INDUCED FRAGMENTATION OF ASTEROIDS DURING CLOSE ENCOUNTERS

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We consider the behaviour of rotating binary asteroids as they pass through Earth's Hill sphere, with primary interest in the effect the tidal force on the interaction between the two components of the binary and their post-encounter trajectories. We focus on contact binary asteroids bound by a regolith bridge, using both direct numerical simulation and analytical approaches to investigate the sensitivity of the system to different parameters. We find that the system is most sensitive to the angle between the binary pair and the orbital path, having a significant impact upon the energy change during a fragmentation event. We also give the results of some basic simulations of a deflection attempt on such an object.

INTRODUCTION

Radar observations suggest that a significant portion of asteroids with Earth-crossing orbits are binary systems, consisting of two components in contact with or in close proximity of each other. As shown by work such as that of Farinella *et al*[1] in the early 1990s, gravitational encounters can significantly alter the orbits and integrity of such binary systems. It is important to be able to accurately predict the motion of these bodies for maximum warning of any possible Earth collision event. The main effect we consider here is the Binary Exchange mechanism, as discussed by Borum *et al*.[2] This is where a rotation of a binary pair is disrupted by the tidal forces, causing the pair to split and altering the orbital energy of each component.

In this work, we consider Contact Binaries; single asteroids formed primarily by two large boulders. We consider both gravitationally bound pairs and those bound by a regolith bridge, as illustrated in Figure 1. The bridge is comprised of many small dust particles; the size of which cause in non-negligble London Dispersion forces between them resulting in adhesive effect between them and the larger components (This mechanism is similar to that discussed by Sanchez and Scheeres[3]). We also model an attempted deflection of the binary system to the close encounter, using and impactor-type method. This work forms a theoretical basis in the event such a deflection mission is required.

The analytical work starts with the full equations of motion for a binary system in a non-inertial reference frame rotating about the centre of the Earth. We use a standard virtual forces method, incorporating the Centrifugal, Coriolis and Euler forces to account for the rotation of the frame. Using a series of relevant approximations, the equations are simplified to give a handle on the factors that have a significant impact on the system. The numerical simulations are performed using a custom multi-body code that includes full inter-particle gravitational interactions, London

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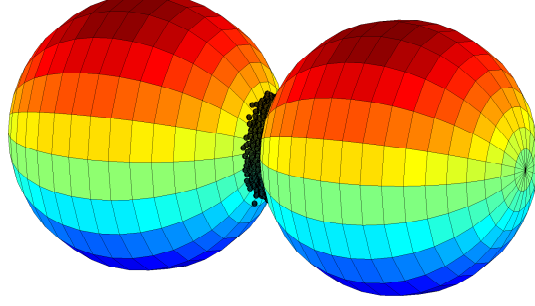[†](massimiliano.vasile@strath.ac.uk) Professor at University of Strathclyde, United Kingdom

Figure 1. Simulation of two large spheres joined by a finer regolith bridge.

dispersion forces and Soft-Body collisions similar to those implemented in *PKDGRAV* by Schwartz *et al*.[4] Previous work on the orbits of binary and rubble pile asteroid system are performed with modeling limited to only the gravitational interactions between the components.

The first set of simulations include only the two binary components and Earth. Simulations are performed for a range of binary system rotation speeds and two-body energies. The first set places the binary components are in a mutual circular orbit with artificially varies force between them to simulate the effects of a regolith bridge or other forces binding the two components. The second set of simulations include a full regolith bridge, incorporating the London Dispersion forces responsible for the binding of the bridge.

In this paper we start by detailing the equations of motion, considering the intial derivation and then re-arragning and simplifying. We then move on to describe our numerical simulation work, first discussing a study of the sensitivity of the parameters indicated from the equations of motion and then considering a series of simulations incorporating a full regolith bridge.

EQUATIONS OF MOTION

The analytical approach to characterising the system is based upon the full equations of motion in a rotating reference frame, with the angular velocity of the reference frame $\vec{\omega}$ varying such that the barycentre of the system remains at a constant angle to the Earth. We define the binary in terms of the position of its barycentre $\vec{R}$, and the vector $\vec{\rho}$ such that the two components are located at $\vec{R} \pm \vec{\rho}$ (the two components are considered to be identical, both of mass m and radius r). Using the standard non-inertial forces for a rotating frame (Centrifugal, Coriolis and Euler), we obtain the following equation:

$$\ddot{\vec{R}} \pm \ddot{\vec{\rho}} = -\frac{GM_{\oplus}(\vec{R} \pm \vec{\rho})}{\|\vec{R} \pm \vec{\rho}\|^3} + \left[\frac{-Gm}{4\|\vec{\rho}\|^3} + F(\vec{\rho})\right]\vec{\rho} + \vec{\omega} \times [2(\dot{\vec{R}} \pm \dot{\vec{\rho}}) + \vec{\omega} \times (\vec{R} \pm \vec{\rho})] + \dot{\vec{\omega}} \times (\vec{R} \pm \vec{\rho}) \quad (1)$$

The resulting equation can be split into two separate parts, detailing the motion of the barycentre and the relative motion of the binary components respectively ($\vec{\rho_{\pm}}$ denotes the acceleration relative to the barycentre of the binary component located at $\vec{R} \pm \vec{\rho}$).

$$\ddot{\vec{R}} = \frac{-GM_{\oplus}}{\|\vec{R}\|^3}\vec{R} + \vec{\omega} \times (2\dot{\vec{R}} + \vec{\omega} \times \vec{R}) + \dot{\vec{\omega}} \times \vec{R} \tag{2}$$

$$\ddot{\vec{\rho}}_{\pm} = \left[\frac{-Gm}{4\|\vec{\rho}\|^3} + F(\vec{\rho})\right]\vec{\rho} + \vec{\omega} \times (2\dot{\vec{\rho}} + \vec{\omega} \times \vec{\rho}) + \dot{\vec{\omega}} \times \vec{\rho} + GM_{\oplus}\left[\frac{\mp\vec{R}}{\|\vec{R}\|^3} - \frac{\vec{R} \pm \vec{\rho}}{\|\vec{R} \pm \vec{\rho}\|^3}\right] \tag{3}$$

