DEVELOPMENT OF A DYNAMIC PROGRAMMING METHOD FOR LOW FUEL CONSUMPTION AND LOW CARBON EMISSION FROM SHIPPING
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ABSTRACT
Fuel saving and low carbon emission in shipping have already become an attractive and important subject in the shipping industry driven by the increasing full oil price and considerations of reduction of green house gas emission. Global shipping consumes 200 million tons of liquid fuel, resoluble for 1.12 billion tons of CO_2 emission in the world. This paper presents the development of a novel forward dynamic programming method to optimise ship's fuel consumption with respect to safety during a voyage. The constraints are weather, sea conditions and the safety constrains defined in the IMO guidance MSC.1/Circ.1228. Compared with the traditional dynamic programming methods, the newly developed weather routing method is more accurate and practical. Simulation results from a case study have shown that the newly developed method is able to save several tons of fuel compared with that of the traditional methods for one North Atlantic voyage. As a result, it can offer much CO_2 emission reduction.
Keywords: Ship weather routing, Dynamic programming, Fuel saving, CO_2 emission reduction

1. INTRODUCTION

Ship weather routing is defined as an optimum track of ship route with an optimum engine speed and power for ocean voyage based on weather forecasts, sea conditions and individual ship's characteristics. The term optimum means a maximum of ship safety and crew comfort, a minimum of fuel consumption and time underway, or any desired combination of these factors. The accuracy in determining the optimum route depends on the following three aspects.

1) The accuracy of the prediction of ship's hydrodynamic behaviour under different weather conditions.
2) The accuracy of weather forecasts.
3) The capability and practicability of the optimisation algorithm used.

The focus of this study is on research and development of an optimisation algorithm. Many optimisation algorithms have been developed for solving ship routing problems in which minimisation of ship fuel consumption and/or passage time is the objective. Most popular methods include calculus of variations (Bijlsma S.J, 1975), modified isochrone method (Hagiwara H, 1989, Hagiwara H & Spaans JA, 1987), 2 dimensional dynamic programming (2DDP) method (De Wit C, 1990, Calvert S et al.,1991), isopone method (Klompstra MB et al., 1992, Spaans JA, 1995) and genetic algorithm (Harries, 2003, 2004, Hinnenthal, 2010, and J. Szłapczy´nska, 2009).

The method of calculus of variations treats ship routing as a continuous optimisation process. Inaccuracy of the solution may arise in functions when second order differentials are used in the optimisation process. Errors could be expanded to an unacceptable level at the end of the calculation.

The modified isochrone method is a recursive algorithm. The route with the minimum passage time is obtained by repeatedly computing isochrones (or time fronts) which are defined as the outer boundaries of attainable regions from the departure point after a certain time. This method offers a route with the minimum fuel consumption by keeping the propeller revolution speed as a constant during the simulation first, then applying the modified isochrone method to determine the minimum time of passage. By varying propeller rotation speed this method is able to find out a propeller rotation speed at which the minimum passage time satisfies the specified arrival time. This minimum time route is then treated as the route of the minimum fuel consumption. Thus, the fuel consumption of this route itself is not minimised.

The 2DDP method based on Bellman's principle of optimality is similar to the modified isochrone method. It also uses a recursive equation to solve the ship routing problem expressed as a discrete optimisation problem. The accuracy of this method depends on the fineness of the grid system used. Compared with the modified isochrone method, an advantage of the 2DDP method is that it allows the operators to take into account of navigation boundaries by means of an appropriate selection of the grid systems. Both the modified isochrone and the 2DDP methods assume that ships sail at a constant propeller rotation speed or a constant engine power for entire voyages in the route optimisation.

The isopone method is an extension of the modified isochrone method. An isopone is the plane of equal fuel consumption that defines the outer boundary of the attainable regions in a three-dimensional space, i.e. geographical position and time. This method

enables the operators to consider a variation of ship engine power in the route optimisation. A weather routing software named SPOS adopted the isopone method at the beginning of the software development. Although the isopone method is mathematically more elegant and theoretically offering better results than that of the modified isochrone method, SPOS had nevertheless abandoned the isopone method and adopted the modified isochrone method in the final products. The main reason for this change over was due to the fact that the isopone method appeared to be more difficult to understand by the operators onboard ships, whereas the modified isochrone method is straightforward and easy to understand.

