A Single-stage Three-phase DC/AC Inverter Based on Cuk Converter for PV Application

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***Abstract* -- This paper presents a new three-phase dc-ac inverter based on the basic Cuk converter. The main feature of the proposed topology is the fact that the energy storage elements as inductors and capacitors values can be reduced in order to improve the reliability, reduce the size, and the total cost. Moreover, the bucking-boosting inherent nature of the Cuk converter, depending on the time-varying duty ratios, provides more flexibility for stand-alone and grid connected applications when the required output AC voltage is lower or greater than the DC side voltage. This property is not found in the conventional current source inverter (CSI) when the DC input current is always greater than the ac output one or in the conventional voltage source inverter (VSI) as the output ac voltage is always lower than the dc input one. Averaged large and small signal models are used to study the Cuk nonlinear operation. Basic structure, control design, and MATLAB/SIMULINK results are presented in this paper. The new three-phase DC-AC inverter is very convenient for PV applications where continuous average input currents are required for appropriate Maximum power Point Tracking (MPPT) operations.**

Keywords* — *DC/DC converters, Cuk Converter, Buck-boost inverter, state space Averaging, PI control, PR control

NOMENCLATURE

*	Reference value of a variable
Abc	Three-phase stationary frame
C	Cuk converter capacitor
d	Cuk converter duty ratio
D	Steady state Cuk converter duty ratio
d-q	Direct and quadrature synchronous frame
E_{in}	Cuk input DC voltage
f	Output voltage fundamental frequency
f_s	Sampling frequency
I_{in}	Total input current
I_{L1}	Cuk converter input current
I_{L2}	Cuk converter output current
L_1 and L_2	Cuk converter input and output inductors
R	Load resistance
t_s	Sampling time
V_c	Cuk capacitor voltage
$V_{c2a,b\ and\ c}$	Cuk output three-phase voltage
V_o	Output load voltage

I. INTRODUCTION

Nowadays, there is an international trend toward modular structured renewable/distributed system concepts in order to reduce the costs and provide high reliability [1]. This affects the DC/AC converter topologies significantly in terms of reducing the size and numbers of passive components of the inverter [2]. For conversion between DC to AC powers, the conventional voltage source inverter (VSI) is the most common converter topology [3]. The instantaneous average output voltage of the VSI is always lower than the input dc voltage. For this reason, a boost dc-dc converter should be used when the required AC peak output voltage is greater than the input DC voltage [4]. This additional boost dc-dc converter can result in high volume, weigh, cost, and losses [5]. In [3], a new inverter is presented as a boost inverter where the required output voltage can be lower or greater than the input dc voltage by connecting the load differentially across two dc–dc converters and modulating the dc–dc converter output voltages sinusoidally. In this topology, both individual boosts are driven by two 180 phase-shifted dc-biased sinusoidal references. The differential connection of the load leads to cancellation of the DC offsets of the output voltage and the peak value of this ac voltage can be lower or greater than the dc input voltage. The main drawback of this structure deals with its control as the control of the AC output voltage requires controlling both Boost converters and hence, the load voltage is controlled indirectly. In [6], a closed-loop sinusoidal PWM-PID control method with real-time waveform feedback techniques is presented. In [7], the simulation of hybrid boost inverter control system is proposed in order to show the DC offsets error. The topologies of buck, boost, and buck-boost inverter have been presented in [8]. In [9], the boost-inverter topology is used to build a single-phase single power stage Fuel Cell system with a backup battery storage unit. Four switches and four diodes are used as well as two output capacitors for each phase. In [10], the authors propose a parallel operation of three-phase AC to DC converter using single-phase rectifier module. The control strategy has high dynamic features, and it can achieve a fast dynamic transient response. However, the proposed configuration includes six Cuk converters with six rectifiers, two single-switch single-diode Cuk converters with two rectifiers for each phase. This all adds to the cost, control complexity and the reliability of the overall system in addition to the use of high value capacitor across the load.

