Fractional Fourier Based Sparse channel estimation for multicarrier underwater acoustic communication system

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***Abstract*--This paper presents a hybrid sparse channel estimation based on Fractional Fourier Transform (FrFT) for orthogonal frequency division multiplex (OFDM) scenario to exploit channel sparsity of underwater acoustic (UWA) channel. A novel channel dictionary matrix based on chirp signals is constructed and mutual coherence is adopted to evaluate its preservation of sparse information. In addition, Compressive Sampling Matching Pursuit (CoSaMP) is implemented to estimate the sparse channel coefficients. Simulation results demonstrate a significant Normalized Mean Square Error (NMSE) improvement of 10dB over Basis Expansion Model (BEM) with less complexity.**

***Keywords*--Fractional Fourier Transform (FrFT), orthogonal frequency division multiplex (OFDM), Dictionary matrix, underwater acoustic (UWA) channel, Compressive Sampling Matching Pursuit (CoSaMP)**

1. INTRODUCTION

Underwater acoustic communication (UWA) [1] suffers from doubly selective (both time and frequency) channel, attributed to long delay spread and serious Doppler spread from high mobility between transmitter and receiver. Orthogonal Frequency Division Multiplexing (OFDM) [2], as a multicarrier communication scenario, is widely used in wireless communications channels. Compared to the single carrier system [3] with complicated equalization, OFDM is superior in resistance to Inter-Symbol Interference (ISI) [4] and low complexity. However, OFDM is more vulnerable to Inter-Carrier Interference (ICI) [4], due to loss in orthogonality caused by high Doppler spread. Recent research has focused on channel estimation of block transmission over UWA channel. Basis Expansion model (BEM) [5] is one of the popular approach which models time variant character of channel via Basis Expansion coefficients and different basis, such as Complex Exponential basis (CE-BEM)[5] and polynomial basis (P-BEM) [6]. In [7], it is shown that DPS-BEM, which is employed to model the time variant channel by providing DPS sequences basis, performs better than other types of BEM, because DPS sequences is set to approximate bandlimited channels over finite time windows, alleviating spectral leakage and truncating problem in CE-BEM. Thus both bandlimited and time-limited nature of channel are taken into consideration. However, more unknown coefficients need to be estimated in UWA channel, than in wireless communication channel due to larger delay spread, bringing about huge complexity. It has been explored the sparse nature of UWA channel, entailing that majority of channel energy is concentrated on a few paths, as is shown in Figure 1.Therefore, Compressed sensing (CS) [8] is applied to estimate the sparse channel information and a good recovery performance attributes to optimization algorithm as well as dictionary matrix construction which to large extent reflects the channel nature, including delay and Doppler spread.

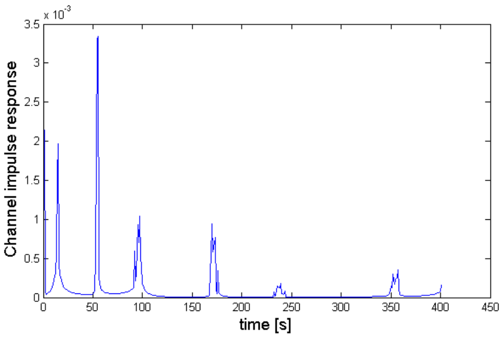

Figure 1: Channel Impulse Response

Fractional Fourier Transform (FrFT) [9], a generalized form of Fourier transform (FT), is applied to modulate OFDM waveform and it plays better performance in suppression of ICI with comparable complexity [10] [11]. However, a sparse channel estimation based on Fractional Fourier Transform for both single and multicarrier communication system was not investigated in the literature

In this paper, a novel sparse channel estimation based on fractional fourier transform (FrFT). A dictionary matrix based on chirp basis is constructed, including the sparseness information of both delay and Doppler. Mutual Coherence, instead of restricted isometry property (RIP) [12] is used to evaluate whether dictionary matrix keeps sparse information well. Due to more energy of channel frequency matrix based on FrFT concentrates along the diagonal than that of FT, leading to less ICI and coherence between columns of dictionary matrix. In addition, Compressive Sampling Matching Pursuit (CoSaMP) [13] Algorithm is adopted to estimate the coefficients of sparse channel paths.

The remainder of paper is organized as follows, Section 2 presents the proposed sparse channel estimation based on FrFT, with FrFT-OFDM system, Fractional Fourier based dictionary matrix, mutual coherence and CoSaMP algorithm. In Section 3, simulation results and discussions are presented. Section 4 gives a conclusion of the paper.

Notation: In this paper, transpose, conjugate and conjugate transpose are denoted as $[.]^T, [.]^*$ and $[.]^H$ respectively. $diag\{.\}$ denotes a diagonal matrix produced by a vector $[\cdot]_{i,j}$ extracts the ith row and jth column element from a matrix. Finally, $<.>$ is the inner product and $|.|$ is the absolute value.