Genetic algorithm as a new global optimization algorithm is widely used in many fields in recently years. Harries (2003, 2004), Hinnenthal (2010), and J. Szłapczy´nska (2009) treated weather routing as a two objectives optimisation problem, i.e. minimise voyage time and fuel consumption simultaneously by using the genetic algorithm. A population of different routes which perturb from a parent route (the great circle mostly) are generated by this method and the preferable ones are chosen during the optimisation procedure. A Pareto frontier providing a feasible domain for decision support is finally constructed by this method.

Besides these methods, there are many other methods that have been used for weather routing in recently years, like iterative dynamic programming algorithm (Kyriakos Avgouleas, 2008), augmented Lagrange multiplier (Masaru Tsujimoto, Katsuji Tanizawa, 2006), Dijkstra algorithm (Chinmaya Prasad Padhy et al., 2008), genetic algorithm (Harries S, Hinnenthal J, 2004) and so on.

Weather routing was first developed for determining shipping courses during a voyage with a minimum of passage time. However, nowadays shipping companies have began to show more interesting in reducing fuel consumption, maintaining a certain time schedule which is specified in the chartering contract of a merchant vessel. In this paper, a new forward 3 dimensional dynamic programming (3DDP) method is presented for minimising fuel consumption and CO_2 emission during a voyage. It is an extension of the traditional 2DDP method, but it allows changes in ships' heading and speed in both time and geographic position, thus, it is able to realise a quasi-global optimum result. Compared with the isopone method, the 3DDP method is more straightforward and easy for programming.

2. 3DDP METHOD

This section introduces a novel 3 dimensional dynamic programme (3DDP) for ship weather routing developed by the authors of this paper.

Detailed advantages and methodology of the method are described below.

2.1 INTRODUCTION TO 3DDP METHOD

Dynamic programming is developed based on Bellman's principle of optimality which can be described as "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957). According to this principle dynamic programming can solve complex problems by breaking them down into many simpler sub-problems. A term of stage is used in the 3DDP method to represent divisions of a sequence of various sub-problems of a system. The decision choosing the control variables in routing optimisation is done at a stage. The procedure is to calculate sub-problems stage by stage. Information from the preceding stage is used to determine the control variables of the next stage. The parameters used in describing a stage must be variables which are monotonically increasing with the progress of ship voyage or route optimisation. In ship routing problems, time, and voyage progress are monotonically increasing during a voyage. Both of them can be used to define a stage. Each stage consists of many states which are defined as a specifically measurable condition of the ship, such as time and geographic location. If time is chosen as the stage variable, the states on the stage can be defined by possible locations where the ship could pass. If voyage progress is chosen as the stage variable, the states should be defined by time and possible geographical positions

Dynamic programming usually uses a backward recursive algorithm described as that a path is optimal if and only if, for any intermediate stages, the choice of the following path is optimum for this stage. However, for a weather routing problem, a forward algorithm offers more convenience in programming. As a contrast with the backward version, a forward algorithm can be interpreted as that: a path is optimal if and only if, for any intermediate stages, the choice of the past path is optimum for this stage.

Chen H (1978) and Calvert S (1990) employed voyage progress as the stage variable and used the backward algorithm. Aligne, F (1998) chose time as the stage variable and used a forward algorithm; the newly developed 3DDP method presented in this paper employs voyage progress as the stage variable together with the use of a forward algorithm.

In the forward algorithm, the initial departure time is fixed; the arrival time at a location where the optimisation calculation has progressed to can be

treated as a flexible parameter. The advantage of this forward algorithm is that it allows a set of route with a minimum fuel consumption to be obtained corresponding to different arrival times at the current location where the optimisation has been concerned. This is a logical process in line with the ship operation along a voyage course.

Compared with the use of time as the stage variable, the advantages of using voyage progress as the stage variable are a less finer grid system is required which will save much computing time. When voyage progress is chosen as the stage variable, the headings of ship are pre-defined by the grid points already, hence the engine delivery power becomes the only control variable.

2.2 GRID DESIGN

Dynamic programming uses a grid system to specify the space layout of stages and states used in the algorithm calculation. Since the great circle is the optimum route under calm water from the departure to the destination, it is chosen as a reference route for the construction of the grid system used in the 3DDP route optimisation.

