STRESS LINEARIZATION CONCEPTS AND RESTRICTIONS IN ELASTIC DESIGN BY ANALYSIS

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ABSTRACT

Stress linearization is widely used in Pressure vessel Design by Analysis based on elastic stress analysis and stress categorization. This paper investigates the structural mechanics basis of stress linearization in the context of limit and shakedown analysis and proposes a new basis for the procedure that relates the stress along a line to the concept of limit load. This removes the need for some conceptual requirements associated with shell analysis from stress linearization, including restriction of SCL location to identified bending planes. It also introduces the concept of selecting stress distributions representative of the limit state to remove the need for some elements of stress categorization in the design procedure.

INTRODUCTION

The ASME Boiler and Pressure Vessel Code Section VIII Division 2 [1] procedures for design based on elastic stress analysis defines three categories of stress associated with specific failure modes: primary, secondary and peak stress. Primary stress is limited to prevent gross plastic deformation under static load. Primary plus secondary stress is limited to prevent ratcheting under repeated or cyclic loading. Primary plus secondary plus peak stress, the total stress, is limited to ensure satisfaction of fatigue life requirements.

Fatigue failure is a local mechanism, characterized by the elastic stress (cycle) at a point in the structure. Gross plastic deformation and ratcheting are global failure mechanisms associated with post-yield stress redistribution. Protection against these is achieved by limiting the allowable values of specific stress categories, based on criteria developed from limit load and shakedown concepts. This requires a detailed elastic stress analysis and specification of through section stress distributions in the form of membrane and membrane plus bending stress. Evaluation of these stress distributions is dependent on the type of stress analysis employed in design and can be particularly problematic for design based on 2D and 3D continuum Finite Element Analysis (FEA).

STRESS CATEGORIES

The ASME VIII Div. 2 elastic stress analysis method for protection against plastic collapse is defined in Section 5.22 [1]. This requires the designer to limit the equivalent stress S_e at locations in the vessel to specified allowable values defined in terms of an allowable stress S for the particular material and design temperature. The design stress basis is the maximum distortion energy yield criterion and the equivalent stress is the von Mises equivalent stress:

$$S_e = \sigma_e = \frac{1}{\sqrt{2}}[\,(\sigma_1 - \sigma_2)^2 + \,(\sigma_2 - \sigma_3)^2 + \,(\sigma_3 - \sigma_1)^2]^{0.5}$$

Section 5.1.2 states that the design by analysis rules are applied to the results from a detailed stress analysis of a component but (5.1.2.3) “Recommendations on a stress analysis method, modeling of a component, and validation of analysis results are not provided… because of the variability in approaches and design processes.” The type of stress analysis employed directly affects how the design by analysis rules are implemented.

The design by analysis procedure requires evaluation of primary stress arising from internal pressure and other mechanical loads, secondary stress arising from mechanical and/or thermal loads and peak stress due to local stress concentration and/or localized thermal loads.

Three sub-categories of primary stress are defined. General Primary Membrane Equivalent Stress P_m is the average value of equivalent stress across the thickness of the section, excluding discontinuities and concentrations. Local Primary Membrane Equivalent Stress P_L is the average stress across the section considering discontinuities but not concentrations. The Primary Membrane Plus Primary Bending Equivalent Stress $(P_L + P_b)$ is the highest value of linearized general or local primary membrane stress plus primary bending stresses occurring on the section, excluding discontinuities and concentrations. The bending constituent is the stress proportional to distance from centroid of a solid section representing a bending plane. Secondary stress Q occurs at structural discontinuities and is defined as a Membrane plus Bending distribution. Peak stress is defined as an increment of stress added to primary or secondary stress by a stress concentration or local thermal load.

STRESS LIMITS

The criteria for protection against gross plastic deformation and ratcheting are incorporated in the design by analysis procedure through limits applied to the various stress categories [2]. Initial development of these procedures adopted the Tresca yield criterion as the design stress basis and Stress Intensity as the equivalent stress. "…The choice of the basic stress intensity limits for the stress categories …was accomplished by the application of limit design theory tempered by some engineering judgement and some conservative simplifications."

