteria to the pipe bend problem; for example, selecting a suitable deformation parameter for combined loading and applying the appropriate graphical construction for the TES and TI methods. These practical problems were seen to introduce possible inconsistency in the characterisation of the plastic load of a bend under combined loading.

In this paper, the pipe bend problem is investigated using two recently proposed plastic criteria based on work and energy characterisation of the plastic response, rather than the load and deformation approach adopted in other criteria. The criteria considered are the plastic work (PW) criterion, proposed by Muscat et al. [17,18], and the plastic work curvature (PWC) criterion proposed by the present writers [19,20]. The proposed advantage of these criteria is that they avoid the problems in defining appropriate load and local deformation parameters encountered in other criteria. The PWC criterion is further intended to incorporate an enhanced representation of the effect of strain hardening on the evolution of the GPD mechanism, leading to a more realistic evaluation of plastic collapse load. Both of these criteria have previously been applied to a limited number of structural configurations, most of which include a single type of load. A better evaluation of the criteria as proposed and recommendations for their development as possible design tools requires a more detailed investigation of their behaviour when applied to more complex configurations. The complex response of pipe bends under combined internal pressure and in-plane bending loads provides a realistic subject for assessing these criteria.

2. Plastic collapse criteria

The twice elastic slope (TES) and the tangent intersection (TI) criteria are both based on a load–deformation curve that is required to characterise the inelastic response of the structural configuration. The TES criterion is shown in Fig. 1a. The plastic load P_P is defined as that corresponding to the intersection of the load–deformation curve and a straight line called the *collapse limit line*, emanating from the origin of the load–deformation curve at angle $\phi = \tan^{-1}(2\tan\theta)$; that is, twice the gradient or slope of the initial elastic response with respect to the y-axis (from which angles θ and ϕ are defined). The tangent intersection (TI) criterion, shown in Fig. 1b, defines the plastic load as that corresponding to the intersection between straight line tangents drawn from the initial elastic response and plastic deformation regions of the characteristic curve. As the characteristic curve is defined in terms of a specified *deformation parameter* (displacement or strain) at a *point* on the structure, the TES and TI criteria essentially characterise the response on the basis of a *local* indicator of plastic collapse. The choice of deformation parameter has a significant effect on the calculated plastic load.

The plastic work (PW) criterion is based on a *global* characterisation of the plastic response. The characteristic curve used in the criterion is a plot of a global load parameter λ, incorporating all the loads (applied in a proportional loading analysis) against the total or global plastic work dissipated in the vessel as the gross plasticity mechanism forms. A schematic load-plastic work curve for a simple strain-hardening structural configuration is shown in Fig. 2a. The initial response of the structure is elastic until the yield load is reached, at which point plastic deformation starts. As the load is increased and the plastic failure mechanism develops, part of the external work done is stored as elastic strain energy and

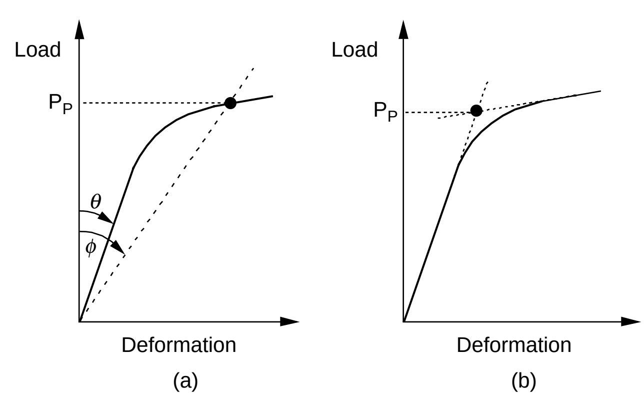

Fig. 1. (a) Twice elastic slope, TES, criterion; (b) tangent intersection, TI, criterion.

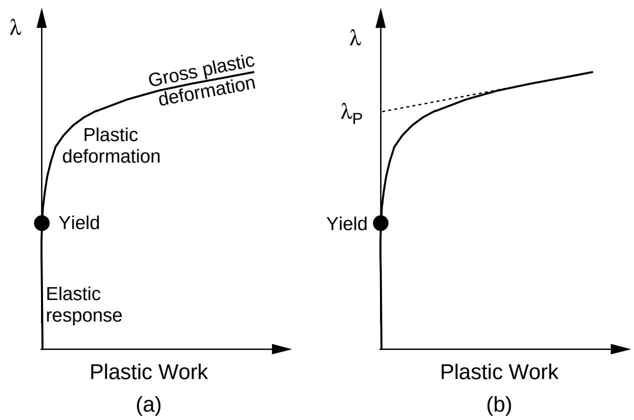

Fig. 2. (a) Development of gross plastic deformation mechanism; (b) plastic work (PW) criterion.

part is dissipated as plastic work. Post yield, the characteristic load–plastic work curve has a non-linear form as the behaviour changes from elastic-dominated to plastic-dominated deformation. Once the plastic failure mechanism has fully formed, the deformation achieves an almost steady plastically dominated state and the characteristic curve becomes almost a straight line. At this stage, the vessel is experiencing GPD and the applied load clearly exceeds the maximum allowed in safe design. The safe plastic load for design purposes lies somewhere between yield and the steady plastic deformation, however, the gradual transition from elastic to plastic deformation makes it difficult to determine the precise plastic load (or GPD load). The PW criterion, shown in Fig. 2b, defines a conservative plastic load λ_P for design purposes as the intersection between a tangent drawn from the steady plastic deformation portion of the characteristic curve and the load parameter axis.