2. SYSTEM MODEL

2.1 FrFT-CS Transceiver

The proposed Fractional fourier based channel estimation is called (FrFT-CS) due to implementation of compress sensing (CS), and FrFT based transceiver is shown in Figure 2. The input signal **s**, modulated by inverse FrFT (IFrFT), goes through the UWA channel with additive Gaussian noise $n(t)$ after appending cyclic prefix. The corrupted received signal $y(t)$, demodulated by FrFT, is processed by sparse channel estimation, including FrFT based dictionary matrix which consists of delay and Doppler representative sets and CoSaMP optimization. The estimated channel information $\widehat{\mathbf{H}}$ is sent to followlling equalization and detection blocks.

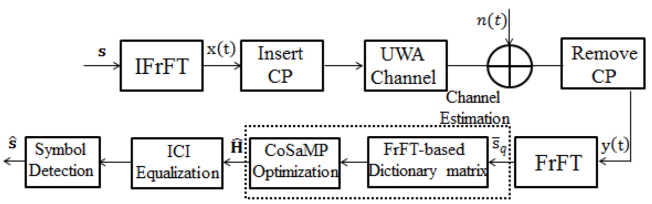

Figure 2: Diagram of FrFT-CS transceiver

2.2 Fractional Fourier Transform OFDM System

In the FrFT based OFDM scenario (FrFT-OFDM), the FrFT and IFrFT blocks are implemented instead of Fourier (FT) and Inverse Fourier Transform (IFT) in conventional OFDM. Initially, a block of data $\mathbf{s} = [s_0, s_1 \ldots s_N]^T$ with N symbols is modulated to N subcarriers, contributing to time domain waveform $x(t)$ via equation [14]

$$x(t) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} s_p \sqrt{\frac{sin\alpha + jcos\alpha}{T}} \exp\left(\frac{j2\pi pt}{T}\right) \exp\left(\frac{-j}{2}\left(t^2 + \frac{4\pi^2 sin\alpha^2}{T^2}p^2\right) cot\alpha\right) \quad (1)$$

where $0 < \alpha < \pi$ is the angle of rotation between time and frequency domain, and p is the data of p^{th} subcarrier. After appending cyclic prefix (CP), the transmitted waveform is corrupted by UWA channel which is doubly selective, contributing to the received signal

$$y(t) = \sum_{i=1}^{L} h_i(t) x(t-\tau_i) \exp(-2\pi f_i t) + n(t) \quad (2)$$

where L, $h_i(t)$, τ_i, f_i and $n(t)$, denote the number of paths, path gain, the delay, the Doppler shift of the i-th path, and complex Gaussian noise respectively.

Then, $y(t)$ demodulated via FrFT after discarding cyclic prefix is expressed as [15]

$$\bar{s}_q = \int_0^T y(t) \sqrt{\frac{sin\alpha - jcos\alpha}{T}} \exp\left(\frac{-j2\pi qt}{T}\right) \exp\left(\frac{j}{2}\left(t^2 + \frac{4\pi^2 sin\alpha^2}{T^2}q^2\right) cot\alpha\right) dt \quad (3)$$

2.3 Compressed Sensing Based Channel Estimation Method

2.3.1 Dictionary Matrix Construction

According to the transmitted and received signal in equation (1) and (3) respectively, the channel frequency response is denoted as

$$[\mathbf{H}]_{m,k} = \frac{1}{T}\left(\frac{\pi^2 sin\alpha^2}{T^2} \exp\left((k^2 - m^2) cot\alpha\right) \sum_{i=1}^{L} \exp\left(-j\left(\frac{2\pi k\tau_i}{T} + \frac{\tau_i^2 cot\alpha}{2}\right)\right) \int_0^T h_i(t) e^{j\left(\frac{2\pi(k-m)}{T} + \tau_i cot\alpha - 2\pi f_i\right)t} dt \quad (4)$$

which could be separated into three parts:

$$\mathbf{H} = \sum_{i=1}^{L} \boldsymbol{\varepsilon}_\mathbf{i} \boldsymbol{\rho}_\mathbf{i} \boldsymbol{\varphi}_\mathbf{i} \quad (5)$$

Where

$$[\boldsymbol{\varphi}_\mathbf{i}]_{m,k} = \int_0^T e^{j\left(\frac{2\pi(k-m)}{T} + \tau_i cot\alpha - 2\pi f_i\right)t} dt = \mathrm{sinc}\left([\boldsymbol{\beta}]_{m,k} T\right) \exp\left(j\pi [\boldsymbol{\beta}]_{m,k} T\right) \quad (6)$$