Limits on primary stress were established by considering the limit state of a straight rectangular beam under combined axial tension and bending as defined in Figure 1. In an elastic analysis, the membrane stress $\sigma_m$and maximum bending stress $\sigma_b$are:

$$\sigma_m = \frac{N}{A} \qquad \sigma_b = \frac{Mh}{I} \qquad A = 2bh \qquad I = \frac{2}{3bh^3}$$

Assuming an elastic-perfectly plastic material model with yield stress σ_Y, the limit state of the beam is reached for any combination of N and M satisfying:

$$\frac{M_L}{bh^2\sigma_Y} + \left(\frac{N_L}{2bh\sigma_Y}\right)^2 = 1$$

Written in terms of the elastic membrane and bending stress distributions, this becomes:

$$\frac{2}{3}\left(\frac{\sigma_b}{\sigma_Y}\right) + \left(\frac{\sigma_m}{\sigma_Y}\right)^2 = 1$$

Plotted this solution on an interaction diagram with axes representing membrane stress and membrane plus bending stress respectively, normalized with respect to yield, gives the limit surface shown in Figure 1.

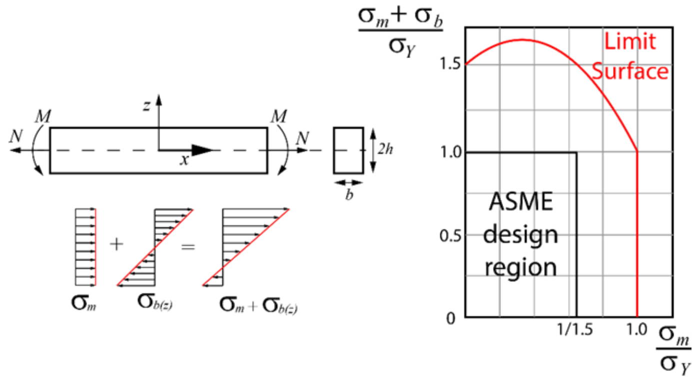

Figure 1. Primary Stress and ASME Design Region

The Code limits on General Primary Membrane stress is the allowable stress S of the material. The value of S is generally the lower of the material yield stress σ_Y divided by 1.5 or the Tensile Strength divided by 2.4. This restricts the allowable value of general primary membrane stress to less than or equal to two-thirds of the material yield stress. Due to the margin between first yield and limit collapse for primary membrane plus bending stress, the stress limit for this category is set at 1.5S. Thus the allowable value of membrane plus bending stress is less than or equal to the material yield stress.

The criterion against ratcheting is based a simple analysis of a prismatic bar subject to an applied thermal strain range ε_R. The residual stress induced in the bar by plastic deformation during the first load cycle extends the elastic range in subsequent cycles provided the strain range is limited such that $E\varepsilon_R \leq 2\sigma_Y$, where E is the material elastic modulus. In design by analysis, $E\varepsilon_R$ is treated as an elastically calculated maximum stress range and limited to twice yield. In terms of the allowable stress S, this corresponds to a limit on the stress range less than or equal to 3S.

STRESS ANALYSIS

The terms *membrane* and *bending* in the various stress categories define the form of the stress distribution required for design by analysis: membrane stress is constant through thickness and membrane plus bending stress varies linearly through thickness, as illustrated for the 1D beam of Figure 1. Interpreting this requirement in analysis of complex structures is a major aspect of the elastic design procedure.

Design by analysis is now routinely based on FEA. The type of element used in the analysis has a major impact on implementation of the procedure. Two approaches are common: analysis using shell elements and analysis using 2D or 3D solid elements.

A shell structure is a doubly curved continuum whose thickness is generally much less than its radii of curvature. The geometry is fully defined by a continuous mid-plane of points (at equal distance from its bounding inside and outside surfaces) and the wall thickness at each point on the mid-surface.