The plastic work curvature (PWC) criterion is based on the same load–plastic work characteristic response curve as the PW criterion but proposes a different interpretation of plastic load [19]. The PWC criterion relates the post yield (elastic-to-plastic) stress redistribution that occurs as the GPD mechanism develops to the curvature of the load–plastic work curve. As load increases above yield, the amount of stress redistribution occurring in the structure is characterised by the rate of change of plastic work with increasing load. Elastic-to-plastic stress redistribution continues until the GPD mechanism forms, after which the stress distribution becomes almost constant with increasing load. The vessel is experiencing GPD (analogous to the limit state experienced when the material is elastic-perfectly plastic). Subsequently, the rate of plastic dissipation becomes near-constant with increasing load and the characteristic load–plastic work curve exhibits a near-constant slope. The load at which this state is achieved is the plastic load of the configuration. However, as stated previously, the gradual transition from elastic to plastic deformation characterised by the load–plastic work makes it difficult to determine precisely when this GPD state is achieved. The PWC criterion seeks to clarify the transition by considering the curvature of the load–plastic work curve. Fig. 3 shows the load–plastic work curve

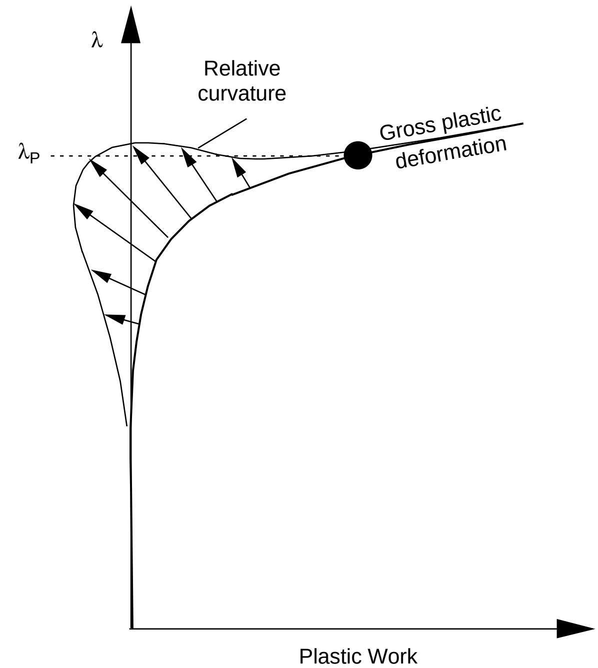

Fig. 3. Plastic work curvature (PWC) criterion.

with the relative or normalised *curvature* at each point superimposed on the curve itself. In the elastic region, the curvature is zero. After yielding, plastic stress redistribution begins and the curvature increases as the plastic deformation mechanism develops. The maximum stress redistribution in the structure occurs at the load corresponding to the maximum curvature. Thereafter, the curvature starts to decrease, indicating a reduction in the amount of stress redistribution. As the curvature decreases to zero, little or no further plastic stress redistribution occurs and the structure has reached a state of GPD. The corresponding load is therefore, designated the plastic load for the component, λ_P.

The PWC was applied to three simple structures under a single load in Ref. [19]: an axially loaded three bar structure, a cantilever beam in bending and a pressurised cylinder. These simple examples showed that the PWC criterion gives useful insight into the formation of the GPD mechanism and represents the constraining effect of strain hardening on the spread of plasticity more consistently and effectively than alternative plastic criteria. However, even for these simple structures the actual structural response is more complex than the simple model shown in Fig. 3. In particular, as the GPD mechanism forms the curvature may tend to a relatively small constant value rather than reach a distinct zero value. Later application of the PWC criterion to a piping branch junction under single and combined (proportional) pressure and moment loading [20] demonstrated the basic concepts for a more complex structure. These examples demonstrated the more complex form of curve and consequently curvature found in real pressure vessel components and highlighted the problem of determining when exactly the curvature plot indicates that a state of GPD has been reached. Guidance on this choice requires greater knowledge of the relationship between the curvature criterion and the structural behaviour of complex pressure vessel components.

In this paper, the PWC criterion is applied to the complex problem of pipe bends under combined pressure and in-plane moment loading. Small deformation and large deformation theory analyses are presented to define limit loads (under proportional loading) and plastic loads (under non-proportional pressure–moment loading). Failure surfaces determined by the PWC are compared with values given by other criteria.

3. Example analyses

The simple piping system considered in the investigation is a 90-degree pipe elbow connected to two straight runs of pipe. Analysis was performed assuming large deformation (non-linear geometry) theory, with elastic–perfectly plastic and bilinear hardening material models. The total elastic and plastic work for structure are obtained by integrating through the volume using an ANSYS macro [19]. The elastic work is

$$W^{e} = \int \sigma_{ij}\, d\varepsilon_{ij}^{e} \tag{6}$$

The plastic work (or energy dissipated) is

$$W^{p} = \int \sigma_{ij}\, d\varepsilon_{ij}^{p} \tag{7}$$

where σ_{ij} is stress component, ε_{ij} strain component, $d\varepsilon_{ij}^{e}$ elastic strain increments and $d\varepsilon_{ij}^{p}$ plastic strain increments.

3.1. Example system layout and dimensions

The bend geometry is mean cross-sectional radius $r_m = 250$ mm, bend radius ratio $R_b/r_m = 3$ and wall thickness $t = 20$ mm, giving a bend parameter $h = R_b t / r_m^2 = 0.24$. Two equal length straight pipe runs were attached to the elbow. Diem and Muller [21] concluded that tangent pipes 1–3 diameters long were sufficient to ensure the stress distribution in the elbow is unaffected by the conditions at the end of the attached straight run. Vernon et al. [22] investigated the effect of tangent length, and chose five times the diameter in their analysis. Herein, the straight lengths are chosen as $10r_m$.