For the modern power conversion applications, continuous input current converters are more attractive solutions for renewable systems. In addition, Maximum Power Point Tracking (MPPT) techniques of Photo Voltaic (PV) systems require the input current to flow continuously [11]–[14]. Generally, there are nine continuous input and output current dc-dc converters of total 33 DC-DC possible single-switch and single-diode dc-dc-converter. These nine converters include two inductors and a one capacitor [14]. Among these converters with continuous input current, Cuk converter has the lowest losses and the best voltage regulation. Moreover, the switched capacitor of Cuk converter increases the voltage boost ability [14]. Because of their buck-boost capability, Cuk converters are used widely in industrial dc power supplies such as in wind energy, Photovoltaic (PV) systems, marine, light-emitting diode driver, compressors, fuel cells, and batteries [15]. Moreover, the current sourcing nature of Cuk converter enables for easy parallel connection. This can be a trend for paralleling many PV arrays on the same Point of Common Coupling (PCC). The DC-DC operation of Cuk converters is studied extensively and reported in the literature. Stability of the open loop and closed loop operation is considered in [16] . Generally, DC-DC converters, including the Cuk, are time-variant systems. This means that the overall transfer function of the converter describing the input-output performance is dependent on the duty ratio as well as the converter parameters. This increases complexity in the control design as the poles and zeros of converter travel through a specified trajectory. Moreover, the time-varying transfer function leads to a distortion in the output voltage and current [17].

This paper proposes a new three-phase DC/AC Inverter base on three bidirectional two-switch two-diode Cuk converters without/or with an optional small DC-link capacitor. The DC/AC inverter is expedient for PV applications where the peaks of the output AC currents are required to be flexible over and below the input DC current for MPPT operation and for providing an easy paralleling at PCC.

II. System Description

The operating modes of a typical Cuk converter can be shown in Fig. 1. The circuitry consists of an input voltage source E_{in}, two switches S_1 and S_2, two antiparallel diodes D_1 and D_2. The energy between the voltage source and the load is transferred through capacitor C. The energy is stored instantaneously in inductors L_1 and L_2.

A. Cuk Converter Modeling Using State Space Averaging method

State space averaging method will be used to model the Cuk converter. Assuming the turn off time of S_1 is T_{off}, turn on time for S_1 = ON is T_{on} and $T_s = T_{on} + T_{off}$, the state space equations during continuous conduction mode of operation can be written as follows:

i. S_1 OFF and S_2 ON ($0 < t < T_{off}$)

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}E_{in} - \frac{1}{L_1}v_c \tag{1a}$$

$$\frac{dV_c}{dt} = \frac{1}{C}i_{L1} \tag{1b}$$

$$\frac{di_{L2}}{dt} = -\frac{R}{L_2}i_{L2} \tag{1c}$$

$$\dot{x}_1 = A_1 x + B_1 E_{in}, \quad v_o = C_1 x \tag{1d}$$

Where $x = [i_{L1} \;\; v_c \;\; i_{L2}]'$

$$A_1 = \begin{bmatrix} 0 & -\frac{1}{L_1} & 0 \\ \frac{1}{C} & 0 & 0 \\ 0 & 0 & -\frac{R}{L_2} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \text{and} \quad C_1 = [0 \;\; 0 \;\; R]' \tag{1e}$$

ii. S_1 ON and S_2 OFF ($T_{off} < t < T_s$)

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}E_{in} \tag{2a}$$

$$\frac{dV_c}{dt} = -\frac{1}{C}i_{L2} \tag{2b}$$

$$\frac{di_{L2}}{dt} = -\frac{R}{L_2}i_{L2} + \frac{1}{L_2}v_c \tag{2c}$$

$$\dot{X} = A_2 x + B_2 E_{in}, \quad v_o = C_2 x \tag{2d}$$

Where

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C} \\ 0 & \frac{1}{L_2} & -\frac{R}{L_2} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \text{and} \quad C_2 = [0 \;\; 0 \;\; R]' \tag{2e}$$

Averaging the state space equations all over the period [$0<t<T_s$] assuming the duty ratio (d) = $\frac{T_{on}}{T_s}$

$$A = A_1(1-d) + A_2 d, \quad B = B_1(1-d) + B_2 d, \quad C = C_1(1-d) + C_2 d \tag{3a}$$

$$\dot{x} = Ax + BE_{in}, \quad v_o = Cx \tag{3b}$$

Where

$$A = \begin{bmatrix} 0 & \frac{-(1-d)}{L_1} & 0 \\ \frac{(1-d)}{C_1} & 0 & \frac{-d}{C_1} \\ 0 & \frac{d}{L_2} & \frac{-R}{L_2} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \text{and} \quad C = [0 \;\; 0 \;\; R]' \tag{3c}$$