As described above, states on a stage are of three dimensions, i.e. time and geographical location with unit spacing ΔY, perpendicularly away from the great circle. The farthest states on a stage from the great circle are the possible locations where the ship may pass to avoid a bad weather or sea conditions. Grids, when crossing islands/rocks en-route of a voyage, should be deleted. Unlike the tradition dynamic programming, the time variable of states in the 3DDP is not fixed. It will be determined as the optimisation procedure is progressing.

Figure 1 shows an example of state projections on a longitude × latitude plane where 16 stages have been allocated (1, 2, 3…16) from the start to the destination of shipping route. The distance between two stages can be equally spaced ΔX. The total number of stages is determined according to the total distance of route and the availability of computing capacity.

2.3 NOTATION DEFINITION

Notations used in the programming are defined as follows.

K: Total number of stage.
N (k): Total number of states' projection on a latitude × longitude plane on stage k, where: k = 1, 2, 3… K. N (1) = 1, N (K) = 1
P (i, k): Position of states' projection on a latitude × longitude plane on stage k, where: i = 1, 2, 3…, N (k). The space between P (i, k) and P (i + 1, k) is
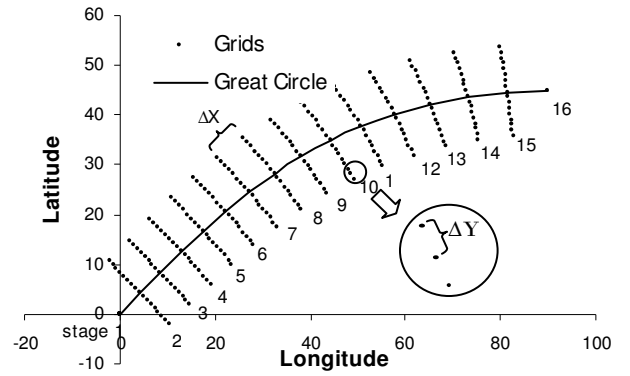

Figure 1: Projections of space grid system on a longitude× latitude plane

ΔY. P (1, 1) is the departure position; P (1, K) is the destination position.
J: Total number of time interval of time axis on a stage.
$\overline{\Delta t}$: Time interval between two adjacent grids on time axis on a stage.
Δt: Time step for fuel consumption calculation between two stages.
X (i, j, k): State on stage k, where: i = 1, 2, 3…, N (k), j = 0, 1, 2 …, J - 1, k = 1, 2, 3…, K. The geographical position of the state X (i, j, k) is defined by P (i, k) on stage k, the time variable of a state is $t_{i,j,k}$, $\overline{t_j} \leqq t_{i,j,k} \leqq \overline{t_{j+1}}$, $\overline{t_j} = \overline{\Delta t} \times j$, $\overline{t_{j+1}} = \overline{\Delta t} \times (j + 1)$. Between point (P (i, k), $\overline{t_j}$) and (P (i, k), $\overline{t_{j+1}}$), there is only one state that has the minimum fuel consumption. A state X (i, j, k) on stage k is in a floating status during the iteration until a point with the minimum fuel consumption is found, then the point is chosen as the state X (i, j, k) before proceeding with following calculation.
X (1, 0, 1): Initial state. Time variable $t_{1,0,1}$ of the initial state is 0.
X (1, j, K): States on the final stage K, j = 0, 1, 2…, J - 1, the position of these states on a latitude × longitude plane is P (1, K).
F_{opt} (i, j, k): The minimum fuel consumption from the initial state to the state X (i, j, k).
u (m): Calm water ship speed, where: m = 1, 2, 3…, M. u (m + 1) - u (m) = Δu, M is the total number of discrete speed between two stages.

2.4 SUB-PROBLEM SOLVING

The procedure of predicting fuel consumption between two stages can be considered as a sub-problem solving in the weather routing optimisation. The predicted optimum fuel consumption during an entire voyage is obtained by adding up of all fuel consumed between two stages along the entire route.

In the execution of the novel 3DDP method, the engine delivery power is taken as a constant value between two stages once it has been optimised

except for the situations when ship has to slow down in certain weather conditions to ensure the ship safety. Since it is convenient to use ship speed in hydrodynamic analysis, determining resistance, ship speed loss, etc, the ship speed in calm water, instead of the engine delivery power, is used as a control variable of stage in the following discussion. Once the ship speed in calm water has been determined, the ship power can be obtained by a one-to-one relationship between engine power and ship speeds. Thus, fuel consumption can be obtained indirectly once ship speed is known. The selected calm water ship speed between two stages is not allowed to be larger than the value corresponding to the maximum continuous rating (MCR) of the main engine.