We start our analysis by working with the equation of motion for $\vec{R}$; as stated earlier, the definition of $\vec{\omega}$ in our reference frame implies that the direction of $\vec{R}$ is constant. Thus, any components of $\ddot{\vec{R}}$ orthogonal to $\vec{R}$ must be zero. This implies the following definiton of the magnitude of $\ddot{\vec{R}}$, along with equation 3 (which is equivalent to the conservation of angular momentum):

$$\ddot{R} = \frac{-GM_{\oplus}}{R^2} + \omega^2 R \tag{4}$$

$$2\vec{\omega} \times \dot{\vec{R}} + \dot{\vec{\omega}} \times \vec{R} = 0 \tag{5}$$

Since we consider no forces acting outside of the orbital plane, the direction of $\vec{\omega}$ must remain constant and orthogonal to $\vec{R}$ at all times. We consider the conservation of angular momentum to obtain an equation for ω (the magnitude of $\vec{\omega}$):

$$\frac{\partial\left(R^2\omega\right)}{\partial t} = 2R\dot{R}\omega + R^2\dot{\omega} = 0 \implies R^2\omega = constant \tag{6}$$

Assuming the initial values R_0 and ω_0 at some time t_0 we get the following:

$$\omega = \frac{R_0^2\omega_0}{R^2} \tag{7}$$

Now, we consider the equation of motion for the position of the binary components relative to the centre of mass, $\vec{\rho}$. We chose to work solely with the equation describing the motion of the component located at $\vec{R} + \vec{\rho}$; an analysis could be performed for the other component but similar results would be expected. We make the assumption that the pair remains in a mutual circular orbit until the fragmentation event occurs. As such, by considering only components of the vector acting in the $\vec{\rho}$ direction we obtain an equation describing the forces responsible for the binding of the pair. We define the Binary-Orbit angle θ as the angle between $\vec{R}$ and $\vec{\rho}$ at a given time.

$$\ddot{\rho} = \left[\frac{-Gm}{4\rho^2} + F(\rho)\right] + 2\omega\dot{\rho} + \omega^2\rho + GM_{\oplus}\left[\frac{\cos\theta}{R^2} - \frac{R\cos\theta + \rho}{(R^2 + 2R\rho\cos\theta + \rho^2)^{\frac{3}{2}}}\right] \tag{8}$$

If we consider the fragmentation event to be instantaneous and that the radius of the binary orbit remains constant up until this point, we can neglect the term involving $\dot{\rho}$ in equation 8. The resulting equation has two important parts; namely the attractive and repulsive terms.

$$\ddot{\rho}_{attr} = \left[\frac{-Gm}{4\rho^2} + F(\rho)\right] \tag{9}$$

$$\ddot{\rho}_{rep} = \omega^2\rho + GM_{\oplus}\left[\frac{\cos\theta}{R^2} - \frac{R\cos\theta + \rho}{(R^2 + 2R\rho\cos\theta + \rho^2)^{\frac{3}{2}}}\right] \tag{10}$$

The attractive force remains constant throughout the orbit, whereas the repulsive force varies with distance to the Earth, relative angle of the pair and orbital angular velocity. The sensitivity of the force to these parameters is investigated with full numerical simulations of such encounters.

3

NUMERICAL SIMULATIONS

Multi-Body Code

As mentioned previously, the numerical simulations are performed using a simple multi-body code that implements soft-body collisions and makes use of a Second-order "Leapfrog" simplistic integration scheme, much like the work of Schwartz et al.[4] The code makes use of OpenCL to allow for the utilisation of multiple processor cores and the possibility of running the code on a GPU.

The simulations incorporate gravitational interactions between all particles, as well as London dispersion forces. For point particles, the attractive and repulsive potentials that characterize the London dispersion interaction are proportional to $1/r^6$ and $1/r^{12}$, respectively. However, the formulation is more complicated for finite sized particles. Hamaker[5] gives a derivation of the attractive potential between two spheres, calculated as the integral of the forces between infinitesimal point-particle elements over both spheres. In principle, this integral could be applied also to the repulsive term. However, since we use a soft-body collision method, including this term would provide no additional accuracy. Instead, we use a Pseudo-potential method (detailed in Equation 9): for distances greater than a given cut-off radius r_{cutoff}, the full value of the Hamaker[5] potential is used; for distances less than r_{cutoff}, a quadratic function is used. The quadratic used is set to have the same value and gradient as the Hamaker[5] potential at the cut-off point, and a global minimum at some point r_{min} that is set to be less than the sum of the radii of the two spheres, thus allowing some overlap and "softening" the potential. In our simulations we use a value of $8 * 10^{-19} J$ for the Hamaker constant A. This value is chosen since it is in the region of the highest values for known elements. As such our simulations give a strongest possible binding for our considered regolith size since any real case would have a lower value of the Hamaker constant (however it should be noted that the strength of the interaction also grows as regolith particle size decreases, and as such it is possible that the strength of binding could actually be stronger if finer regolith were present).

$$V(r) = \begin{cases} -\frac{A}{6}\left(\frac{2R_iR_j}{r^2-[R_i+R_j]^2} + \frac{2R_iR_j}{r^2-[R_i-R_j]^2} + \log\frac{r^2-[R_i+R_j]^2}{r^2-[R_i-R_j]^2}\right) & : r > r_{cutoff} \\ \frac{V'_{cutoff}}{2(r_{cutoff}-r_{min})}(r-r_{min})^2 + V_{cutoff} - \frac{V'_{cutoff}(r_{cutoff}-r_{min})}{2} & : r \leq r_{cutoff} \end{cases} \quad (11)$$

Simulation Setup and Results

Using the code described, a series of simulations are performed to explore the dynamics of close encounters of binary asteroids with Earth-sized bodies. Before discussing specific results, we outline the setup considered and the nature of the output of the code.