From 3c, the voltage transfer function of the Cuk converter [$G_v = \frac{V_o}{E_{in}}$] can be written as:

$$G_v = \frac{Rd(1-d)}{CL_1L_2s^3 + CL_1Rs^2 + s(L_2 - 2dL_2 + d^2L_2) + (R - 2dR + d^2R)} \tag{3d}$$

From 3d, it can be noticed that the dynamics of output voltage depends on the duty ratio (d). At the steady state, ($s \rightarrow 0$) and when $d = D$ is constant, the transfer function tends to:

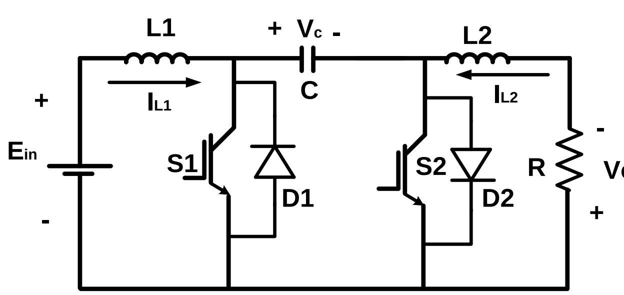
$$G_{v,ss} = \frac{D}{1-D} \tag{3e}$$

For the parameters shown in Table I, the poles and zeros of G_v are derived and plotted in Fig. 2a in order to study the dynamic behavior. The duty ratio is varied from 0.1 to 0.85. It can be concluded that increasing the duty ratio, leads the dominant poles of the real axis to move to the slower region, towards the origin, and the system dynamics become slower. This can be verified from the step response in Fig. 2b as the system gets slower with increasing the duty ratio. To show the meaning of the previous analysis, a MATLAB simulation is used when the duty ratio is varied according to G_v to draw a sinusoidal output voltage with a DC offset. The input voltage is set to 20V. Fig. 2c shows the difference between the reference and the actual output voltages because of the variation of dynamics with the value of duty ratio.

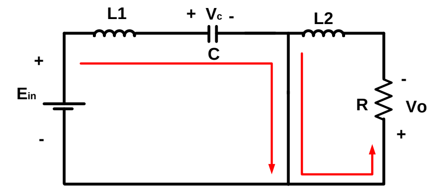
Table I. System Parameters

L_1	1 mH
C	10 μF
L_2	1 mH
R	1 Ω
f	50 Hz
f_s	50 KHz

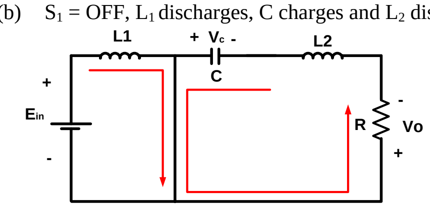
The proposed three-phase DC/AC inverter based on Cuk converters is shown in Fig. 3. As a current source, the proposed system can be paralleled easy for any further power extension. Each Cuk converter builds a sinusoidal output voltage, specifically current, with a DC-offset. Because of the balanced energy operation of the three phases, it is expected that the DC offsets of each phase are cancelled and the three-phase load expert pure sinusoidal voltages and currents. Fig. 4 shows the open loop operation of the system in Fig. 3 with the parameters in Table I and 10 nF optional output shunt capacitors. The expected output voltages at points V_{c2a}, V_{c2b} and V_{c2c} are sinusoidal voltages of magnitude 20V peak and 40V DC offset. The duty ratios of the three Cuk converters, d_a, d_b and d_c are calculated from (3e) and have been shown in Fig. 4a. However, the output voltages in Fig. 4b are distorted. It can be seen clearly from the output currents in the synchronous rotating frame in Fig 4c that a 2nd harmonic appears because of Cuk non-linearity. This is because the varying dynamics at each point, which is explained in Fig. 2.