3.2. Material models

Two material models were considered: elastic–perfectly plastic and bilinear hardening, with Young's modulus, $E = 200$ GPa, yield stress $\sigma_y = 300$ MPa and Poisson's ratio $\nu = 0.3$. By definition, the plastic modulus used in limit analysis is $E_P = 0$ GPa. Two different values for plastic modulus were used in the strain hardening analyses. For in-plane closing moment, the plastic modulus was assumed to be 5% of the elastic modulus, $E_P = 10$ GPa. After an initial investigation of in-plane opening moment loading using 5% strain hardening, it was decided to use a lower value of plastic modulus of 2% for all the results presented here. This was essentially to aid presentation and discussion of results, as the effect of 5% strain hardening combined with geometric strengthening gave an extensive plastic response that, when plotted with the other results, obscured the detail of the plastic mechanism formation part of the curves. A plastic modulus of $E_P = 4$ GPa was therefore, used for the in-plane opening moment loading analysis.

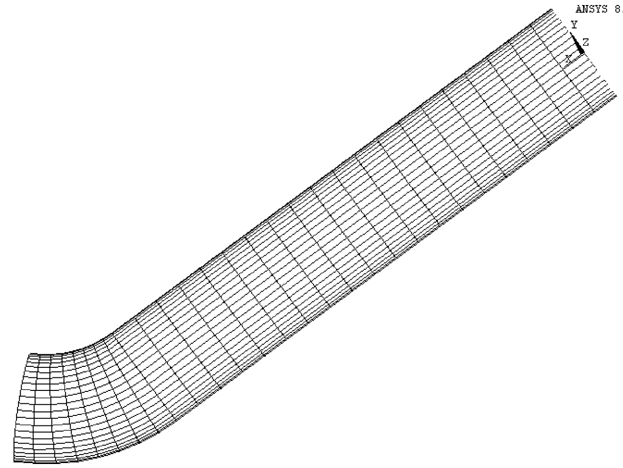

Fig. 4. Shell finite element model.

3.3. Finite element model

The geometry and boundary conditions of the systems have two planes of symmetry, so only one quarter of the elbow with appropriate symmetry boundary conditions is modelled. The piping system was modelled in ANSYS8.0 [23] using plastic shell elements SHELL181. After convergence studies, a mesh of 672 elements was chosen, comprising 28 elements in the axial direction (20 along the straight/8 along the bend) 24 elements around the circumference, as shown in Fig. 4. Elastic beam elements, BEAM4, were used to model the flanges terminating the straight runs. Moment loading was applied to the flange by applying a point moment to the central node of a web of radial beam elements from the centre of the pipe-end to the flange.

4. Results

Large deformation analysis plastic load interaction diagrams were obtained for the example system using the TES, TI, PW and PWC criteria. Application of these criteria for pressure only and moment only loading is presented and discussed in Section 4.1. Plastic load interaction diagrams are presented and discussed in Section 4.2.

4.1. Pressure and moment only loading

Under pressure-only loading, first yielding occurs in the middle of the bend at the inside surface of the intrados. As pressure is increased the plastic zone spreads axially towards the junction with the straight run and circumferentially towards the extrados. Large deformation effects were not found to be significant in pressure-only loading. Applying the TES and TI

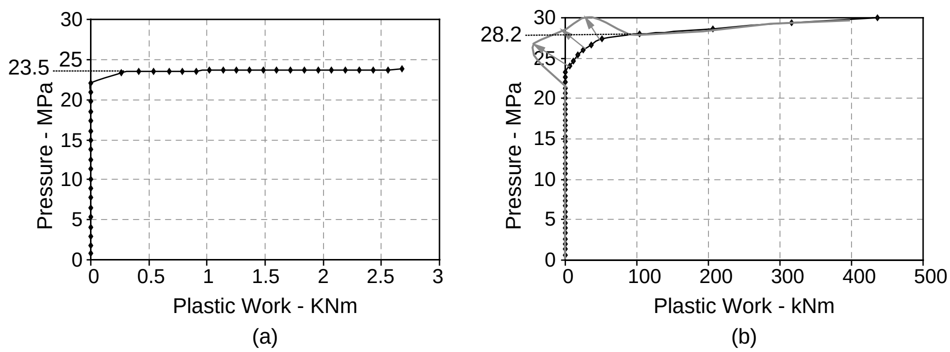

Fig. 5. Pressure versus plastic work plots for pressure-only loading for (a) elastic-perfectly (b) bilinear strain hardening material models.

criteria to the pressure characteristic load–deformation plots gives plastic pressures of $P_{TES}=26.5$ MPa and $P_{TI}=26$ MPa for the strain hardening material and for the elastic–perfectly plastic material the TI gives plastic pressures of $P_{TI}=$ 23.5 MPa, however, TES is not applicable as there is no intersection between the curve and twice elastic slope line.

The pressure versus plastic work curve for the elastic–perfectly plastic pipe bend is shown in Fig. 5(a). As the bend is relatively thin, the plastic zone rapidly spreads across the wall thickness after first yield at pressure $P_y=20.9$ MPa. The change from elastic to GPD occurs over only two load steps in the incremental plastic analysis and the curve is piecewise linear between the elastic and plastic response regions. When the load reaches 23.5 MPa, the curve becomes almost a straight line and the curvature tends to zero. This indicates that the bend is experiencing GPD. As the slope of the steady plastic deformation region is approximately zero, the PW and PWC criteria give the same value of plastic pressure, $P_{PW}=P_{PWC}=$ 23.5 MPa.