No matter how efficient the optimisation algorithm is, the optimised results would become no sense without careful considering ship hydrodynamic behaviour and real ship propeller characteristics. In this paper, to make this new 3DDP method practical and functional a simpler and easier approximate method established by Kwon (2008) is used for the prediction of involuntary drop in speed in irregular waves and wind with the assumption that the engine of the selected ship is able to provide a constant power output under different weather conditions and an IMO guideline (2007) is used to ensure the ship safety during the optimisation procedure.

The procedure of determining fuel consumption for a given ship speed in calm water between two stages is as the following:

Step 1: Setting the calm water ship speed.
Step 2: If the calm water ship speed is smaller than the minimum allowed value, skip out the calculation for the given heading. That means the navigation for this given heading is not available under this calm water ship speed or engine delivery power.
Step 3: Calculation of ship speed. Ship speed loss is calculated based on the Kwon's method.
Step 4: If ship operations violate the set constraints, ship speed in calm water will be reduced by Δu, then go back to step 2.
Step 5: Calculation of propeller rotation speed according to real propeller characteristic curve.
Step 6: Calculation of ship position and fuel consumption for the next time interval Δt.
Step 7: Execute step 1 to 6 repetitively with the fixed time interval Δt between two stages until the ship (simulation step) arrives at the next stage or the final destination.

The time interval Δt for calculation is normally chosen at the frequency of reception of weather forecasting which is usually every 3 or 6 hours. Figure 2 shows the procedure described above.

2.5 RECURSION PROCEDURE

The recursion procedure of the forward dynamic programming can be described as follows:
Step 1: Set stage variable k = 1. F_{opt} (1, 0, 1) = 0 and all other F_{opt} (i ≥ 1, j ≥ 0, k ≥ 2) is set to infinite.
Step 2: Iterate steps 3 to 7 below for each state X* = X (i, j, k) on stage k, where: i = 1, 2, 3…, N (k), j = 0, 1, 2…, J-1. Parameters attached with symbol *
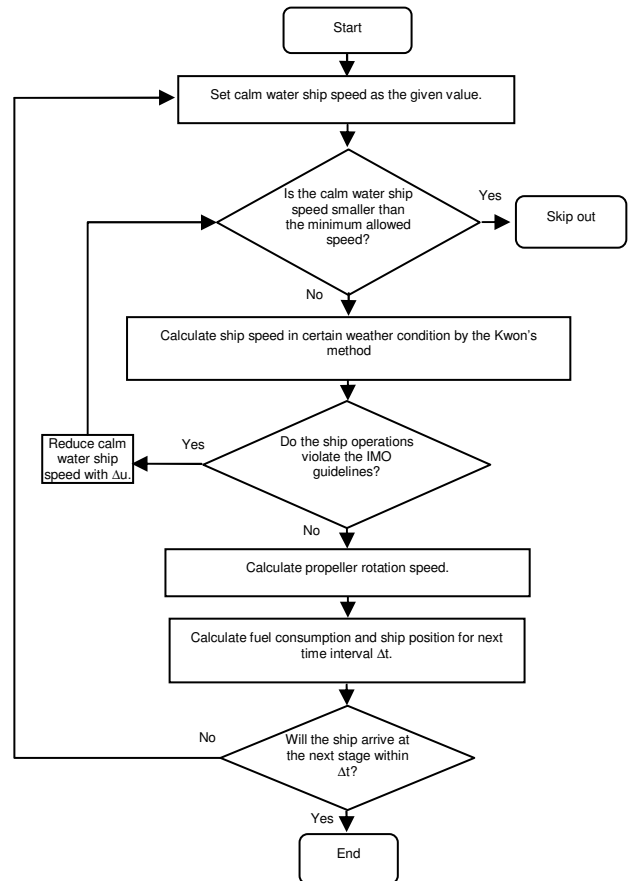