The majority of our simulations detailed in this section consider a parabolic encounter with a closest approach distance of 124km. Our code takes a list of initial positions and velocities in Cartesian co-ordinates as the input, and outputs a track of positions, velocities, kinetic and potential energies for each body considered. To generate the input parameters to give the desired orbit path, we first simulate a single body (with the same mass as the sum of the two binary components) starting at the closest approach point with velocity orthogonal to the direction to the Earth, with the simulation running for half the final desired time. We then take the resulting final output position and velocity, reverse the velocity and use this as the initial position and velocity for the barycentre of the binary for the main simulation. In all our simulations we consider a binary comprised of two 2 metre diameter spheres, each of mass 10 tonnes, with an initial mutual orbital radius of 1.01 metres from the barycentre and rotating about the barycentre with an angular speed of 1.7032 $mrad \cdot s^{-1}$.
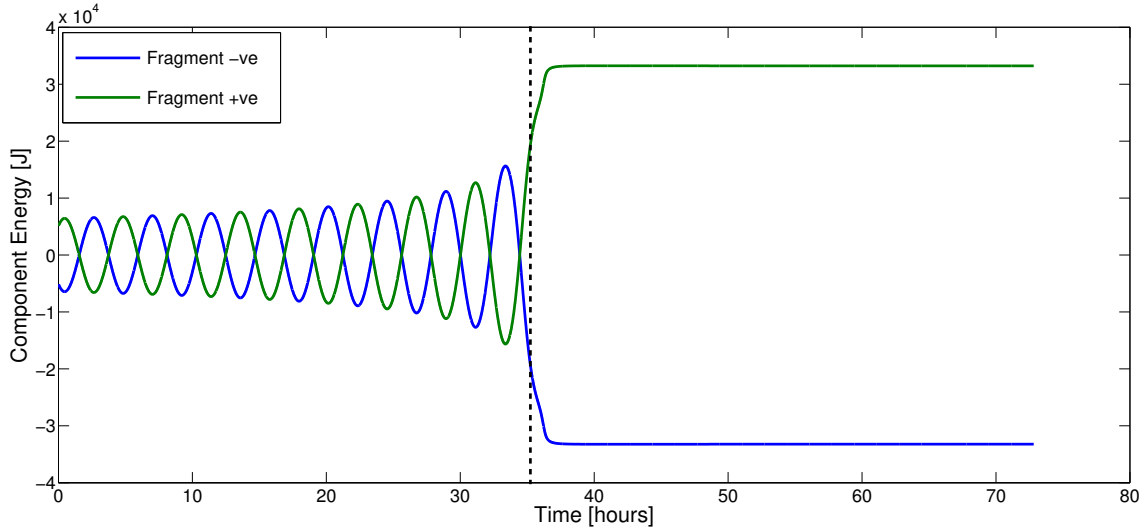
Figure 2. Plots of the Gravitational potential and kinetic energy of both components of the binary from a numerical simulation with a close approach distance of 124km. The vertical dashed line indicates the time at which fragmentation occurs.

The two data outputs we primarily consider are the sum of kinetic and potential energies for each component individually and the acceleration of each component relative to the barycentre; Figure 2 shows a plot of this total energy for the two binary components produced by a simulation, and figure 3 the relative acceleration. After the fragmentation one components is captured (having its orbital energy reduced below zero) and the other escapes (gaining orbital energy). The break-up is observed as the energy trace transitions from the oscillating regime to a steady state; we define the fragmentation energy as the difference between the average total energy of a binary component and the final energy state.

Analytical Comparisons

Using the set of equations derived earlier, we can make estimates for the behavior of the binary system. By running a series of simulations with varied initial conditions, we gauge the relationship between the fragmentation energy and the parameters θ, ω and R. Firstly, however, we attempt to verify the accuracy of some of the assumptions applied in the derivation of the equations; namely the constant rotation rate and radius of the binary pair prior to the fragmentation event. To do this, we run a simulation with a close approach distance of 124km and an orbital angular speed at closest approach of $1.703\ mrad/s$.Figure 4 shows the radius and angular velocity of the binary pair over the course of an orbit. Prior to the fragmentation event, the fluctuation in angular speed is approximately 1% of its magnitude.

Now, we consider the sensitivity of the fragmentation energy and peak component acceleration (relative to the barycentre) to the parameters θ, ω and R. θ is varied by changing the starting orientation of the binary pair, whilst ω and R are changed by altering the orbital parameters. Assuming the rotation rate is balanced with the attractive force between the binary components such that the
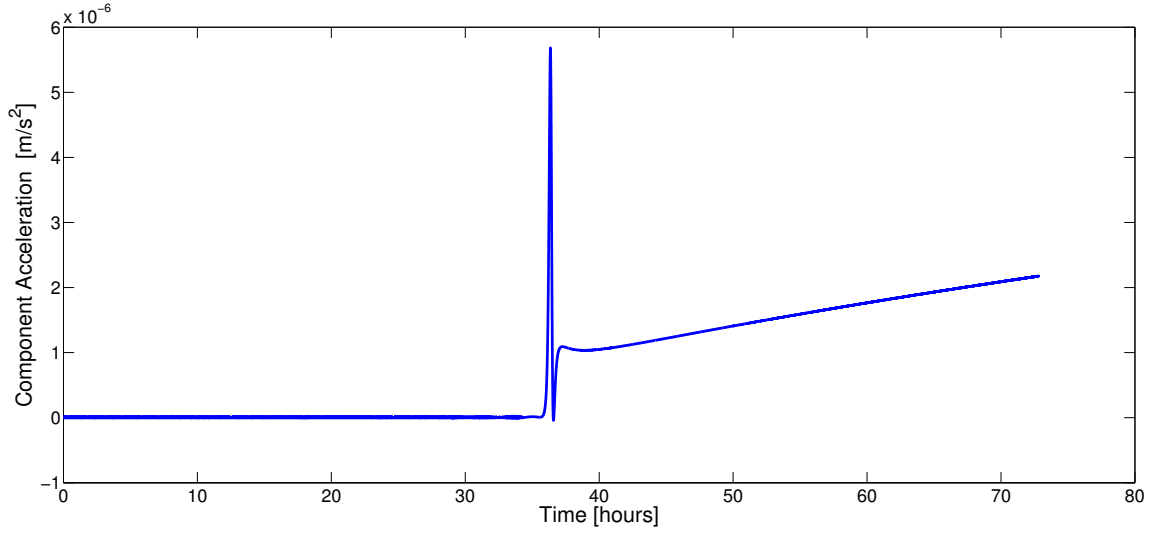
Figure 3. **Plot of the acceleration of one binary component relative to the barycentre from a numerical simulation with a close approach distance of 124km.**

orbit is circular when in free space (as is the case in all simulations presented here), any effective repulsive force between the two components introduced by the influence of a third body would alter the orbit into an elliptical one or if strong enough eliminate the possibility of any stable orbit. If the strength of the repulsive force is greater than that of the attractive, the net effective force between the pair is repulsive and as such fragmentation is inevitable.