The pressure versus plastic work curve with superimposed normalised curvature distribution for 2% strain hardening is shown in Fig. 5(b). The maximum pressure applied in the analysis was 30 MPa. In the strain hardening bend, the bilinear hardening material model inhibits the spread of the plastic zone through the thickness of the pipe and a more gradual transition from elastic to plastic deformation occurs. Applying the PW criterion gives a plastic load of $P_{PW}=27.5$ MPa. The normalised curvature decreases from a maximum at the transition between elastic and plastic response to zero at a pressure of 28.2 MPa. This pressure is therefore, specified as the plastic work criterion plastic pressure $P_{PWC}=28.2$ MPa.

The pressure only normalized plastic pressures are summarised in Table 1, which also shows the first yield, limit and instability pressures (as appropriate). Non-linear geometry effects are not apparent in the bend under pressure loading and the limit and instability load for small deformation perfectly plastic analysis are similar. The plastic loads given by the TI, PW and PWC criteria for the elastic–perfectly plastic material are equal to the limit load. When a 2% strain hardening material is considered, the PWC criterion gives the highest plastic load followed by PW, TES and TI criteria.

The behaviour of a pipe bend in the elastic range is the same under in-plane closing and opening moments. First yield occurs in the middle of the bend at the inside surface of the crown in both cases. As the load is increased beyond yield, the plastic zone spreads axially along the crown towards the straight run and circumferentially outwards, towards the extrados and the intrados. When large deformation effects are included, the plastic strain distributions are markedly different for opening and closing moments: a closing moment gives rise to through-thickness plastic deformation at the crown and an opening moment causes the whole bend cross-section to experience plastic deformation.

Moment–rotation characteristic curves are shown for large deformation opening and closing moments in Fig. 6. In the case of the geometrically weakening closing moment, the perfectly plastic large deformation instability load occurs at around 80% of the limit load (as given by perfectly plastic small deformation analysis). In the geometrical strengthening opening moment case, the perfectly plastic instability load is significantly higher than the limit load.

The TES and TI criteria were applied to the large deformation characteristic curves of Fig. 6. For the closing moment and elastic–perfectly plastic material, the TES plastic moment M_{TES} is undefined (no intersection between the collapse limit line and moment–rotation curve) and the TI criterion plastic moment $M_{TI}=600$ kN m. For the closing moment bilinear material, $M_{TES}=M_{TI}=700$ kN m. The TES

Table 1
Pressure only plastic loads

	$\bar{P}_Y$	$\bar{P}_L$	$\bar{P}_{TES}$	$\bar{P}_{TI}$	$\bar{P}_{PW}$	$\bar{P}_{PWC}$	Normalized instability
Perfectly plastic	0.87	0.98	–	0.98	0.98	0.98	0.98
2% bilinear hardening	0.87	–	1.104	1.08	1.15	1.17	–

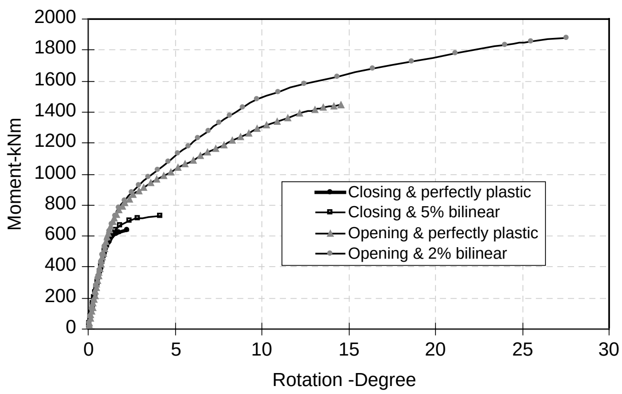

Fig. 6. Large deformation theory analysis characteristic moment–rotation curves.

can be applied to both the opening moment models, giving $M_{TES}=900$ kN m for the elastic–perfectly plastic material, and $M_{TES}=980$ kN m for the 2% bilinear hardening material. However, care has to be taken when applying the TI criterion to the opening moment curves. The perfectly plastic curve has a long region of essentially steady state response but a distinct change in slope is observed just prior to instability failure. Taking the tangent from the first steady response region gives a value of $M_{TI}=810$ kN m for the opening moment elastic–perfectly plastic material. In the bilinear hardening curve, two steady regions are observed. Examining equivalent plastic strain contour plots of the analysis showed that first yield occurs at the crown of the bend and the plastic zone grows with increasing moment. When the moment reaches 880 kN m, the middle of the first 'knee' on the curve, a second plastic zone forms at the intrados of the bend. As the load increases, these plastic zones grow and the curve exhibits a steady plastic response until plastic deformation starts to spread to the attached straight pipes at around 1500 kN m, corresponding to the second 'knee' of the curve. The steady response between the two knees therefore, indicates GPD of the bend itself and the tangent line should be draw from this region. For the opening moment and bilinear material case, $M_{TI}=810$ kN m.

The large deformation analysis moment–plastic work curves for the pipe bend, with superimposed curvature distributions, are shown in Figs. 7 and 8 for closing and opening moments, respectively. The von Mises equivalent plastic strain distributions in the bends at the calculated PWC failure load are also shown.