Figure 2: Estimate of fuel consumption between two stages

are used as interim parameters on stage k for iterating calculation. If a state X* = X (i, j, k) is unattainable due to constraints, the calculation of the state is given up and go to the calculation of the next state.
Step 3: Calculate the ship heading H* from the position P (i, k) of state X* to the next stage position P (i', k + 1) as shown in Figure 3, where: i' = 1, 2, 3…, N (k + 1). A rhumb line is used as the ship route between position P (i, k) and P (i', k + 1). Iterate steps 4 to 7 for each H*. If certain ship heading H* violates the set constraints, skip out the calculation for this chosen heading and go to the next heading calculation.
Step 4: Iterate steps 5 to 7 for each u* = u (m), where: m = 1, 2, 3…, M.
Step 5: Calculate the fuel consumption $\Delta f_{m, i'}$ and the voyage time $\Delta t_{m, i'}$ between state X* on stage k and a state on stage k + 1, If a ship speed u*

violates the set constraints, skip out of the calculation of the chosen speed and go to the calculation of next speed. $t_{m, i'}$ is denoted as the arrival time (simulation progression) at position P (i', k + 1) from the initial state, $t_{m, i'} = t_{i, j, k} + \Delta t_{m, i'}$, as shown in Figure 4. Position X'= (P (i', k+1), $t_{m, i'}$) is to represent a new possible state on stage k + 1, The total fuel consumption from the departure to X' is f', $f' = F_{opt}$ (i, j, k) + $\Delta f_{m,i'}$. Parameters attached with symbol ' are used as interim parameters on stage k + 1 for iterating calculation.

Step 6: Calculate j' which satisfies $\overline{t_{j'}} \leqq t_{m, i'} \leqq \overline{t_{j'+1}}$. If f' < F_{opt} (i', j', k + 1), then replace the old X (i', j', k + 1) and F_{opt} (i', j', k + 1) by X' and f' respectively to form new X (i', j', k + 1) and F_{opt} (i', j', k + 1) as shown in Figure 5. It can be seen that, only one possible state which has the minimal fuel consumption between the time interval $\overline{t_{j'}}$ to $\overline{t_{j'+1}}$ at the position P (i', k + 1) on stage k + 1 will be treated as the state X (i', j', k + 1). The time variable of the state X (i', j', k + 1) is not fixed. The benefit of using non-fixed time for possible (float) states is that it eliminates the interpolation in the calculation which is usually used in dynamic programming problems. Thus, it can save computing time. When the weather in $\Delta\overline{t}$ time does not change too much using the floating technique will not influence the accuracy of the optimised results.

Step 7: The departure state X* on stage k, from which the ship (simulation step) arrives at the state X (i', j', k + 1) on stage k + 1 and the corresponding control variables are saved for tracing the optimum route by a backward procedure at the end of the calculation.

Step 8: Let k = k + 1, go to step 2 until k = K.

Once the state on the final stage K has been obtained, a backward calculation procedure will be used to identify a route with the optimised fuel consumption. This route is the optimised route resulted from the 3DDP.

This optimisation procedure described above will be repeated every time when weather forecast or ship position is updated at sea.

3. CASE STUDY AND COMPARISON

To illustrate the advantages of the novel 3DDP method, a comparison between the 2DDP method, the novel 3DDP method and the genetic algorithm is given in this section. All these methods use the same constraints during the optimisation.

The data for the case ship, route and weather conditions offered by National Oceanic and Atmospheric Administration (NOAA) are as the following,

Case ship: A 54,000 DWT container ship.

Departure from: Le Havre.
Arrival at: New York.
Departure time: 03:00 P.M. 25/ 01/ yyyy.
Time interval between states on a stage: $\Delta\overline{t}$ = 0.1 hour. (Just used by the 3DDP method)
Time step for fuel consumption calculation between two stages Δt = 6 hours.
Calm water ship speed: u = 5 to 25.4 knots.
Ship speed change step: Δu = 0.1 knots.
Total stage number: K = 14.
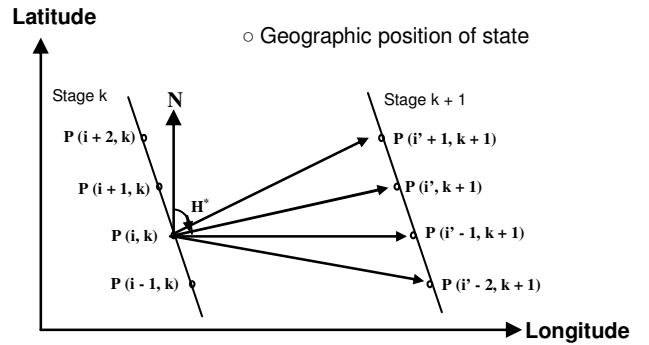Stage space: ΔX = 212 miles.