The main load-bearing mechanism of a shell structure is *membrane* action of internal forces acting parallel to the curved mid-plane. Membranes are thin sheets of material with negligible bending stiffness that support transverse loads through a system of tensile stresses acting in the (curved) plane of the membrane. These stresses are constant through the thickness of the membrane and the state of stress is fully defined by three components acting on the mid-plane, two normal tensile stresses and one shear stress. The concept of membrane stress in shell structures is more general in that the shell can support compressive as well as tensile membrane stresses. Shell structures are also required to support shear forces, bending moments and twisting moments. These tend to be more significant at global discontinuities and in the vicinity of local loads and supports.

Shell theory and FEA using shell elements simplifies analysis by making assumptions about the form of shell deformation. Figure 2a shows an element cut from a generally loaded shell of uniform thickness *h* defined in orthogonal co-ordinate system *t-l-r* with origin at the center of the mid-surface and *r* in the direction of the mid-surface outward normal ***n*** in. In general, the outer surface is subject to traction $\boldsymbol{t}^{(\boldsymbol{n})}$ and the inner surface to traction $\boldsymbol{t}^{(\boldsymbol{n})}$, as in Figure 1b. In pressure vessel design, these tractions are usually external and internal pressure respectively.

In first order approximation elastic thin shell theory [3], equilibrium of the shell element requires a total of 10 internal forces and moments acting on the cut surfaces: four in-plane forces N_t, N_l, N_{tl}, N_{lt} as shown in Figure 1c and, shown in Figure 1d, two out of plane forces N_{tr}, N_{lr}, two bending moments M_t, M_l and two twisting moments M_{tl}, M_{lt}.

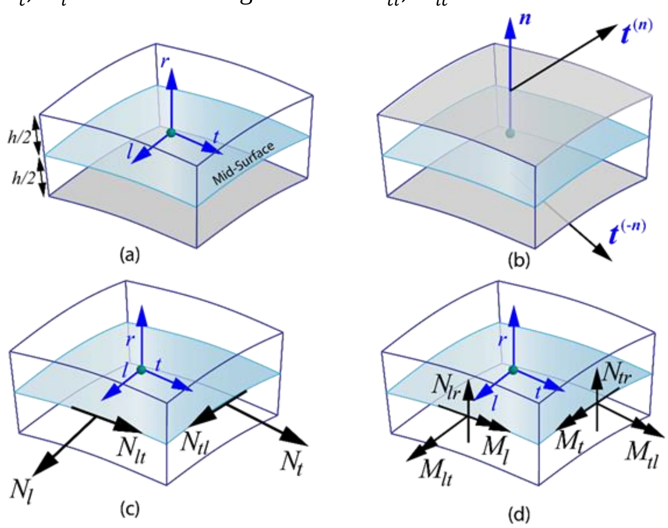

Figure 2 Element of shell and section forces.

First order theory assumes deformation of the shell conforms to the Kirchhoff hypothesis that straight lines initially normal to the mid-surface remain straight and normal after deformation and the shell thickness remains constant during deformation. This implies that all strain components in the direction normal to the mid-surface must be zero: that is, $\gamma_{lr} = \gamma_{tr} = \varepsilon_r = 0$. Further, the in-plane direct strains ε_l and ε_t and shear strain γ_{lt} are constrained to vary linearly with location *r* through the thickness of the shell. The corresponding state of stress for a homogeneous elastic material can be obtained from the 3D elasticity constitutive equation. As the transverse shear strains are zero, this leads to zero transverse shear stresses τ_{lr} and τ_{tr}. It is further assumed that the transverse normal stress σ_r is negligible and can be neglected. The remaining in-plane stresses σ_l, σ_t, and τ_{lt} are generally non-zero and constrained to vary linearly through the thickness of the shell.

The linear through thickness stress distributions can be treated as the superposition of two constituent stress distributions: a constant through thickness distribution and a linearly-varying distribution with zero value at the mid-surface. This form of stress distribution, illustrated for normal stress component σ_l in Figure 3a, conforms to the stress distribution required in design by analysis. The constant through thickness stress distribution σ_{lm} is treated as a *membrane stress* and the linearly varying constituent σ_{lb} as a *bending stress*.