Fig. 7a shows the closing moment, perfectly plastic material response. Applying the PW criterion gives $M_{PW}=600$ kN m. Considering the PWC criterion, the curvature increases to a maximum at 500 kN m and falls to zero at the instability load, giving a plastic moment of $M_{PWC}=638$ kN m. Applying the PW criterion to the closing moment bilinear hardening plot of Fig. 7b gives $M_{PW}=700$ kN m. The plot shows that the curvature reaches a maximum at 600 kN m then reduces to zero at load $M_{PWC}=720$ kN m. However, it is noted that a distinct discontinuity in curvature occurs just below the specified plastic moment, at $M=700$ kN m.

Fig. 8a shows the opening moment, perfectly plastic material moment–plastic work curve. The initial transition from elastic to elastic–plastic response, the first 'knee' in the curve, is followed by a steady plastic response up to a distinct change in slope just prior to instability. This was previously observed in the corresponding moment–rotation curve shown in Fig. 6. Applying the PW criterion tangent to the initial steady plastic response region gives a plastic moment of $M_{PW}=$ 840 kN m. The superimposed normalised curvature plot highlights these regions of stress redistribution. Equivalent plastic strain contour plots show that the first peak on the curvature plot indicates stress redistribution as the bend response changes from elastic to plastic response. The subsequent decrease in curvature indicates decreasing stress-redistribution as the GPD mechanism forms and plastic action dominates the bend response. In this case, the curvature does not decrease to zero but passes through a minimum and starts to increase again as the global instability mechanism forms. By

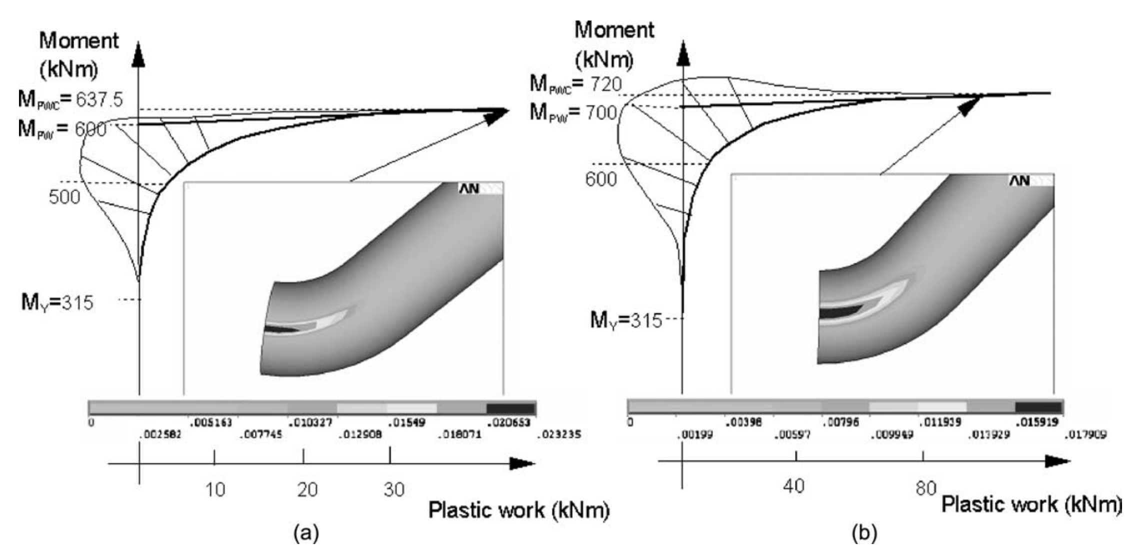

Fig. 7. In-plane closing moment large deformation analysis (a) perfectly plastic (b) 5% bilinear hardening, with von Mises equivalent plastic strain contour plots at specified PWC plastic load.

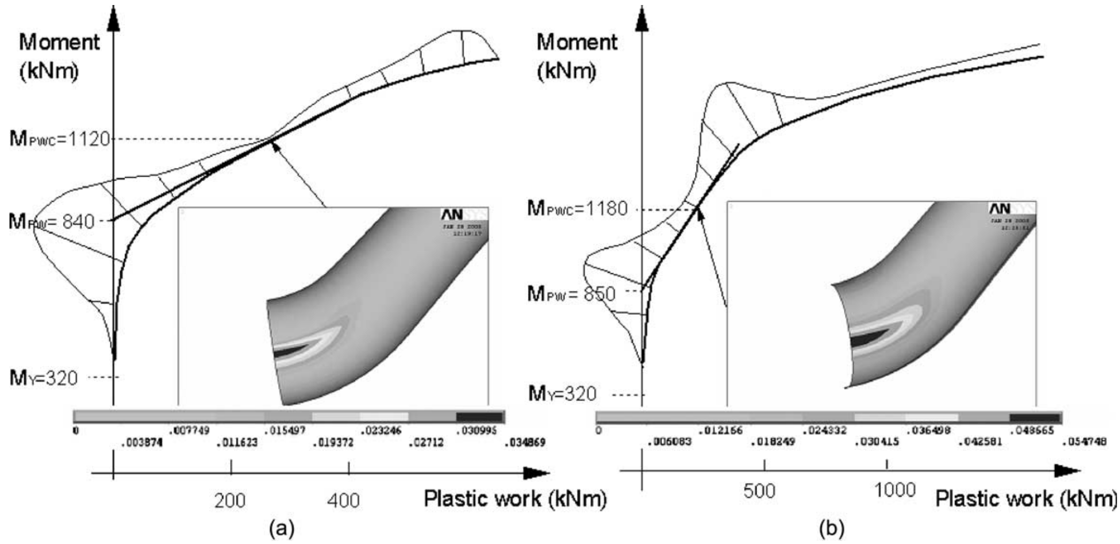

Fig. 8. Opening moment large deformation analysis (a) perfectly plastic (b) 2% bilinear hardening, with von Mises equivalent plastic strain contour plots at specified PWC plastic load.

this stage the bend itself is experiencing GPD and therefore, the local minimum is specified as representing the plastic load, giving $M_{PWC}=1120$ kN m.