To investigate the sensitivity of the fragmentation to θ we ran a series of simulations as discussed above, witha close approach distance of 124km and orbital angular velocity at close approach of 1.703 $mrad/s$. We varied the starting orientation in 30 uniform steps between 0 and π. The results of these simulations are illustrated in figures reffig:phaseand 7. Figure 5 shows a surface of the total energy one component in a binary system as a function of time and angle θ (Since θ is a function of time, we consider its value immediately prior to the fragmentation in our plots here). As can be seen there is a significant variation in the final energy, with some cases getting captured while others escaping. The discontinuities present in some curves are due to collisions between the components which occur as their orbit is destabilized. The relative velocity of these collisions is of the order of $10^{-4}m/s$, so no destruction or deformation on the boulders is expected. As the angle varies steadily, the fragmentation energy initially varies as a small offset. However, at a certain point the energy varies more rapidly as the components swap (the one which gained orbital energy during the encounter for lower values of θ instead loses energy and vice versa). Figure 5 shows the peak acceleration experienced over the same range of θ values. The values exhibit the general periodicity predicted in equation 10. However, the region for theta between 1 and 2.1 radians exhibits very rough and discontinuous behaviour. For values of theta in this region, there are significant collisions between the two components, so we would not expect our equations to be valid in these cases. These collisions can be identified as the jumps in the value of component energy present in figure 6

For the analyses of both the ω and R sensitivity, the setup is somewhat more complicated in principle as it is not possible to scale one without altering the other throughout the rest of the orbit. However, since the effects of the disruption will be greatest at the closest approach point,
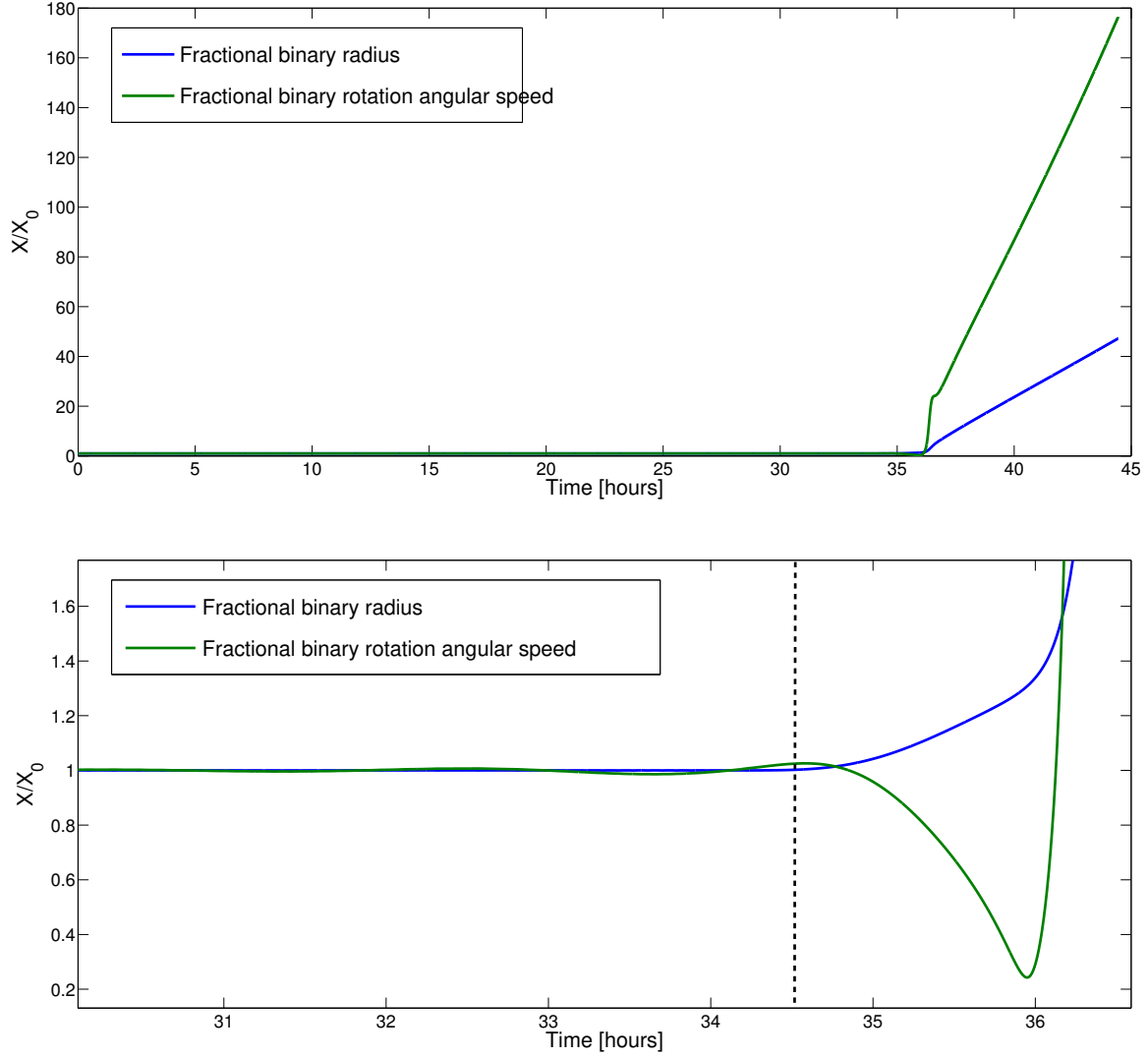
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Figure 4. **Plots of angular velocity and radius of a binary pair as fractions of the initial values (full encounter duration and close-up of fragmentation). The vertical dashed black line in the close-up plot indicates the fragmentation event time.**
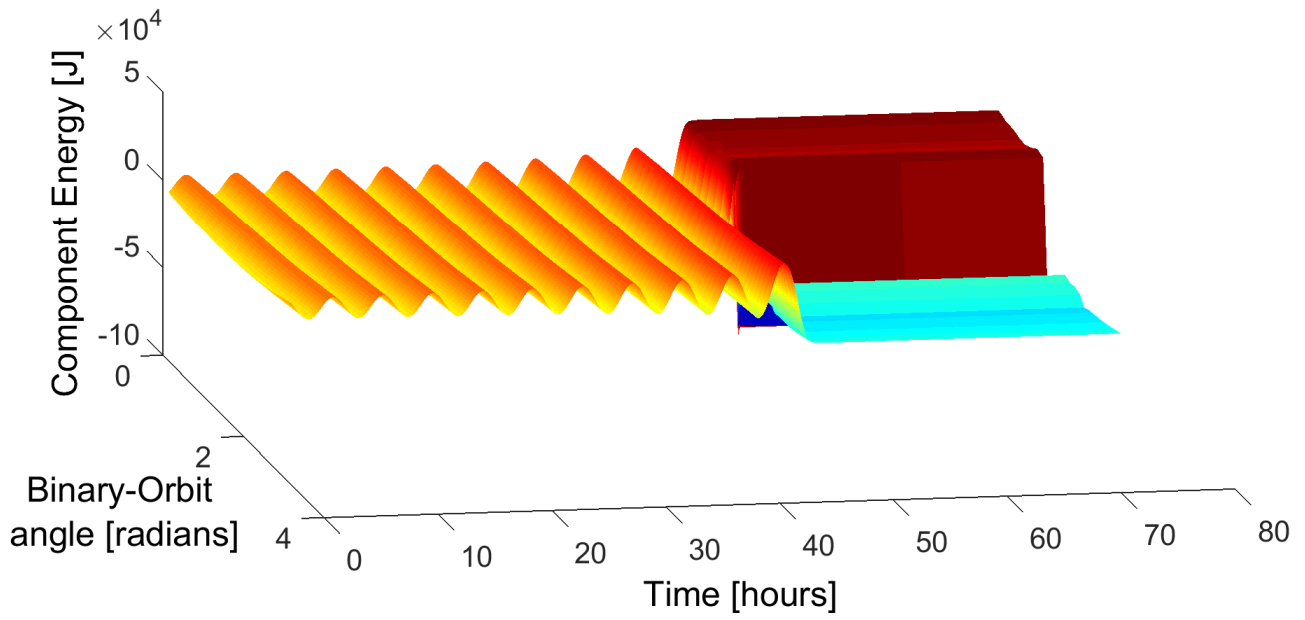
Figure 5. Surface showing the energy of one component for varying angle θ over the duration of the encounter.
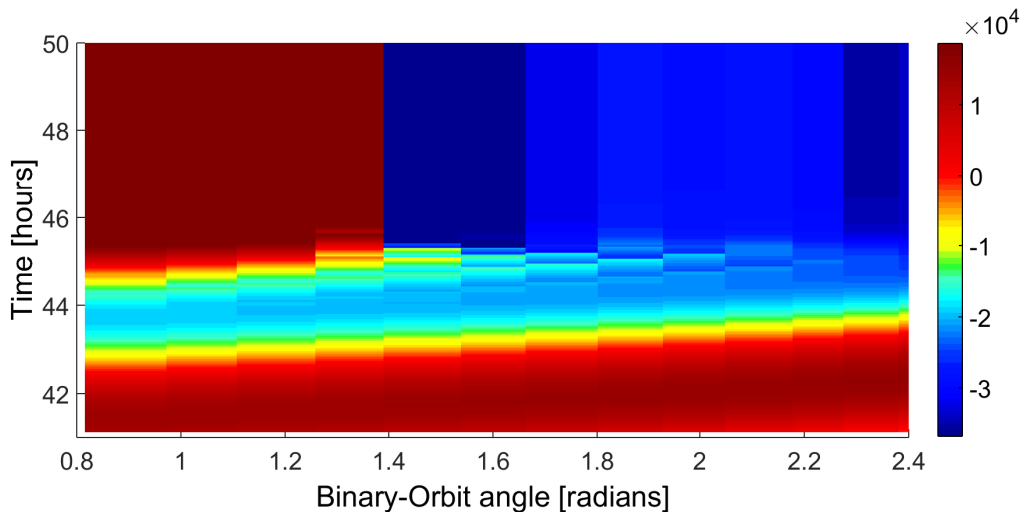