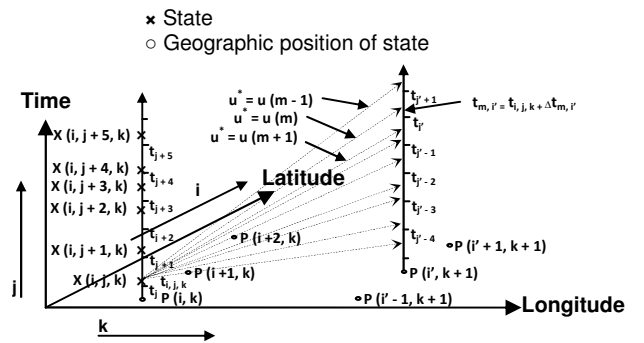Figure 3: Calculate ship's heading between two stages

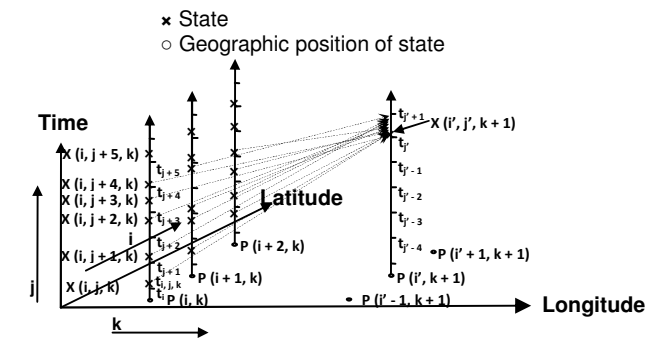Figure 4: Calculation of possible states on stage k+1

Figure 5: Determine the state

State space: ΔY = 46 miles.
Maximum continuous rating of a main engine (MCR): 48598 kW (corresponding to 25.4 knots in calm water).
Specific fuel oil consumption: 170 g / kWh.
The ratio between CO_2 and fuel by weight: 3.2: 1.

Safety constraints: The maximum wave height is 7 m and compliant with the IMO guidance (2007).
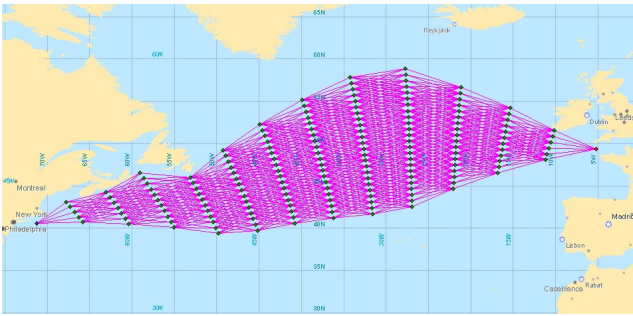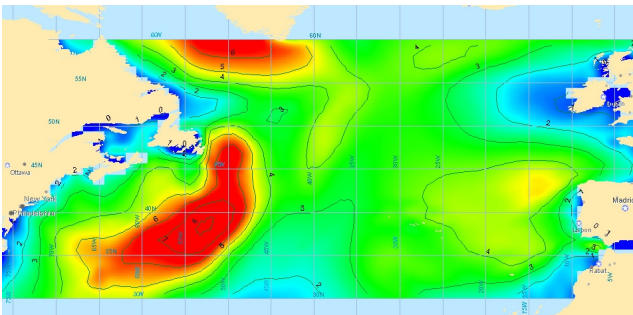

Figure 6: The grids system

Figure 7: Weather forecast at 3 A.M. 25 / 01 / yyyy for the Northern Atlantic region

Figure 6 shows the grid system used in the simulation. Figure 7 illustrates the weather forecast. A significant wave height 8 m at 3 A.M. 25 / 01 / yyyy exists at the Northern Atlantic region. The storm weather is moving towards northeast. Significant waves with a height more than 10 m occur at times. The ship must avoid this storm area for safe operation.

The 2DDP method treats the engine delivery power as a constant during the voyage except that the vessel reduces her speed when encountering certain storm weather conditions. The minimum time routes are calculated under every discrete calm water ship speed by this method.