The opening moment bilinear moment–plastic work curve, Fig. 8b, has two distinct peaks and two steady–state regions. The peaks are associated with the formation of plastic zones discussed in connection with the moment rotation curves of Fig. 6: the first peak indicates plastic deformation of the bend and the second indicates plastic deformation of the attached straight run. According to previous discussion, the calculation of plastic loads should not be based on the second plastic deformation stage and the PW criterion tangent line is consequently drawn from the first steady–state plastic deformation region of the load–plastic work curve, giving $M_{PW}=845$ kN m. Considering the PWC criterion, the normalised curvature does not decrease to zero after formation of the bend plastic deformation mechanism but reaches a local minimum as the second plastic deformation zone arises in the straight run. In this case, the plastic load is specified at the local minimum between the two peaks, $M_{PWC}=1180$ kN m.

The moment only normalized plastic moments are summarised in Table 2, which also shows the first yield, limit and instability moments (as appropriate).

4.2. Pressure–moment loading

In large deformation analysis, the calculated structural response of a pipe bend under combined loading is highly dependent on the loading sequence [16]. Here, non-proportional loading is applied. First, internal pressure is applied to a specific value then held constant as the moment is applied and increased until the solution fails to converge. The load axis of the characteristic PWC curve in this case is not a load parameter λ representing all applied loads but rather the moment load that initially causes yield and eventually causes gross plastic deformation. The combined load at the last convergent solution is specified as the instability load. Normalised pressures, Eq. (4), values from 0.0 to 1.0 in steps of 0.1 or 0.2 were considered.

A typical load–plastic work curve for combined pressure and closing moment with an elastic–perfectly plastic material is shown in Fig. 9 (the von Mises equivalent plastic strain distribution at the designated PWC failure load is also shown). An initial pressure of $P=14.4$ MPa (i.e. below yield for pressure only loading) is applied and held constant as the moment is subsequently applied. As the moment increases, first yield occurs at the crown. At the outside surface of the bend,

Table 2
Moment only plastic loads

		$\bar{M}_{Y}$	$\bar{M}_{L}$	$\bar{M}_{TES}$	$\bar{M}_{TI}$	$\bar{M}_{PW}$	$\bar{M}_{PWC}$	Normalized instability
Closing moment	Perfect plasticity	0.21	0.54	–	0.40	0.40	0.43	0.43
	5% bilinear hardening	0.21	–	0.47	0.47	0.47	0.47	0.49
Opening moment	Perfect plasticity	0.21	0.54	0.6	0.54	0.56	0.75	0.96
	2% bilinear hardening	0.21	–	0.65	0.54	0.56	0.79	1.25

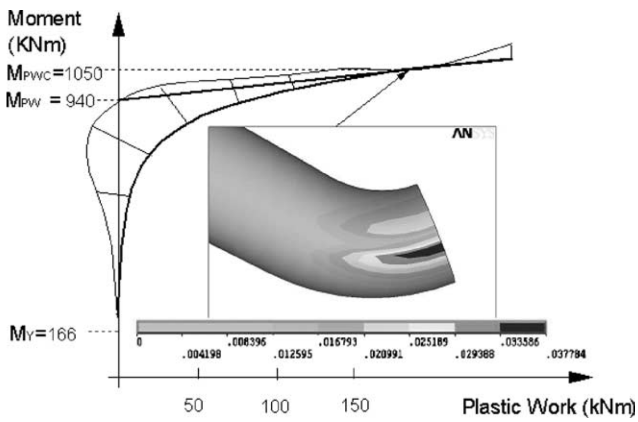

Fig. 9. Combined internal pressure ($\bar{P}=0.6$) and in-plane closing moment loading example for a perfectly plastic material, with equivalent plastic strain contour plot at specified PWC plastic load.

the plastic zone spreads both axially along the crown towards the straight run and circumferentially towards the extrados and the intrados, but the maximum plastic strain remained in the middle of the crown. At the inside surface of the bend, the intrados became plastic first, then the crown. For elastic–perfectly plastic material, first yield occurs at $M=166$ kN m around outside surface of crown, the inside surface of intrados yields at $M=230$ kN m and yield across the wall thickness at the crown occurs at $M=510$ kN m. Another plastic zone close to the extrados occurred at $M=630$ kN m. When the moment reaches 920 kN m, almost the entire inside surface of the bend has yielded. Applying the TES and TI criteria to a moment rotation plot gave $M_{\mathrm{TES}}=884$ kN m and $M_{\mathrm{TI}}=928$ kN m. The plastic moment given by applying the PW criterion to Fig. 9 is $M_{\mathrm{PW}}=940$ kN m and the plastic load given by the PWC criterion is $M_{\mathrm{PWC}}=1050$ kN m.

Plastic load curve given by the TES, TI, PW and PWC criteria for large deformation analysis are compared with limit load curves (the load corresponding to the last equilibrium solution for small deformation theory perfectly plastic analysis) and instability load (the load corresponding to the last equilibrium solution for large deformation theory perfectly plastic analysis) in Fig. 10. Curves obtained from the plastic load solutions of Chattopadhyay et al. [14], Eqs. (1) and (2), based on the TES criterion and multi-linear hardening analysis are also presented. The Chattopadhyay multi-linear hardening model differs from the bilinear hardening model used in the present analyses but Eqs. (1) and (2) are presented for general application in Ref. [14] and included here for comparison. Pressure is normalised with respect to the limit pressure of a thin cylinder, Eq. (3), and moment is normalised with respect to the limit moment of a straight pipe under pure bending, Eq. (4). The normalised pressure range is limited by the pressure only behaviour of the models. In small deformation theory analysis, instability occurs for pressures of $\bar{P}>0.8$ as soon as any moment loading is applied. In the hardening analysis, pressures of up to $\bar{P}=0.9$ could be applied.