(a) Cuk Converter

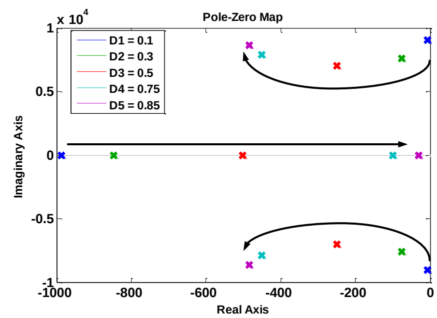


(b) S_1 = OFF, L_1 discharges, C charges and L_2 discharges

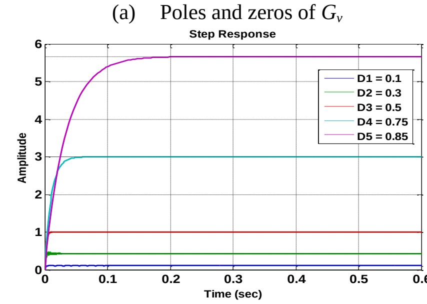


(c) S_1 = ON, L_1 charges, C discharges and L_2 charges

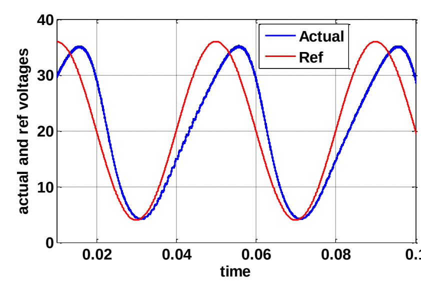
Fig. 1 The operating modes of a typical Cuk converter



(a) Poles and zeros of G_v



(b) Step Response of G_v



(c) Cuk with time varying duty ratio

Fig. 2 Frequency and time analysis of Cuk Converter

In the next section, a control strategy is proposed for the Cuk inverter system in order to deal with the nonlinearity, control the desired output current, and eliminate the predefined distortion.

III. Control Design

The control objective is to track a predefined sinusoidal output voltage. The control structure for the converters is shown in Fig. 5. V_d, V_q and V_{dc} are the direct, quadrature, and DC offset components of the output voltage at V_{c2a}, V_{c2b} and V_{c2c}.

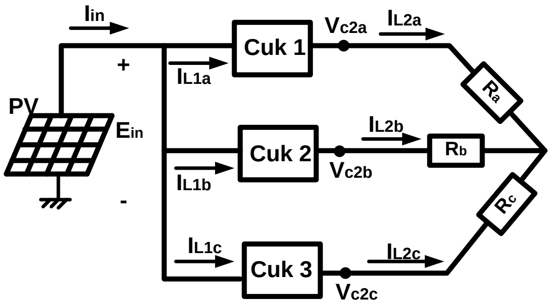
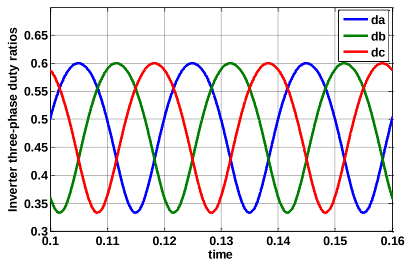
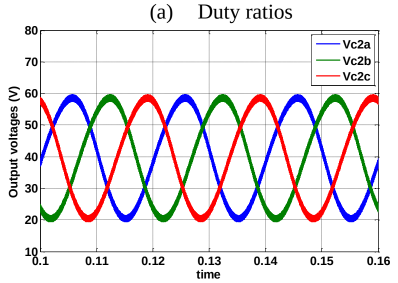


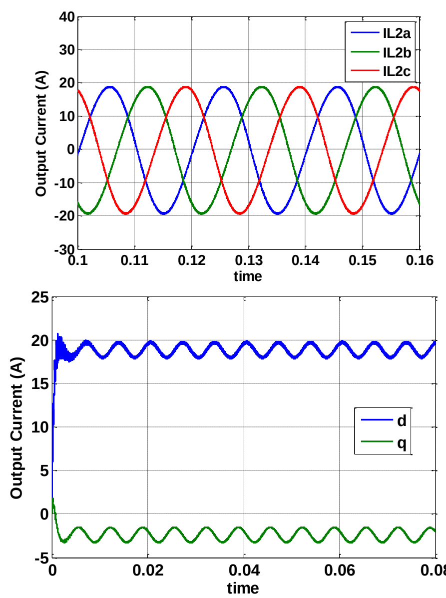
Fig. 3. Proposed Cuk based DC/AC three-phase Inverter



(a) Duty ratios



(b) Output voltages at V_{c2a}, V_{c2b} and V_{c2c}



(c) Output currents (three-phase and dq components)

Fig. 4. Open loop operation of the proposed system in Fig. 3.

The subscript '*' is referred to a reference value. K_p and K_i are the proportional and integral gains of the PI controller. The detailed mathematical analysis of the non-linear Cuk converter is out scope of this paper and will be considered in further publication.