Figure 6. Surface showing the energy of one component for varying angle θ over the duration of the encounter (close-up of fragmentation time).
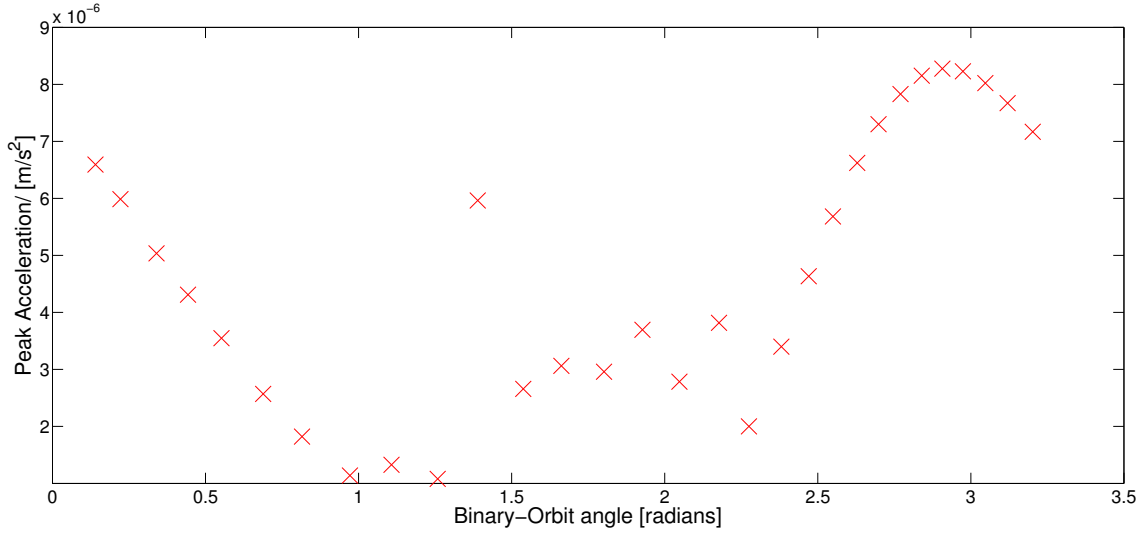
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Figure 7. Plots of the Peak acceleration relative to the barycentre for varying angle θ.

we change each parameter in uniform steps whilst keeping the other constant at this point. Using this procedure, the sensitivity of the acceleration during fragmentation to the Earth-Binary distance R is investigated. This is done by varying the closest approach distance whilst keeping the ratio between R and v at closest approach (and hence ω) constant. We increase the close approach distance in increments of 20km, starting at 124km, keeping a constant orbital angular speed at close approach of $1.703 \; mrad/s$. As can be seen in figure 8, the peak component-barycentre acceleration experienced during the fragmentation decreases smoothly as close approach distance increases; this is as expected from equation 10.