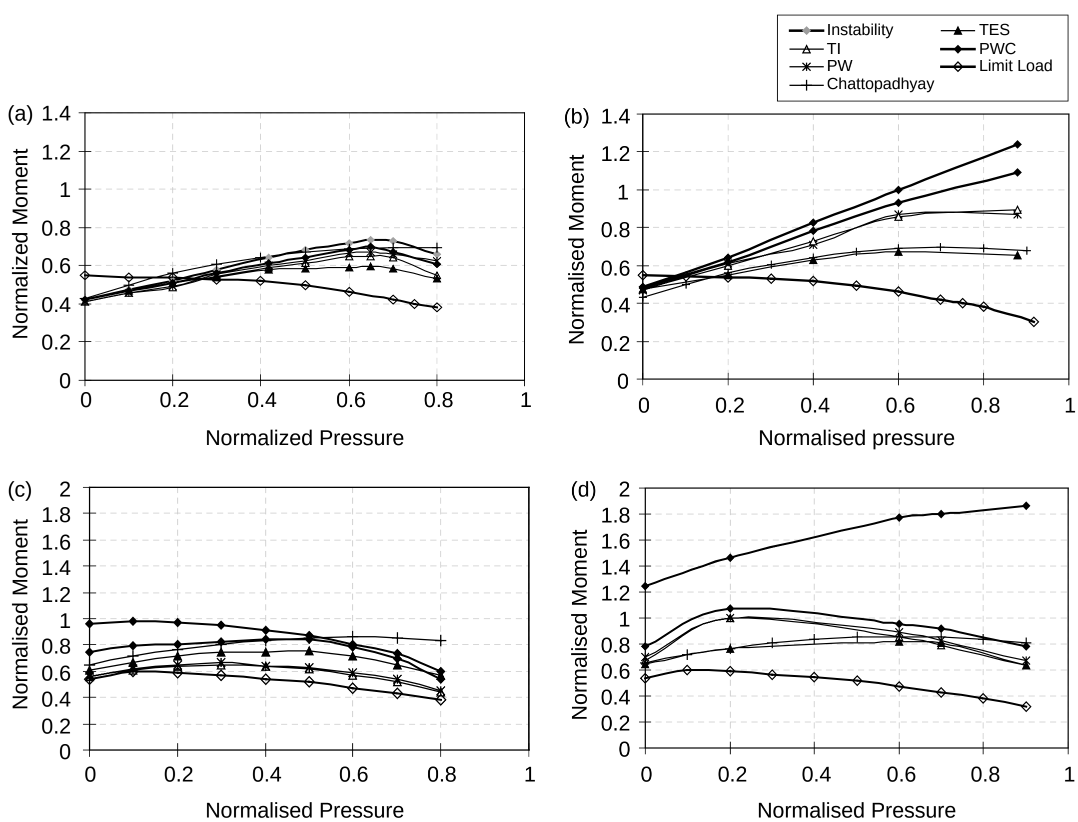

Fig. 10. Plastic loads interaction diagrams for large deformation combined pressure and in-plane moment loading (a) closing moment perfectly plastic, (b) closing moment 5% bilinear hardening, (c) opening moment perfectly plastic, (d) opening moment 2% bilinear hardening.

Fig. 10a and b shows the plastic load surfaces calculated for combined internal pressure and closing in-plane moment large deformation analysis (except the limit load curve, which assumes small deformation theory). Fig. 10a shows the results for an elastic-perfectly plastic material and show that limit analysis is not conservative when the loading is bending-dominant (as is well known from the literature). However, at greater pressures, pressure-bending coupling leads to geometric strengthening and the instability load is significantly greater than the limit load. In the geometrically weakening region, all of the criteria considered give plastic load curves similar to the instability curve, with the TI criterion giving the most conservative values of plastic load. The Chattopadhyay equation gives plastic loads above the instability load. Fig. 10b shows the corresponding plastic load curves for the 5% strain hardening material model (the limit load curve is by definition based on an elastic–perfectly plastic material). The strain hardening response reduces the pressure range over which weakening is observed with respect to the limit load. The TES plastic load curve is the most conservative (and similar to the curve given by the Chattopadhyay equation) indicating it takes the least account of the effect of strain hardening on static strength. The TI, PW and PWC criteria all give plastic load curves similar to the instability curve but are slightly more conservative. At higher pressures, the combined effects of geometric strengthening and strain hardening lead to plastic loads considerably greater than the limit load. The TES criterion gives significantly lower loads than the other criteria in this region. The PWC gives the highest values of plastic load. It follows the form of the instability curve but indicates more conservative plastic loads. The TI and PWC criteria both give similar results, with values of plastic load between the TES and PWC values.