From equation 3b, the control input is considered as the input voltage E_{in}. However, normally, the voltage of the PV is constant over a long period, depending on the MPPT operation, and hence the control input should be written in terms of the time varying duty ration d. The small signal equations of the Cuk converter can be driven from equation 3c by considering the small signal deviations $\hat{x}, \hat{y}$ and $\hat{u}$. where;

$$\begin{gathered} \hat{x} = x - X \\ \hat{v}_o = v_o - V_o \\ \hat{d} = d - D \end{gathered} \tag{4a}$$

X, V_o and D are the steady state values of x, v_o and d

$$\begin{gathered} \dot{\hat{x}} = a\hat{x} + b\hat{d} \\ \hat{v}_o = c\hat{x} \end{gathered} \tag{4b}$$

$$a = \begin{bmatrix} 0 & \frac{-(1+D)}{L_1} & 0 \\ \frac{(1-D)}{C_1} & 0 & \frac{-D}{C_1} \\ 0 & \frac{D}{L_2} & \frac{-R}{L_2} \end{bmatrix}, b = \begin{bmatrix} \frac{V_c}{L_1} \\ \frac{-(I_{L1}+I_{L2})}{C_1} \\ \frac{V_c}{L_2} - \frac{R}{L_2} I_{L2} \end{bmatrix} \text{ and}$$

$$c = [0 \quad 0 \quad R]'$$

where; $V_c, I_{L1} and\ I_{L2}$ are the steady state values of $v_c, i_{L1}\ and\ \ i_{L2}$ respectively

In order to ease the control design process, a point at the middle of the trajectory in Fig. 3a, where $d = 0.5$, is chosen to be an intermediate operating point. The poles loci of the closed loop system of Eq. (4b) are plotted in two different ways. In Fig. 6a, K_i is held constant at (0.7) and K_p is varied in the range [0.1:0.8]. In the same way, Fig. 6b shows when K_p is held constant and K_i varies from [0.1:0.8]. From Fig. 6a, it can be noticed that increasing the proportional value will drive the poles toward the right hand side. And from Fig. 6b, the imaginary poles are stuck in their loci while the real poles move away from the origin to the left hand side. The gain values are selected by compromising between the both cases. From Fig. 6a and b, selecting the values of $K_p = 0.3$ and $K_i = 0.4$ provides preliminary proper dynamic performance and stability margin from the imaginary axis.

IV. Simulation Results

The proposed three-phase Cuk inverter is simulated in MATLAB/SIMULINK with the selected parameters and gain values. Fig. 7 shows the results for the voltage response. The output voltages are shown in Fig. 7a with 40V DC offset and 20V peak-to-peak. However the dq components in Fig. 7b show that the actual output voltages and currents have second harmonic components. This can be explained by the nonlinear nature of the Cuk converters as described in the previous section. By increasing the parameters of the Cuk Converters (L_1, L_2 and C), the trajectory of the poles in Fig. 2a becomes shorter. Hence, the effect of Cuk nonlinearity becomes less and the 2^{nd} order harmonic will decrease in the output currents and voltages. However, increasing the converter parameters will affect the size, cost, losses and will add to the control complexity. As a solution, the controller is modified as in Fig. 8. A band pass filter tuned at the 2^{nd} harmonic, 3^{rd} harmonic from the dq frame point of view, to extract its components in the output voltage.