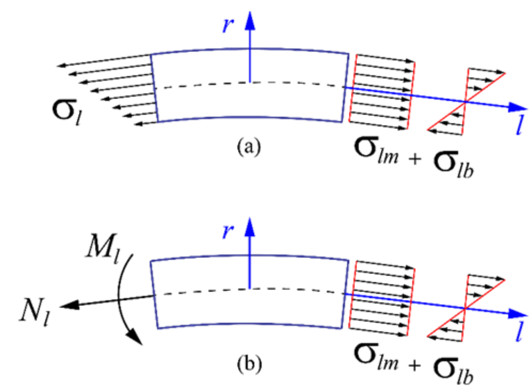

Figure 3. Membrane and bending stress constituent distributions.

The use of *membrane* and *bending* terminology is suitable for the constant and varying constituents of the normal stresses σ_l and σ_t. The in-plane shear stress τ_{lt} can also be defined in terms of constant and linear constituents, $\tau_{lt} = \tau_{ltm} + \tau_{ltb}$ however the terminology "shear bending stress" has no coherent meaning in stress analysis.

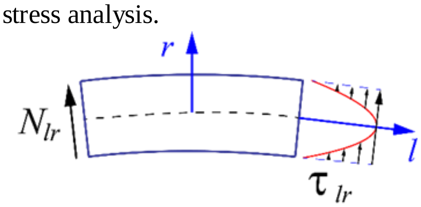

Figure 4. Parabolic out of plane shear stress distribution.

Considering an element of shell as shown in Figure 2, the action of the linearly varying in-plane stresses σ_l, σ_t, and τ_{lt} on the cut surfaces can satisfy equilibrium with the in-plane forces, bending moments and twisting moments. However, these stresses do not contribute to equilibrium with the two out of plane forces N_{tr}, N_{lr}, which requires non-zero shear stresses τ_{lr} and τ_{tr} acting on the cut surfaces. Following the deformation assumptions of first order shell theory and the elastic constitutive relationships, the transverse shear stresses reduce to zero. However, they are re-introduced to meet equilibrium requirements in a manner similar to that of beam theory by assuming a parabolic distribution of transverse shear (with zero value at the outer surfaces). These transverse shear stress distributions are not amenable to decomposition into constant and linearly varying components.

Thin shell theory explicitly defines the through thickness normal stress as zero. However, when the external surfaces are subject to pressure loading, equilibrium requires the radial stress at the inside and outside surfaces to be equal and opposite to the applied pressure. The distribution of radial stress through the thickness of the shell is undefined but for thin shells it would be approximately linear and therefore amenable to expression as constant and linear through thickness constituents. However, the terms membrane radial stress and bending radial stress are not meaningful in mechanics: in particular, the linearly varying component does not act about a center of bending.

First order shell theory is one of several shell theories based on different assumptions relating to geometry and deformation proposed for different applications. For example, thick shell theory retains the assumption that that straight lines initially normal to the mid-surface remain straight after deformation but they are not required to remain normal. This assumption, referred to as the Mindlin hypothesis, introduces transverse shear deformation directly into the deformation model. Shell finite elements in commercial FE codes may conform to a standard shell theory or be based on a 3D isoparametric formulation with either the Kirchhoff or Mindlin hypothesis defining through-thickness deformation.

Shell analysis and shell-based FEA is applicable to a wide range of pressure vessel component geometries. However, many configurations are not amenable to shell analysis. These vessels tend to be relatively thick and include features not captured by the approximations of shell theory. Components of this form now tend to be modelled using 2D and 3D continuum FEA. This enables more accurate modelling of complex features and boundary conditions but introduces problems in reconciling the calculated continuum stress results with Code membrane and bending stress distribution requirements.