For checking the performance of the genetic algorithm, 3 kinds of classical method, NSGA II (Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal et.al, 2002), SPEA (Carlo R. Raquel, Prospero C, et.al, 1999), MOPSO_CD (E. Zitzler, 1999) are chosen. Two object functions, minimum fuel consumption and minimum passage time, are used. Unlike other researchers who choose ship route by perturbing a parent route when employ the genetic algorithm, a same grid system as it used by the 2DDP and 3DDP method is applied in this paper. The advantage of using this grid system by the genetic algorithm is that the unavailable routes which cross the land can be deleted before the optimisation procedure starts, so some computing time can be saved. Once the routes have been chosen according to this grid system the design

variables of the genetic algorithm are the index of the geographic position of states on a stage and the calm water ship speed (engine power) between two stages. For example, if the total stages number is 16, 29 (14 indexes of the geographic position and 15 calm water ship speed settings) design variables will be used. Both NSGA II and SPEA adopt the real-coded method by using the simulated binary crossover (SBX) operator and polynomial mutation (K. Deb and R. B. Agrawal, 1995). The parameters setting for the genetic algorithm are listed in Table 1.

In a comparison with the number of parameters used by the 3DDP method, much more parameter settings are needed by the genetic algorithm. These definitely give some inconvenience to the operators, and how to set these parameters is another problem for the operators. The impact and sensitivity studies of these parameters used by the genetic algorithm are out of the scope of this paper and will be given no further studies. Another disadvantage of the genetic algorithm is that, it is a kind of stochastic method and can not ensure the same results can be achieved in every run.

Table.1: The parameters setting for the genetic algorithm.

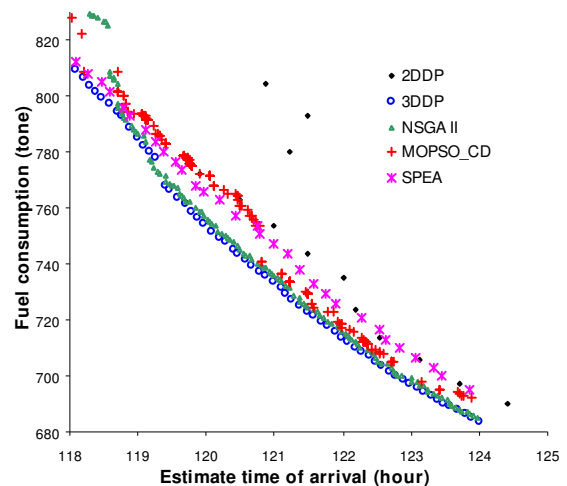|	Crossover distribution indexes η_c	mutation distribution indexes η_m	crossover probability p_c	mutation probability p_m	external archive number	population size	generation
NSGA II	20	20	0.9	0.3	---	200	10000
MOPSO_CD	---	---	---	0.3	500	200	10000
SPEA	20	20	0.9	0.3	50	200	10000

Figure 8: Fuel consumption for different ETA

Figure 8 shows the optimised results of fuel consumption for different estimate time of arrival (ETA) from 118 to 125 hours. For the genetic algorithms, the results are the PARETO frontiers. It can be clearly seen that, the 3DDP method offers the best results with a very uniform distribution. The results optimised by the 2DDP method are the

worst for most arrival times and with a much dispersed distribution. As shown in Figure 8, the results from the 2DDP can not cover the whole range of the ETA. The 3 genetic algorithms presented above can only offer local optimised results. Actually after 5000 generations, further calculation has little improvement to the accuracy of the results by the genetic algorithms.

It can be seen from Figure 8, amongst the 3 generic algorithms, NSGA II is the best method. For most ETA, this method offers more accurate results and a finer distribution of fuel consumption against arrival time than these calculated by MOPSO_CD and SPEA. The optimisation capabilities of MOPSO_CD and SPEA are well matched, they are better than the 2DDP method and worse than the 3DDP method and NSGA II.

Generally speaking, in a medium sea condition, the differences between the optimised results by each method are insignificant, but in a rough sea condition, the 3DDP method can show much more advantages in seeking the real global optimised results. After 30 independent runs for every genetic algorithm under 3 rough sea conditions, it is found that no results are better than that of the 3DDP method, but most time they are better than that of the 2DDP method.
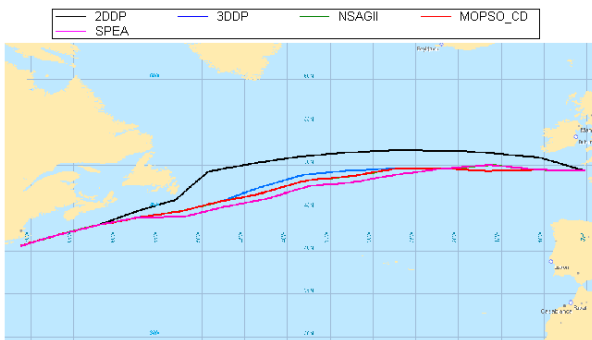