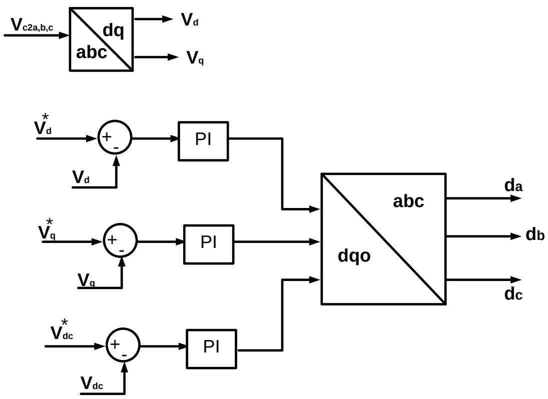
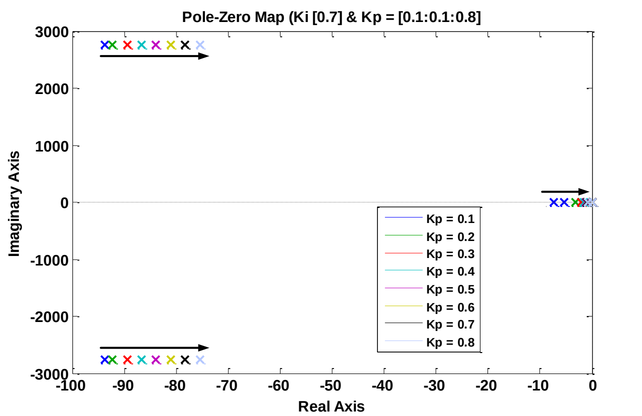
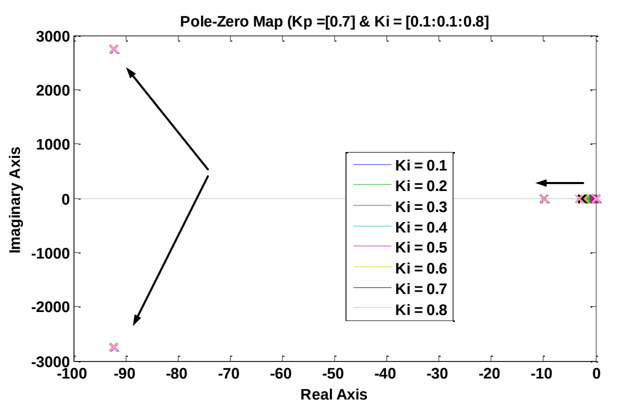


Fig. 5 Control Structure



(a) Pole-zero map of (4b) when K_i is held constant at (0.7) and K_p is varied in [0.1:0.8].



(b) Pole-zero map of (4b) when K_p is held constant at (0.1) and K_i is varied in [0.1:0.8].

Fig. 6. Root loci for a fixed K_i and a range of K_p or vice versa.

A proportional-resonant (PR) controller is inserted to force this component to zero. The values of PR controller are chosen to be very small as they do not interrupt the main PI loop. The results are shown in Fig. 9 where the PR controller is able to suppress the 2nd harmonic components from the voltages and currents. Fig. 10 shows the experimental results for Fig.9 using TMS320F280335 DSP.

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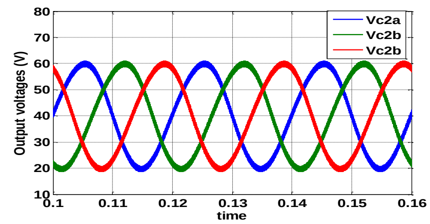
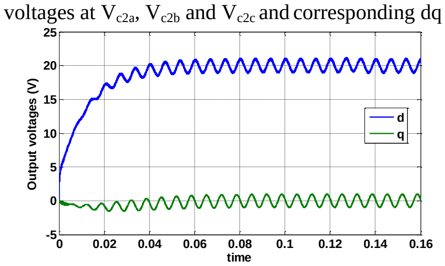
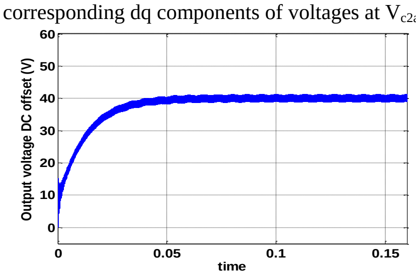


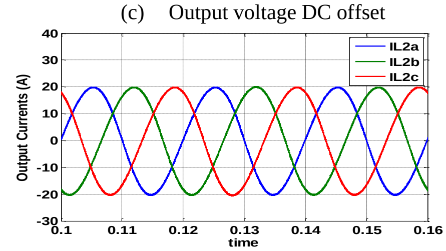
(a) Output voltages at V_{c2a}, V_{c2b} and V_{c2c} and corresponding dq components



(b) corresponding dq components of voltages at V_{c2a}, V_{c2b} and V_{c2c}



(c) Output voltage DC offset



(d) Output Currents

Fig. 7 The proposed system under PI control

V. CONCLUSION

Due to its inherent current sourcing nature, Cuk converter is considered as an attractive alternative for the conventional dc-ac converters in PV applications. The reason for that is the continuous input current, which enables for direct MPPT techniques, and the ability of paralleling more than dc-ac converter at the same PCC. Moreover, because of the small input current ripples, no capacitor is required across the PV array or even used; it's a small plastic capacitor instead of electrolytic one. In this paper, a three-phase DC-AC Cuk converter based current source inverter has been proposed and studied. The state space averaging method was used to design the control structure. The Cuk converter inherent nonlinearity is a main reason for output currents and voltages distortion. The effect of this nonlinearity can be relieved by increasing the Cuk converter inductors and capacitors values. However, this will affect the total cost, size and control complexity. In this work, an additional control loop is proposed to reduce the distortions with the minimal passive element values. Satisfactory results in terms of reduced 2nd order harmonic components in the output currents and voltages were obtained and verified by MATLAB/SIMULINK.