In continuum mechanics, the 3-D state of stress at a point is fully defined by the six independent components of the Cauchy stress tensor σ_{ij}. Defined with respect to an orthogonal co-ordinate system *xyz*, Figure 5, the second order Cauchy stress tensor in matrix form is:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

There exists a particular orientation of the tensor reference co-ordinate system, the principal directions 1, 2, 3, at which the shear stress components are zero, Figure 5b. The normal stresses in these directions are the principal stresses σ_1, σ_2, σ_3 and the principal stress tensor is:

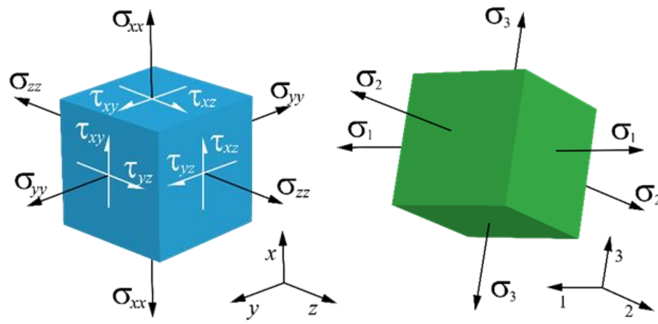

Figure 5. Stress tensor components and principal stresses

$$\sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

The finite element solution defines (approximately) the state of stress throughout the model domain. This may be specified in terms of stress components in any arbitrary co-ordinate system or in terms of principal stresses (and directions). The program also evaluates functions of stress such as von Mises equivalent stress and Stress Intensity.

In general, the stress distribution in a solid FEA model does not conform to the membrane plus bending form required in design by analysis. It is therefore necessary to post-process the calculated stresses to obtain stress distributions suitable for Code assessment. Three methods for linearization of FEA results are given in Annex 5.A *Linearization of Stress Results for Stress Classification*. Here, consideration is limited to 5.A.4 *Stress Integration Method* (stress linearization).

STRESS LINEARIZATION

Stress linearization is a mathematical technique used to derive membrane and membrane plus bending stress distributions suitable for Code assessment from continuum FEA stress distributions. Stress linearization was proposed by Kroenke [4,5] and Gordon [6] in the 1970s. Although widely used in industry thereafter, specific guidance was not included in ASME B&PV Code Section VIII Div. 2 until 2007 [7]. Code procedures for stress linearization are defined in Annex 5.A.4. These draw on outcomes from an extensive Pressure Vessel Research Council (PVRC) project investigating three-dimensional stress criteria by Hechmer and Hollinger, reported in detail in [7] and summarized in [8].

The stated objective of the Code stress linearization procedure is to determine membrane and bending stress distributions on planar cross sections through the thickness of a component called Stress Classification Planes (SCP). This is achieved by determining a linear stress distribution statically equivalent to the total (continuum) stress evaluated by FEA: i.e. giving rise to equivalent net membrane and bending actions at the SCP location. The SCP concept has been identified as problematic in both concept an application [9].

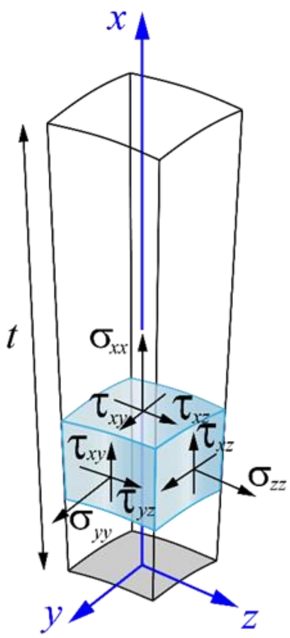

Figure 6. Hechmer and Hollinger SCL construction.

In Kronke's linearization procedure for axisymmetric FEA [4], a straight Stress Classification Line (SCL) is defined through the thickness of the model. This implicitly defines a plane of revolution about the rotational symmetry axis. Forces and moments acting on this plane can be defined by integrating the stress distribution along the SCL over the surface area and related to statically equivalent membrane and membrane plus bending stress distributions. No such plane is defined by a SCL through a 3D FE model and an alternative rationale is required to justify stress integration along the SCL in this context.

In [8] and [9], Hechmer and Hollinger define planar faces of infinitesimal width, nominally normal to the hoop and meridional directions, with the SCL. These, with curved planes representing the inside and outside faces, define an infinitesimal volume of material, as illustrated in Figure 6a, that "…should conform to the type of shell through which it passes, i.e., singly or doubly curved. Due to the curvature, the differential element appears to have an increasing width as the SCL progresses through the thickness…" [9]. Each point on the SCL is represented a differential "cube" on which the component stresses calculated by FEA act and the internal faces of the volume can serve as SCPs for stress integration.