$\boldsymbol{\varphi}_\mathbf{i}$ is the contribution of ICI caused by Doppler frequency in $\boldsymbol{\beta}$.

$$[\boldsymbol{\beta}]_{m,k} = \frac{k-m}{T} + \frac{\tau_i cot\alpha}{2\pi} - f_i \quad (7)$$

$$[\boldsymbol{\rho}_\mathbf{i}]_{m,m} = \exp\left(-j\left(\frac{2\pi k\tau_i}{T} + \frac{\tau_i^2 cot\alpha}{2}\right)\right) \quad (8)$$

$\boldsymbol{\rho}_\mathbf{i}$ is a diagonal matrix the delay of i^{th} path among all the subcarriers and ε_i denotes the complex path gain, it is clear from (5) that each discrete path with delay τ_i and Doppler frequency shift f_i contribute to the Inter carrier interference (ICI). It can be seen from (7) that in comparison with conventional OFDM, ICI of FrFT-OFDM is reduced by the increase of $\tau_i cot\alpha$. In other words, more energy of **H** is concentrated along the main diagonal of $\boldsymbol{\varphi}_\mathbf{i}$.

Due to the sparse character of UWA channel, both the multipath delay and Doppler shift can be discretized to build the dictionary representation matrix, denoted as $\bar{\tau}$ and f_r respectively. [16]

$$\bar{\tau}\epsilon\left\{0,\frac{T}{\vartheta B},\frac{2T}{\vartheta B},\dots T_g\right\}$$

$$f_r\epsilon\{-f_{max},-f_{max}+\Delta f,-b_{max}+2\Delta f,f_{max}\}$$

in which ϑ is the oversampling factor, and $f_{max}$is selected as residual Doppler shift after process by FrFT.

The received signal in (3) can be simplified and written in

$$\mathbf{Y}_p = \boldsymbol{Ah} + \boldsymbol{n_p}$$

$$= [\boldsymbol{\gamma}(\boldsymbol{f_1}),\dots\boldsymbol{\gamma}(\boldsymbol{f}_{\mathbf{N_f}})],\begin{bmatrix}\varepsilon_1\\ \vdots\\ \varepsilon_{N_f N_\tau}\end{bmatrix}+\boldsymbol{n}_p \quad (9)$$

where $\boldsymbol{\gamma}(\boldsymbol{f_1}) = [\boldsymbol{\rho_1\varphi_1}\mathbf{s} \quad \dots\dots \; \boldsymbol{\rho}_{N_\tau}\boldsymbol{\varphi}_{N_\tau}\mathbf{s}]$

$\boldsymbol{\varphi_1}$is a the number of columns of the measurement matrix is much greater than that of the row, namely $N_f N_\tau \gg L$.

The Dictionary matrix **A** should meet the requirements of restricted isometry property (RIP) [12], explained as below:

Matrix A with unit-norm columns one can define the restricted isometry constants $\boldsymbol{\delta_s}$ as the smallest number such that, $(\mathbf{1}-\boldsymbol{\delta}_s)|x|^2 \le |Ax|^2 \le (\mathbf{1}+\boldsymbol{\delta}_s)|x|^2$ for any x that is s-sparse.

However, it is difficult to calculate the isometry constants $\boldsymbol{\delta_s}$

Another method which measures the mutual coherence of the dictionary matrix is expressed below:

$$\mu(\mathbf{A}) = \max_{1\le i,j\le,i\ne j}\frac{|<a_i a_j>|}{\|a_i\|\|a_j\|} \quad (10)$$

where $\boldsymbol{a_i}$ and $\boldsymbol{a_j}$ represent the i^{th} and j^{th} column of **A** respectively. The equation (10) is defined to calculate the maximum absolute correlation between any two normalized columns. We attempt to create **A** with minimum $\mu(\mathbf{A})$. In equation(5) that **A** consists of $\boldsymbol{\rho}_\mathbf{i}$, $\boldsymbol{\varphi}_\mathbf{i}$ and $\mathbf{s}$, where $\boldsymbol{\rho}_\mathbf{i}$ a diagonal matrix. The coherence between any two column of **A** depends on$\boldsymbol{\varphi}_\mathbf{i}$, which is affected by ICI. As is considered in **Section 2.21** that channel frequency response **H** based on FrFT have a better performance of energy concentration along the diagonal, contributing to less coherence. In addition, the randomly placed pilots [8], which has been largely agreed as one kind of optimal method for sparse channel estimation, is implemented.

2.3.2 Compressive Sampling Matching Pursuit (CoSaMP) Algorithm

In order to solve the sparse problem of equation (9), various kinds of sparse recovery algorithms have been proposed. Among them, the basis pursuit (BP) [17] and OMP [18] algorithms are the most popular in the application of UWA. OMP, compared with BP, has two advantages. Firstly, it is more convenient to implement OMP which is a greedy iterative algorithm than BP based on convex optimization. Secondly, the computational complexity of BP is much more intensive than that of OMP, especially when the subcarrier number is large. Consequently, in this paper, CoSaMP, an improved algorithm of OMP is implemented to search for the optimal result.