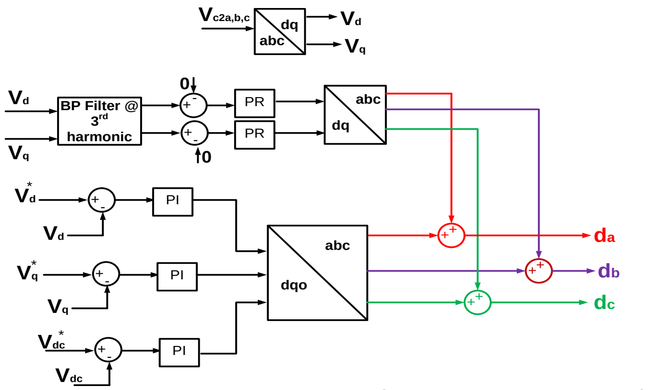
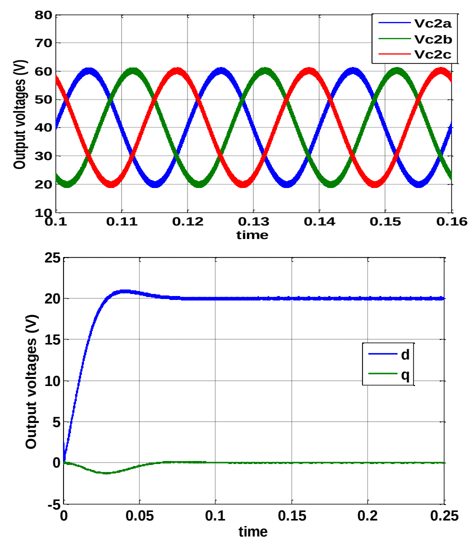
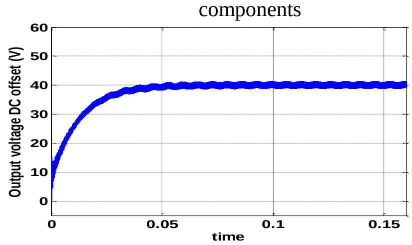


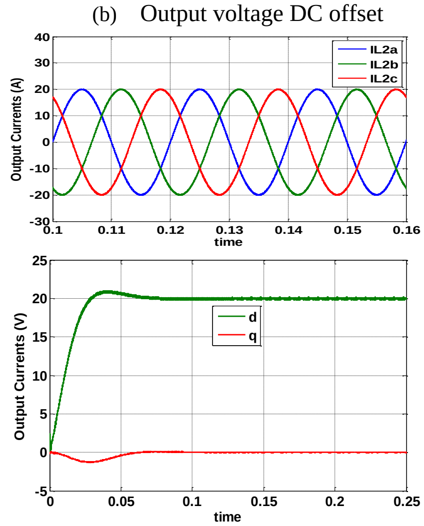
Fig. 8 Control structure with eliminating the 3rd harmonic in the dq frame (2nd in the stationary)



(a) Output voltages at V_{c2a}, V_{c2b} and V_{c2c} and corresponding dq components

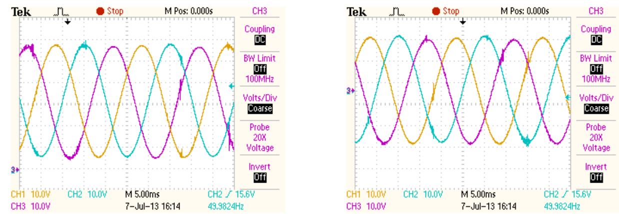


(b) Output voltage DC offset



(c) Output Currents and corresponding dq components

Fig. 9 The proposed system under PI + PR control



(a) Voltages at V_{c2a}, V_{c2b} and V_{c2c} (b) Load Voltages

Fig. 10 Experimental Results

REFERENCES

[1] Johanna M.A. Myrzlk, " Novel Inverter Topologies for Single-phase Stand-Alone or Grid-Connected Photovoltaic Systems," IEEE PEDS 2001 Indonesia, vol. 1, no. 1, pp. 103 - 108, Oct 2001.