SCL LOCATION

Annex 5-A.3 states "...For the evaluation of failure modes of plastic collapse and ratcheting, Stress Classification Lines (SCLs) are typically located at gross structural discontinuities…". The guidelines provided for selection of SCL location and orientation imply that that linearization of membrane plus bending stress is appropriate only in regions where bending stress predominates, hoop and meridional stresses vary monotonically through thickness and the shear stress distribution is parabolic or negligible. These conditions broadly conform to shell-type structural deformation. In [10], Hechmer and Hollinger state "…Code limits were developed for two-dimensional, axisymmetric geometries and loads, analyzed with shell theory, where bending planes can be defined and the planes remain plane. Thus, before determining the membrane and bending stresses, the potential failure plane is chosen. Under 3D conditions, a single bending plane for all loads may not exist or a chosen plane may not remain planar during loading...". The Code recommendations essentially restrict linearization to regions where a valid bending plane can be identified, excluding regions such as transitions between structural elements where shear and warping are significant in comparison with bending.

The ASME SCL local co-ordinate system for 3D and 2D solid models is shown in Figures 7a and 7b respectively. Here, the SCL is defined in a local orthogonal *x*-*y*-*z* co-ordinate system, where *x* is in the through thickness (ASME Tangential) direction, with the origin at the inside surface. The system is orientated such that the *y* axis lies in the hoop direction and the *z* axis in the meridional direction if these concepts are appropriate to the geometry considered: in general, they are arbitrary orthogonal in-plane directions).

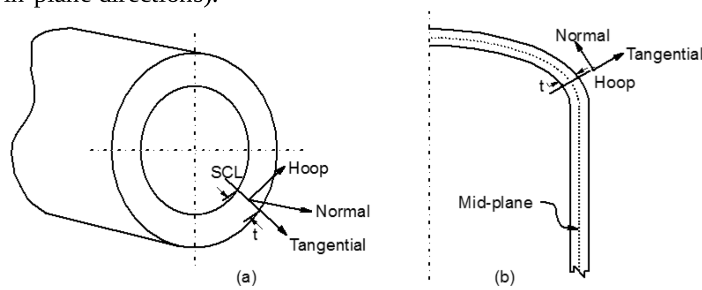

Figure 7. ASME local co-ordinate systems for (a) 3D and (b) 2D solid models.

The total stress at a point in the vessel calculated by FEA is defined by the six independent components of the stress tensor σ_{ij}: three normal stresses and three shear stresses. In general, the six stress components vary (non-proportionally) with location *x* over the length *t* of the SCL.

5-A.4.1.2 defines the membrane stress tensor as the tensor comprised of the average of each stress component over the stress classification line. This is evaluated by integrating the total stress distribution over length *t*, (5 A.1):

$$\sigma_{ij,m} = \frac{1}{t}\int_0^t \sigma_{ij}\, dx$$

The basis of this equation is the membrane stress distribution gives the same net section force as the total stress distrubution. The equation is valid if the width of the SCP associated with σ_{ij} has uniform width with respect to position *x*. It is not strictly valid for a SCP that varies in width, such as the planes associated with the SCL of Figure 6.

Bending stress distributions are evaluated only for specific stress components. The bending stress is determined by equating the total turning action or moment about the mid-point of the SCL with a linear distribution with zero value at the mid-point. The maximum value of bending stress will occur at one of the ends of the SCL and is given by (5.A.2):

$$\sigma_{ij,b} = \frac{6}{t^2}\int_0^t \sigma_{ij}\left(\frac{t}{2} - x\right)dx$$

Again, this is valid if the width of the SCP is constant with position *x*.