3. SIMULATION RESULTS AND DISCUSSION

The proposed FrFT-CS is simulated, compared with DPS-BEM. The Normalized Means Squre Error (NMSE) and channel impulse response (CIR) estimated by both methods are display.

3.1 Simulation set up

3.1.1 Parameter of OFDM

The number of subcarriers is N=256, of which 64 pilot symbols are randomly allocated. The duration of OFDM symbol T is 104.5ms, while the bandwidth of baseband signal B is 9.77 kHZ. The guard interval is 24.6ms. The continuous signal is sampled at baseband sampling frequency. The optimal order of FrFT-OFDM is set as one.

3.1.2 Parameter of UWA channel

The UWA channel is designed with $N_p = 7$ discrete paths. The multipath delay are distributed exponentially with mean $\mathrm{E}[\tau_{p+1}-\tau_p] = 1ms$, leading to the duration of maximum delay spread is approximately 6ms. The average power reduces exponentially with delay, while the amplitudes are Rayleigh distributed. The Doppler spread is set as$f_d = 0.10\Delta f$. The signal to noise ratio ranges from 0 to 30db. [19]

3.1.3 Simulation performance analysis

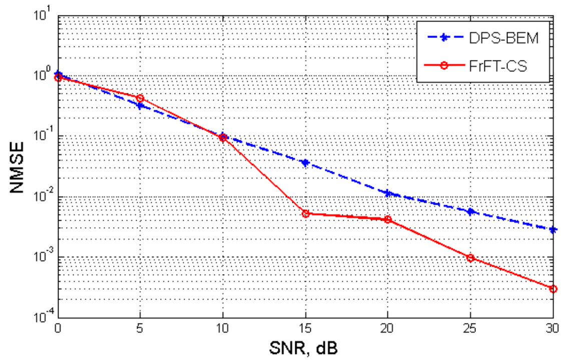

Figure 3: Comparison of MSE of DPS-BEM and FrFT-CS

Figure 3 compares the Normalized Mean Square Error (NMSE) performance between real and estimated channel based on the approaches of DPS-BEM and FrFT-CS. The optimal order is chosen as 0.95 and the oversampling factor ϑ is set at 2. It is obvious that the performance of FrFT-CS is superior to that of DPS-BEM with NMSE improvement of approximately10dB when SNR is 30dB. The reason of better results is that the discrete prolate spheroidal basis (DPS), a deterministic basis is not able to model the channel shape accurately as the increase of unknown coefficients. In addition, a bias from the discrete grids of sparse delay causes off-grid effect, which is exacerbated in UWA channels. FrFT-CS, in comparison, only extracts the significant taps via coherence between columns of dictionary matrix and the oversampling factor alleviate the off-grid effect, enhancing the accuracy of channel estimation. The Channel Impulse response (CIR) of FrFT-CS and DPS-BEM via 64 delay samples is demonstrated in Figure 4 below. However, as the Doppler spread becomes larger than$0.10\Delta f$, the performance of FrFT-CS will become worse, because the increase of $cot\alpha$ aggravate the error between different τ_i , which will be solved in the future research.

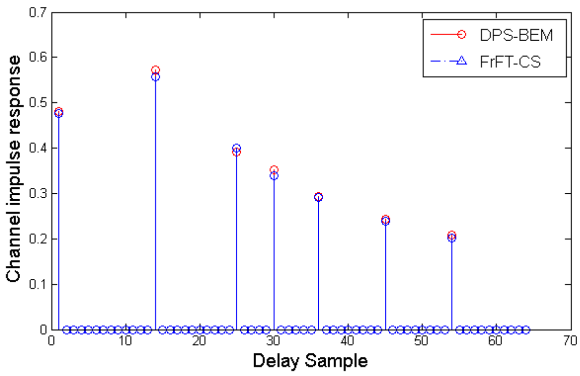

Figure 4: Channel impulse response estimated by DPS-BEM and FrFT-CS

4. CONCLUSION

In this paper, a novel sparse channel estimation based on Fractional Fourier Transform (FrFT) for orthogonal frequency division multiplex (OFDM) scenario over time-variant underwater acoustic (UWA) channels is developed. It recovers the channel coefficients of sparse taps via FrFT based dictionary matrix and CoSaMP greedy algorithm. The mutual coherence, instead of RIP is employed to evaluate its preservation of sparse information. The simulation results demonstrate that FrFT based sparse channel estimation compared to DPS-BEM at less complexity achieving an improvement in Normalized Mean Square Error (NMSE).

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