Finally, we consider the sensitivity of the acceleration during fragmentation to the orbit angular speed ω. We maintain a constant closest approach distance of 124km but increase the orbital energy (and hence both the tangential and angular orbital velocity at this point). We start with an orbital angular speed of $1.703 \; mrad/s$ and increase in steps of $0.01538 \; mrad/s$. Figure 9 shows a plot of the energy traces of a single component for a range of values of ω. As can be seen the peak acceleration decreases steadily as ω increases; this is contrary to what we would expect from equation 10. One possible cause of this discrepancy is that as the value of ω increases, the time for which the binary is in the strongest region of the gravitational field decreases. Since the tidal force is proportional to the distance between the two binary components, it may be that the components do not drift sufficiently far apart whilst in close proximity to the Earth, hence reducing the peak acceleration experienced.

Regolith-Bound Binary

Following the sensitivity analysis, numerical simulations of two boulders bound by a regolith bridge on a Parabolic trajectory are performed. For the small grain sizes considered in our regolith ($100\mu m$), the dominating force between particles are London Dispersion forces. Ideally, we wish to consider binary components with radii over 1 metre bound by a regolith bridge consisting of small ($100\mu m$) and medium ($1cm$) particles. However, the number of small particles that would need to be considered would make simulation of such a system not feasible. To overcome this, we first consider the binding effect between the small and medium particles. We perform a simulation with
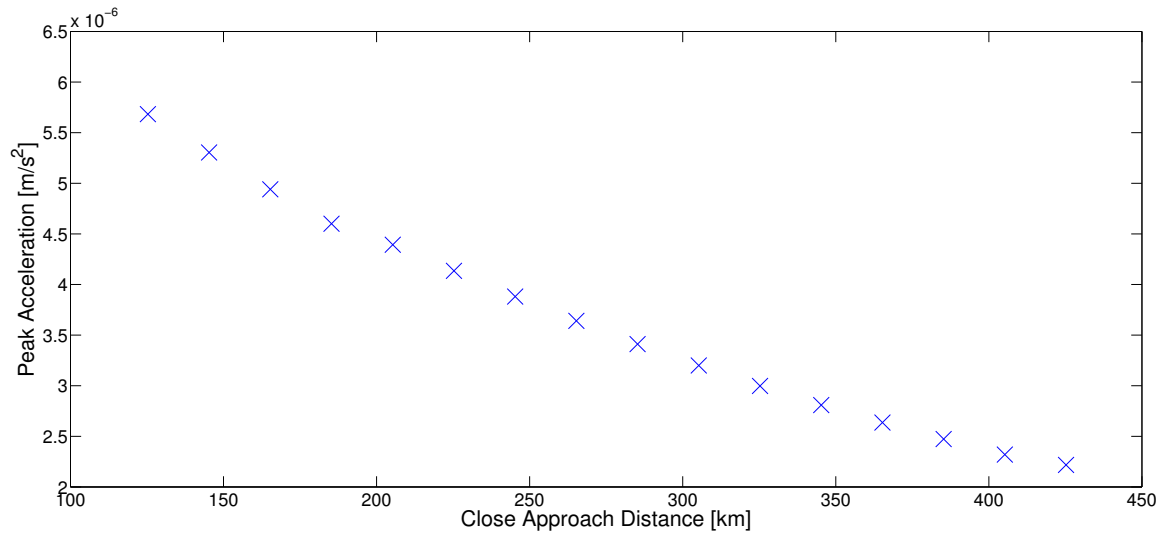
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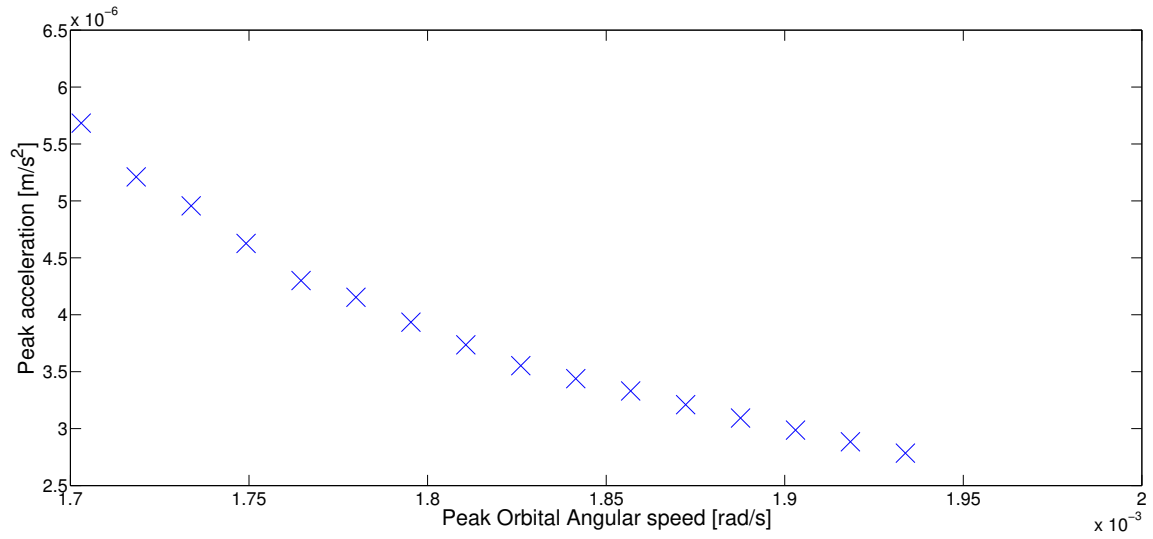**Figure 8. Plots of the total energy for varying closest approach distance R.**

Figure 9. Plots of the total energy for varying orbital angular speed ω.
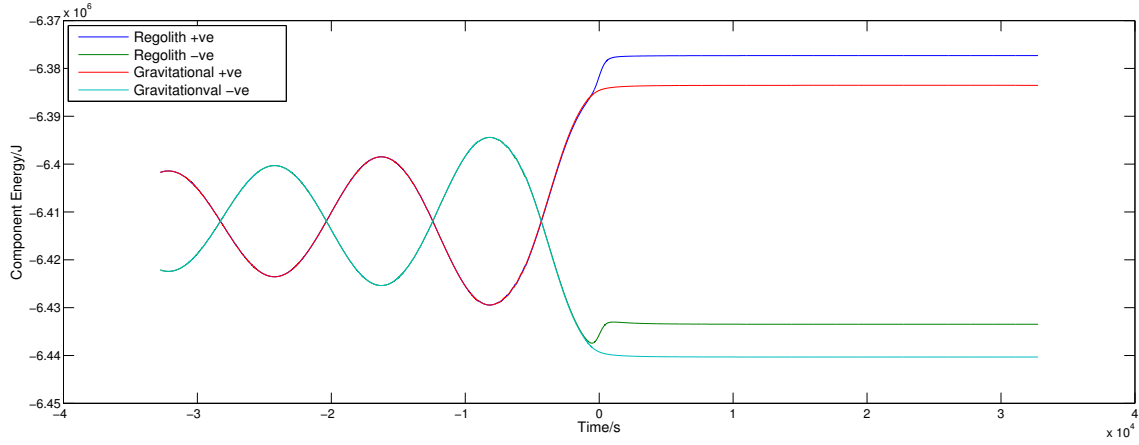
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Figure 10. Plots of the Total energy of both components of the binary from numerical simulations; both for the purely graviational and the regolith-bound cases.