Bending stress is evaluated for the hoop and meridional stress components (i.e. σ_{yy} and σ_{zz}). These are the stress components associated with structural bending effects in shell theory. Bending stress is not calculated for the direct stress in the through-thickness direction, σ_{xx}, or for the in-plane shear stress τ_{xy}. Linear stress distributions are calculated for the two out-of-plane shear stresses using (5.A.2) but these are not referred to as "bending" stresses. The bending stress tensor therefore contains the maximum values of linear through thickness distributions corresponding to σ_{xx}, σ_{yy}, τ_{xy} and τ_{xz} only.

The stress linearization procedure therefore defines values of membrane stress for each of the six stress components and values of maximum membrane plus bending stress for four of the stress components suitable for application of 5.2.2.4 *Assessment Procedure*.

LIMIT LOAD BASIS

The Code stress categorization and stress linearization procedures draw extensively on concepts from shell analysis to define a coherent assessment procedure. This approach can therefore be expected to work reasonable well for thin shell-like structures. However, the main purpose of employing continuum FEA and stress linearization is to address problems not amenable to shell analysis. Hechmer and Hollinger [10] observed that "As conditions become less axisymmetric (structures or loading), the stresses along a line become similarly less related to shell-of-revolution stresses… [and] the need for a reference plane of bending raises questions on the relation of the method to predicting failure. The central issue is whether stress along a line, rather than on a plane, can be related to limit load or shakedown."

An alternative basis for stress linearization can be defined by considering the procedure in terms of lower bound limit load theorem, which states: *if, for a given load, there exists a statically admissible stress field in which the stress nowhere exceeds yield then that load is a lower bound on the limit load of the structure*.

Consider a SCL defined in a local orthogonal *x*-*y*-*z* co-ordinate system, with *x* is in the through thickness direction. A prismatic element of material width *dx* and *dy* is defined about the SCL, as illustrated in Figure 7. As *dx* and *dy* are made vanishingly small, variation in vessel thickness with respect to *x* and *y* becomes negligible and the element may be assumed to be a cuboid of length *t*.

The state of stress at any point within the element is defined by the six stress components in the *xyz* co-ordinate system. For a finite cross-section, the state of stress varies continuously throughout the element. However, for vanishingly small *dx* and *dy*, variation with respect to *x* and *y* becomes negligible and the stress distribution in the element varies with respect to *z* only. The state of stress within the element is therefore fully defined if the distribution of the six stress components along the SCL is known.

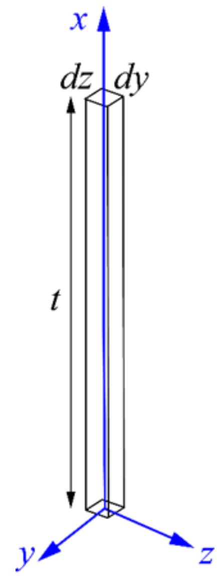

Figure 8. Isolated prismatic element of material for lower bound limit analysis.

If the prismatic section is cut from the wall of the vessel, the stress system must be such that it satisfies equilibrium with the internal forces and moments acting on the cut faces and the external surface tractions acting on the top and bottom surfaces.

The section forces can be evaluated from elastic FEA by integrating the appropriate stress components over the cut surfaces of the element of material. The surface tractions acting on the inner and outer faces of the element are most commonly internal pressure at the inside and zero pressure at the outside. From the lower bound limit load theorem, any stress field in equilibrium with these section forces and surface tractions which does not exceed yield at any location corresponds to a lower bound on the limit load of the structure.

In constructing a lower bound solution, the elastically calculated component stress distributions can be replaced by statically equivalent alternative distributions. Following the ASME stress linearization procedure, statically equivalent linear stress distributions can be adopted for selected stress components using equations (5.A.1) and (5.A.2). These can then be used in conjunction with the elastic stress distributions not chosen for linearization to fully define the state of stress along the SCL, from which the distribution of equivalent stress along the SCL can be calculated. The full value of through thickness normal stress should be specified at the inside and outside surfaces to satisfy the external surface traction boundary conditions of the problem. In the Code linearization procedure, this component is only considered in the membrane stress check. No linear variation is included in the bending stress tensor on the basis that this stress does not contribute a bending effect. However, this stress is required for equilibrium with the external load and, when included, limits the magnitude of the bending stress components of the in-plane normal stresses within the constraints of the yield criterion.