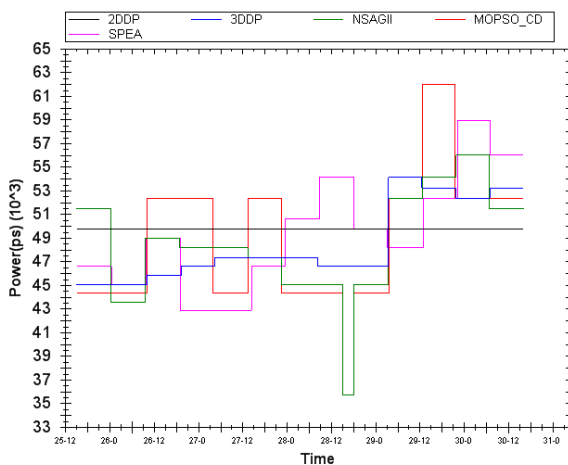Figure 9: Optimised route, ETA = 121 hours
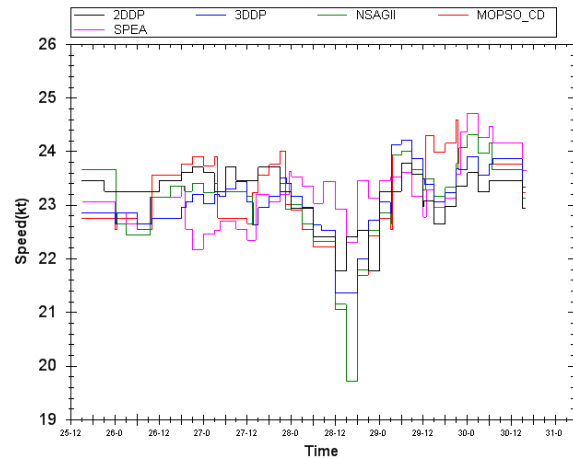

Figure 10: Engine delivery power

Figure 11: Ship speed over ground

Figure 9 shows the optimised routes with a specified ETA at 121 hours. Table 2 presents a summary of the optimised routes. Figure 10 and 11 show engine delivery power and ship speed over ground of the routes optimised by different methods. The routes calculated by the 3DDP, NSGA II and MOPSO_CD are shorter than that of 2DDP and SPEA. Compared with the results from the 2DDP method, the results from the 3DDP show about 20 tons of fuel saving and 64 tons of CO_2 emission reduction. The routs optimised by NSGA II and MOPSO_CD are nearly the same but the strategies of engine delivery power are not the same. As a result, NSGA II can save 4.2 tones of fuel and reduce 13.44 tones of CO_2 emission compare with these of the 2DDP method. The route optimised by SPEA is the longest one compared with others, however, because the engine delivery power can be changed during the voyage, it can still save 6.5 tons of fuel than that of the 2DDP method and reduce 20.8 tons of CO_2 emission.

Table.2: Summarise of optimisation results.

	2DDP	3DDP	NSGAII	MOPSO_CD	SPEA
ETA (h)	121	121	121	120.8	121
Distance (sm)	2806.9	2791.7	2791.5	2791.5	2807.2
Fuel (tone)	753.3	733.4	736.4	740.6	746.8
CO2(tone)	2410.56	2346.88	2356.48	2369.92	2389.76

4. CONCLUSION

In this paper, an overview of different methods developed for weather routing has been presented. Advantages and disadvantages of each method have been discussed. A novel forward dynamic programming method for weather routing has been introduced in a great detail. A case study has shown that, the newly developed 3DDP method can offer a quasi-global optimal routing with a number of advantages than the 2DDP and the genetic algorithm. Compared with the 2DDP method, the newly developed method allows changes of

shipping course and engine delivery power during the optimisation process. Compared with the genetic algorithm, this method needs less parameters and produces excellent performance. Its programming is straightforward and easy to understand.

As the fuel oil price has been increasing to ever high and emission regulations are becoming more and more stringent, fuel saving from shipping is of a significant importance to shipping companies and long term impact to the global environment. The use of the newly developed weather routing programme is able to contribute to the international complain on fuel saving and reduction of green house gas emissions.

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