two medium sized granules bound by a bridge of 510 small particles. The system is placed under rotation, initially with the rotation speed matching that of a purely gravitational circular orbit for the two medium granules. The rotation rate is gradually stepped up until the binding of the granules fails. The value of the Hamaker[5] constant used in the simulation is then increased such that two medium sized granules have the same maximum rotation speed without the bridge as when bound by the small particulate bridge with the real value of the Hamaker[5] constant. A factor of approximately 200 was found to be sufficient; this is roughly equal to the number of small particles in contact with each medium granule. The system considered for the regolith-bound binary simulations is comprised of the same larger binary components as in previous simulations, with the addition of a bridge comprised of 510 medium sized granules; the additional effect of the small particles is approximated by increased strength of the London Dispersion force.

As with the previous encounter simulations, a closest approach distance of 124km is used, and the binary asteroid considered has a total diameter of 4 metres. Figure 10 shows the total energy results from the simulation, compared to those from a simulation of the same binary components and close approach distance but without the regolith bridge. The curves match closely before the fragmentation event; however when fragmentation occurs both binary components gain additional energy from the collapse of the binding regolith bridge. It can be seen that the amount of energy gained from the collapse is about 30% of the difference in total energy between the two components.

Deflection attempt

Lastly, we present here a few simulations of attempted deflections for a binary asteroid on a collision course with Earth. The validity of these simulations has two main limitations. Firstly, the soft-body collision model does not allow for any permanent deformation, fragmentation or similar phenomena; due to the high velocities required for an impactor-based deflection some degree of deformation would be expected in reality. The second limitation is that of size; due to computational time constraints, the binary pairs we consider in all our simulations are significantly smaller than any which would need to be deflected (having a total diameter of 4 metres and a mass of 20 tons). Still, these simulations indicate certain effects and phenomena which may be present in a more realistic deflection scenario.

To select an appropriate value for the spring constant and damping constants for the soft-body collisions, we first simulate a collision between two 1 metre radius spheres at 1 m/s. We adjust the parameter such that the system loses roughly 20% of its Kinetic energy in this collision, thus matching the experimental data of the coefficient of restitution for metre-scale granite spheres as measured by Durda *et al.*[6] Our impactor is modelled as a 60cm diameter sphere of mass 1 ton; the impact velocity is 840 m/s, with the impact occuring parallel to the $\vec{\rho}$ direction.

We ran simulations for several cases: an undeflected binary, a deflected single-body with the same mass as the binary and both components of a deflected binary. Figure 11 shows the orbit paths produced as the results of these simulations. It can be seen that the post-deflection paths of the two components of the binary are greatly different to that of a single body of equal mass. This is likely due to a Newton's Cradle type effect in the collisions; as such the component which is struck by the impactor passes all of the additional momentum from the impact to the second, leaving its momentum largely unchanged by the deflection attempt and as such it still impacts the Earth (the binary is also fragmented during the impact). This effect is most prevalent when the two components are of roughly equivalent mass.

We also simulate the effect of an impact on a full regolith-bound binary; both with the same impact speed as the previous case and with a lower speed impact (50 m/s). Figure 12 shows the positions of the particles in the simulation at 3 time steps (1.221 ms prior to impact, 2.685 ms and 19.287 ms following the impact) for the high speed case: It can be seen that a large amount of the regolith is discharged and the bridge fails completely, causing the pair to fragment immediately. The outcome is very much the same for the low speed case.

CONCLUSION

We present a methodology to model contact binary asteroids bound by regolith during an encounter with a large body such as Earth. Using an analytical approach we have attempted to identify which parameters should impact upon the energy involved in the fragmentation of such an object and the extent of such sensitivity. We find that there the dynamics of the fragmentation event are highly sensitive to the angle of the binary pair with respect to the orbital path, whilst the sensitivity to the closest approach distance and speed of the orbit are much more smooth and in line with the analytical predictions. Also we find that there is a notable change in the fragmentation energy when a full regolith bridge is considered.

Additionally, we give the results of some basic simulations of a deflection attempt. These suggest that the dynamics of an impact-based deflection differ greatly based on whether the object is a single body or a contact binary.