The lower bound limit load P_L for the specific class line is then obtained from proportionality:

$$P_L = \frac{\sigma_{e\,max}}{\sigma_Y}$$

where $\sigma_{e\,max}$ is the highest value of equivalent stress at any point on the SCL. However, as plastic collapse is a global failure mechanism, the response of an appropriate number of SCLs at appropriate locations would be required to establish a valid lower bound limit load for a component.

The SCL limit load calculation implicitly assumes all component stress distributions are primary. In addition, it does not in this form discriminate between primary membrane and primary membrane plus bending distributions, which should have different margins against limit collapse (as seen in Figure 1). The suitable margins for these distributions can be addressed relatively simply by constructing separate limit solutions for membrane only and membrane plus bending distributions and specifying appropriate design factors. The distinction between primary and secondary stress requires further intervention in the procedure.

SHAKEDOWN AND SECONDARY STRESS

The internal forces acting on the cut faces of the prismatic element in the limit analysis approach are calculated from the elastic stress distribution. The actual limit state internal force distribution may differ significantly from the elastic distribution due to post yield stress redistribution. If a moment reduces to zero after stress redistribution, the corresponding elastically calculated bending stress component is by definition secondary and should not be included in a limit load assessment. Such distributions can be identified by applying the established stress categorization procedure and removed from the limit model.

A possible alternative to stress categorization is to assume forms of stress distribution for limit analysis that better represents the limit state stress distribution. Candidate functions can be defined in terms of two parameters, such as the two coefficients of a linear polynomial in conventional linearization. Two other two-parameter forms are illustrated in Figure 9. These represent the form of stress distribution found in the limit states of components under bending and combined bending and torsion. Adopting these in a limit analysis would remove the need for identifying secondary stress distributions in the assessment procedure. Whilst this may be useful, it would be difficult to devise a general procedure in this form. For example, the bending-type distributions would not provide a good model the behavior of thick cylinders or spheres under pressure loading: continuum mechanics analysis of such components would suggest a logarithmic variation function would be appropriate.

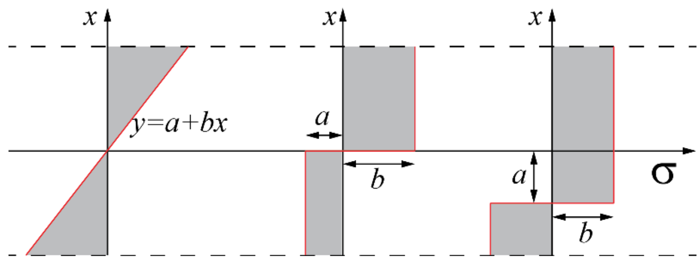

Figure 9. Candidate two-parameter stress distribution models for limit analysis.

CONCLUSIONS

The type of stress analysis employed in pressure vessel design based on elastic analysis and stress categorization can have a major effect on how Code procedures are applied in practice, in particular how the calculated stresses are related to the Code requirement for membrane and membrane plus bending distributions.

Design based on shell analysis incorporates the concepts of membrane and bending stress in the analysis method and is consequently relatively simple to implement. However, not all pressure vessel configurations are amenable to shell analysis and the wish of designers to employ more detailed stress analysis based on 2D and 3D continuum FEA raises a number of conceptual and practical problems in application of Code procedures.

Stress linearization is an established method for reconciling the continuum stress distribution evaluated in solid FEA with the membrane and bending forms required by Code criteria. The basis of the stress linearization procedure has conventionally been related to the bending response of shell structures, in particular the required form of stress distributions and definition of reference bending planes for equilibrium calculations. These considerations have influenced the guidelines for appropriate SCL location and orientation given in the Code, as well as which stresses should and should not be linearized.

The basis for stress linearization proposed here directly relates the stress along a line to the concept of limit load. This removes the need for some of the conceptual requirements associated with shell analysis from the procedure, such restriction of SCL location to identified bending planes.

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