[2] W. Kleinkauf, J. Sachau, H. Hempel: Developkts in inverters for photovoltaic systems - Modular power conditioning and plant technology,1 Ith E.C. Photovoltaic Solar Energy Conference, Montreux, 1992.

[3] Ram´on O. C´aceres and and Ivo Barbi, A Boost DC–AC Converter: Analysis, Design, and Experimentation," IEEE Trans. on Power Electronics, vol. 14, no. 1, pp. 134 - 141, Jan 1999.

[4] F. Barzegar and S. Cuk, "Solid-state drives for induction motors: Early technology to current research," in Proc. IEEE Region 6 Conf., Anaheim, CA, Feb. 15–18, 1982.

[5] S. B. Kjær, J. K. Pedersen, and F. Blaabjerg, "A review of single-phase grid-connected inverters for photovoltaic modules," IEEE Trans. Ind. Appl., vol. 41, no. 5, pp. 1292–1306, Sep./Oct. 2005.

[6] Yaosuo Xue and Liuchen Chang, "Closed-Loop SPWM Control for Grid-Connected Buck-Boost Inverters," 2004 35th Annul IEEE Power Electronics Specialists Conference, vol. 5, no. 5, pp. 3366-3371, June. 2004.

[7] T. D. Rachmildha, Y. Haroen, A. Muqorobin, and E. Rijanto "Single Phase Boost Inverter Using Hybrid Modelling Approach," IEEE Conference on Power Engineering and Renewable Energy 2012, pp. 1–6, Jul. 2012.

[8] B. Prasad, S. Jain, and V. Agarwal, "Universal Single-Stage Grid-Connected Inverter," IEEE Trans. Energy Conversion., vol. 23, no. 1, pp. 128–138, March. 2008.

[9] M. Jang, M. Ciobotaru, and V.G. Agelidis, "A Single-Phase Grid-Connected Fuel Cell System Based on a Boost-Inverter," IEEE Trans. Power Electronics. Appl., vol. 28, no. 1, pp. 279–289, Jan. 2013.

[10] V. Chunkag and U. Kamnarn, 'Parallelling three-phase AC to DC converter using CUK rectifier modules based on power balance control technique', IET Power Electron., 2009, vol. 3, no.4, pp. 511–525

[11] Y. Xue, L. Chang, S. B. Kjær, J. Bordonau, and T. Shimizu, "Topologies of single-phase inverters for small distributed power generators: An overview," IEEE Trans. Power Electron., vol. 19, no. 5, pp. 1305–1314, Sep. 2004.

[12] L. Quan and P. Wolfs, "A review of the single phase photovoltaic module integrated converter topologies with three different DC link configurations," IEEE Trans. Power Electron., vol. 23, no. 3, pp. 1320–1333, May 2008.

[13] J. M. A.Myrzik andM. Calais, "String andmodule integrated inverters for single-phase grid connected photovoltaic systems—A review," in Proc. IEEE Bologna Power Tech Conf., Jun. 23–26, 2003, vol. 2, p. 8.

[14] B. W. Williams, "DC-to-DC Converters With Continuous Input and Output Power," IEEE Trans. Power Electron., vol. 28, no. 5, pp. 2307–2317, May. 2013.

[15] H. S. H. Chung, K. K. Tse, S. Y. R. Hui, C. M. Mok, and M. T. Ho, "A novel maximum power point tracking technique for solar panels using a SEPIC or Cuk converter," IEEE Trans. Power Electron., vol. 18, no. 3, pp. 717–724, May 2003.

[16] F. A. Himmelstoss and C. M. Walter, "A simple Cuk converter derivated two-quadrant DC motor controller," in Proc. Int. Symp. Power Electron. Electr. Drives Autom. Motion, Jun. 2010, pp. 1108–1112.

[17] J. Mahdavi, A. Emadi, and H.A. Toliyat, "Application of State Space Averaging Method to Sliding Mode Control of PWM DC/DC Converters., IEEE Industry Applications Society Annual Meeting New Orleans, Louisiana vol. 2, no. 2, pp. 820 - 827, Oct. 1997.