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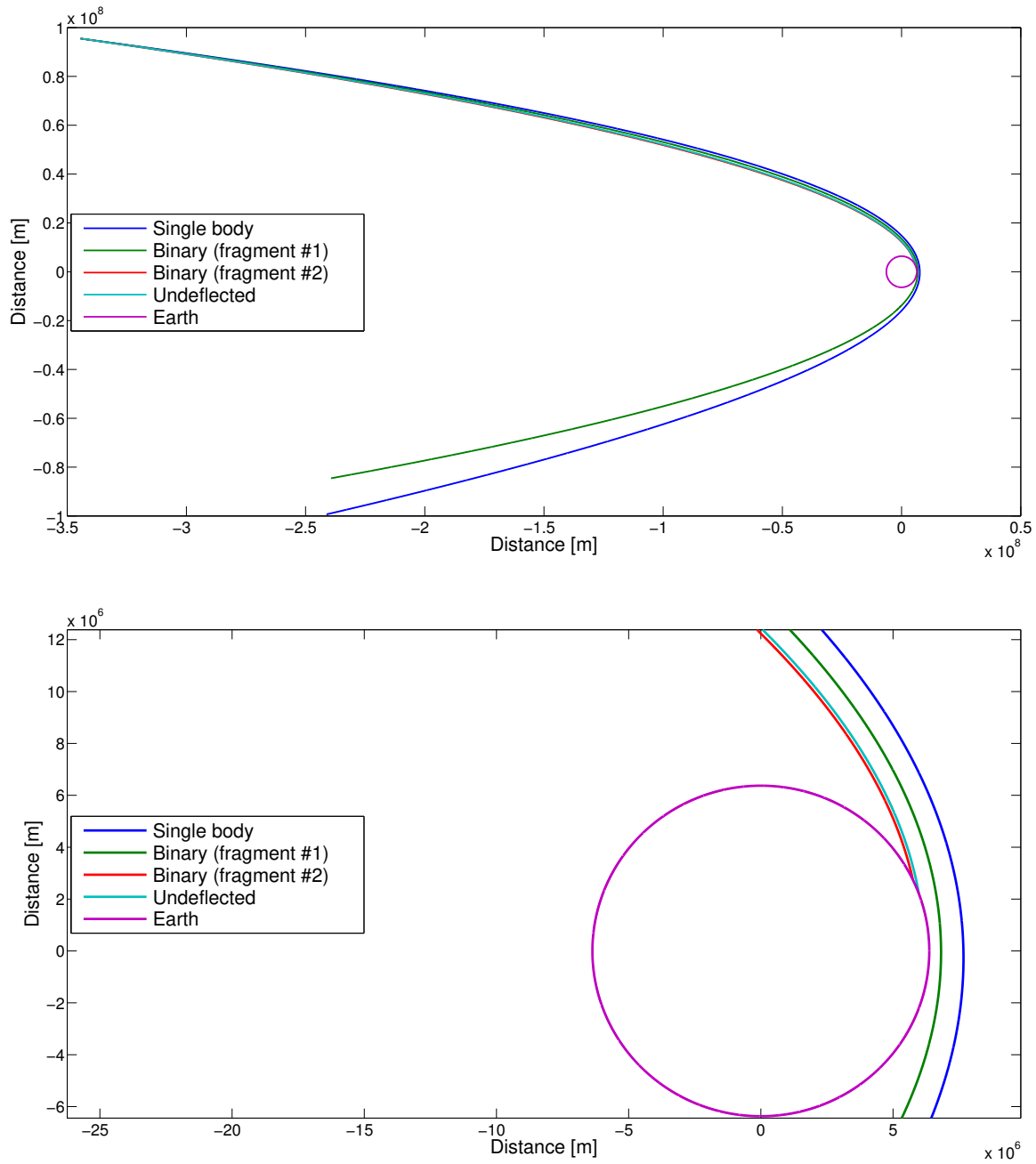
Figure 11. Orbital paths of binary and single body asteroids pre and post deflection (full orbit and close-up of close encounter/impact).
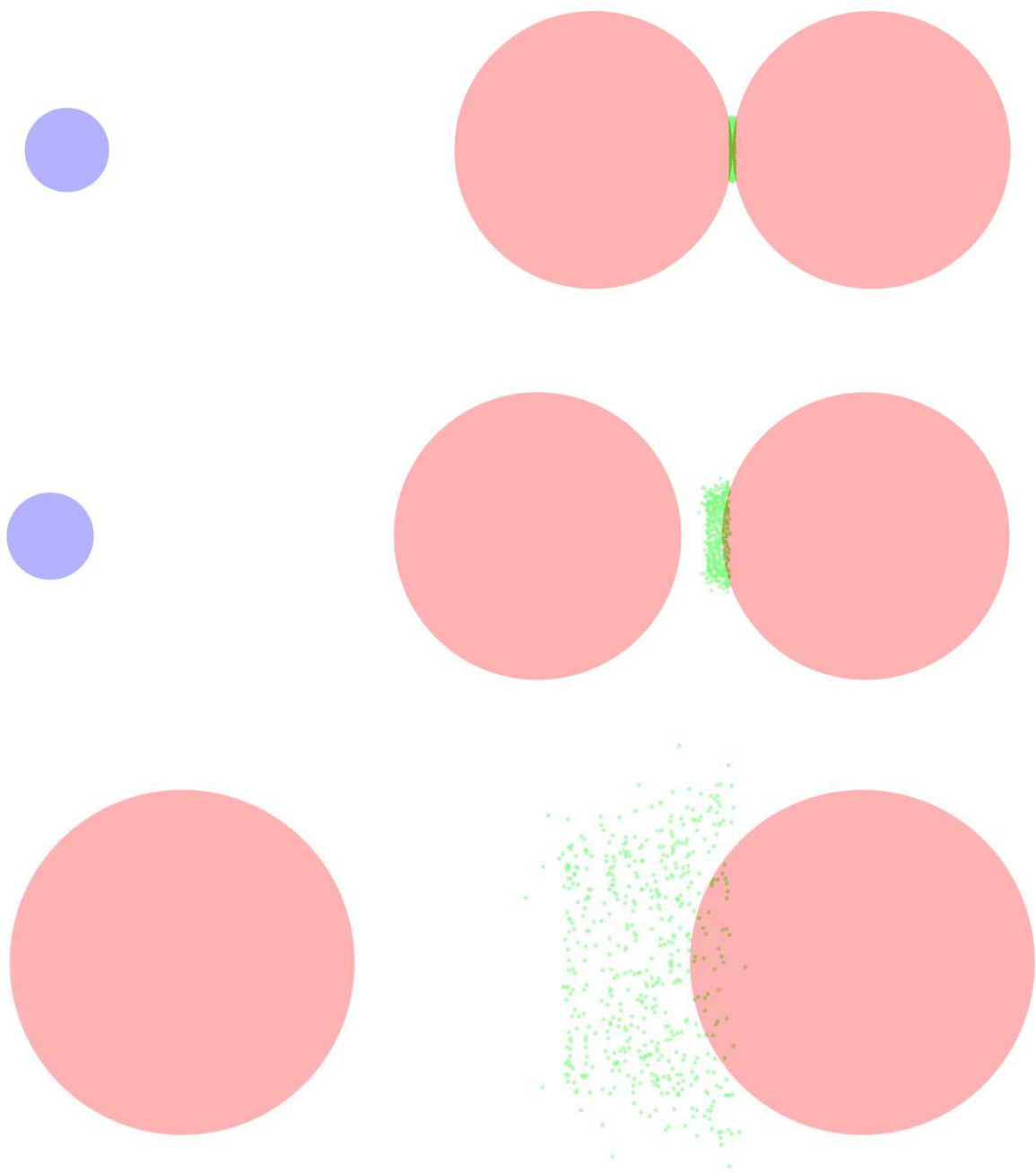
Figure 12. Particle positions during simulations 1.221 ms prior to impact, 2.685 ms and 19.287 ms following the impact. The impactor is blue, the binary components are red and the regolith particles are green.

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