Fig. 10c and d show, the results for combined pressure and opening in-plane moment and indicate that geometric strengthening occurs for this type of combined load (as is known from the literature). In the elastic–perfectly plastic analyses, Fig. 10c, the TI and PW criteria give the most conservative values of plastic load for the entire loading range. The highest values are given by either the PWC criterion, but lower than the instability load. The TES criterion plastic loads are between the PWC and TI values for the entire range. The plastic load curve given by the Chattopadhyay equations are not conservative for high pressure loads. This is because the equation is based on a strain hardening response. Fig. 10d shows the corresponding results for the 2% strain hardening analyses. Clearly, the combination of large deformation theory and strain hardening lead to very high instability loads, especially in pressure dominated loading. The plastic pressure curves given by all the criteria are considerably lower than the instability curve, indicating that GPD occurs well below the instability load. The lowest plastic pressures are given by the TES criterion (and the similar Chattopadhyay equation curve). The other criteria show a greater significance of strain hardening on static strength. The TI, PW and PWC criteria give similar forms for the plastic load curve, with the highest values given by the PWC criterion.

5. Discussion

The PW and PWC criteria both characterize the gross plastic collapse of a structure in terms of plastic work dissipated with increasing load. The PW criterion applies a simple geometric construction to a load–plastic curve work curve to define the plastic load. The actual load–plastic work characteristic curves obtained in Section 4 for single and combined loads usually have a gradual transition or 'knee' in the curve. This is a characteristic of the stress redistribution occurring in the component as the as behaviour changes from elastic to GPD response. The PW criterion essentially replaces the actual curve with an ideal curve in which the response is elastic up to the Plastic load λ_P and thereafter exhibits a linear GPD response, as shown in Fig. 11. This model is similar in principle to that underpinning the TI criterion and has the advantage that it is a simple criterion to apply in design, provided the appropriate point on the characteristic curve from which to draw the steady GPD response line can be identified. The results presented indicate that the PW criterion gives similar values to plastic load to the TI criterion.

The PWC criterion is not as simple to apply as the PW criterion but it allows the designer to follow the development of the GPD mechanism, and hence, define the plastic load, in greater detail. The criterion is based on the assumption that the curvature of the load–plastic work curve indicates the amount of elastic-plastic stress redistribution occurring with increasing load. The plastic pressure, or GPD pressure, is indicated by a reduction in curvature (or stress redistribution) to a relatively small constant value or zero, after which the external work done on the structure is dissipated as plastic work in the GPD mechanism. The curvature plots for the pipe bend

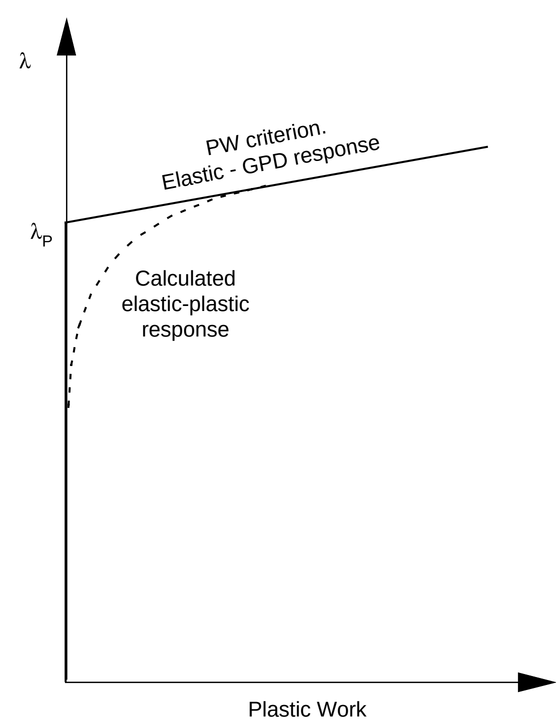

Fig. 11. Plastic work criterion elastic-GPD response model.

configurations show that the actual behaviour is complex and identifying the point at which redistribution can be regarded as insignificant, and hence, GPD commences, is perhaps more subjective than the PW approach. Considering the results in more detail, in many cases the PW criterion indicates GPD is occurring whilst the PWC criterion clearly shows that significant stress redistribution is still occurring in the bend. It can therefore, be argued that the PW criterion is over-conservative, as is also the case for the TES and TI criteria. This is not to say that these criteria are unsuitable for design, in which measured conservatism is generally welcome, but it may indicate that they do not lead to the best utilisation of material strength.

6. Conclusions

The example analyses presented show that the PWC criterion fully characterizes the development of the gross plastic deformation mechanism in pipe bends under combined pressure and in-plane moment loading. The criterion represents the strength enhancing effect of a strain hardening material model more significantly than alternative criteria. Chattopadhyay's closed form equations, (1) and (2), are based on the TES criterion and are relatively insensitive to the strain hardening behaviour of the material. They can therefore, be used as general equations for design purposes but they do not fully represent the strengthening effect of material strain hardening. Alternative equations based on the PWC criterion would indicate higher plastic loads but would be applicable only to specific classes of material with similar post-yield (strain hardening) behaviour to the material model used for developing the equations.

The results presented show that the pipe bend response results in complex PWC plots, in which the curvature does not necessarily fall to zero after the GDP mechanism has formed. It is therefore, difficult to define the plastic load in terms of the magnitude of steady state curvature. If the PWC criterion is to be incorporated in pressure vessel design, a distinct definition of plastic load in terms of the relative curvature is required. It is straightforward to identify the maxima in curvature and to relate these to formation of individual plastic zones and mechanisms. The maxima themselves are not good indicators of plastic collapse, as significant stress distribution still occurs with increasing load. However, the decrease in relative curvature after the maximum associated with the GPD mechanism has occurred may be specified as the GPD state; for example, the plastic load is when the curvature falls to 10% of the maximum. Such a procedure would be acceptable in design provided it is established that the calculated plastic load is conservative and satisfies the intent of the relevant design Code.

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