Peridynamics: A Novel Approach for Material and Structural Modelling

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Solid mechanics is an interdisciplinary subject mainly deals with deformation of materials and structures under external loading conditions. In many applications, safety is an important concern. For instance, a small crack can cause catastrophic damage to an airplane which can result in loss of many lives (see Fig. 1a). Moreover, such incidents can cause significant damage to the environment which might take years for complete recovery process such as the effect of oil spill due to a damaged tanker (see Fig. 1b), corroded pipeline, etc. In order to prevent from such undesired incidents, several approaches are commonly followed within the solid mechanics framework. One common approach is to perform experiments to test the durability of materials and structures. Although such an approach can provide us realistic data, such experiments are not always possible and they are mostly expensive. Today, engineers and researchers commonly use theoretical or computational approaches as an alternative to analyse the problems that they are working on. Theoretical approaches mostly depend on various assumptions to simplify the calculations. Moreover, they are restricted to specific geometries and loading conditions. Hence, it is essential to use a technique which does not have such limitations and is also economically feasible. Computational techniques are very good candidates for this purpose and they are commonly used both in industry and academia.

(a) **(b)**

Fig. 1. (a) Damaged airplane fuselage and (b) oil spill from a tanker

There are currently various computational approaches available and some of these techniques are implemented as part of commercial softwares. Amongst these, finite element method (FEM) is the most common and popular approach. In FEM, the material or structure to be analysed is divided into “finite” number of small regions called “elements” (see Fig. 2). All elements are connected to each other through “nodes”. FEM basically tries to solve a governing equation which is a different representation of well-known Newton’s second law and it is a partial differential equation from mathematical point of view. Although FEM is a very successful technique to analyse many problems that engineering community is dealing with, it encounters a difficulty when predicting failure or crack initiation and propagation in materials and structures due to its mathematical structure. For such special cases, a newly introduced approach called “peridynamics” can be utilized and it is the main focus of this paper.

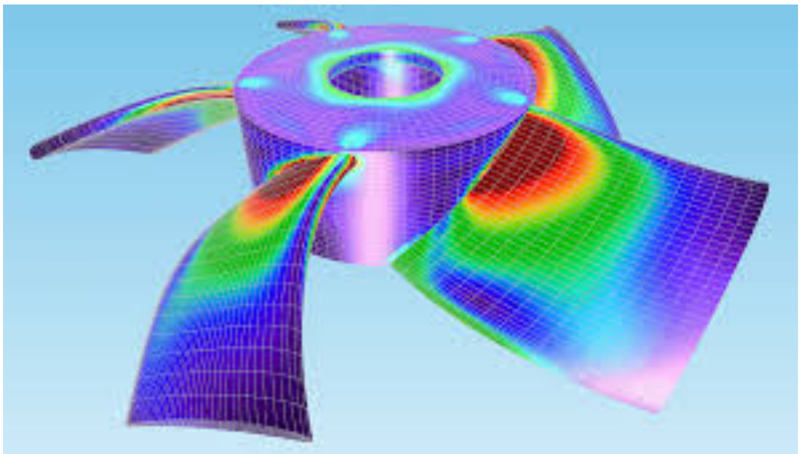

Fig. 2. Finite element analysis of a cooling fan (Comsol) [1]

Peridynamics

Peridynamics was introduced by Dr. Stewart Silling from Sandia National Laboratories in USA. The first scientific paper was published in 2000 [2]. The word “peridynamics” is composed of two Greek words “peri” and “dynamics” which mean “horizon” and “force”, respectively. Peridynamics is a new continuum mechanics formulation which uses integro-differential equations for governing equations as opposed to partial differential equations which are used in classical continuum mechanics and FEM approach as mentioned earlier. The main advantage of peridynamics is that it doesn’t use spatial derivatives in its formulation which causes problems when predicting failure. Moreover, it is a non-local formulation due to its length scale parameter called “horizon”. Hence, “non-local” effects, which can be seen at very small scales, can be captured [3]. “Horizon” defines the range of interactions between material points. “Material point” is the basic object of a continuum formulation which represents an infinitesimal volume.

There are currently mainly three different peridynamic formulations which can be found in the literature. These are “bond based peridynamics”, “ordinary state based peridynamics” and “non-ordinary state based peridynamics” [4-7]. The main difference between these three types is the forces that material points are acting on each other. “Non-ordinary state based peridynamics” is the most general form in which the peridynamic forces between material points can be in any direction and magnitude (see Fig. 3a). “Ordinary state based peridynamics” is based on the assumption that the peridynamic forces between two material points are along the “bond” direction between material points, but can have a different magnitude (see Fig. 3b). “Bond based peridynamics” is the original peridynamic formulation that Silling introduced in 2000 [2] and based on the assumption that peridynamic forces between two material points are equal and opposite to each other (see Fig. 3c). Moreover, peridynamic forces only depend on the deformation behaviour of material points which are associated with them as opposed to the other two “state based” formulations in which peridynamic forces also depend on the deformation behaviour of other material points within their horizons.

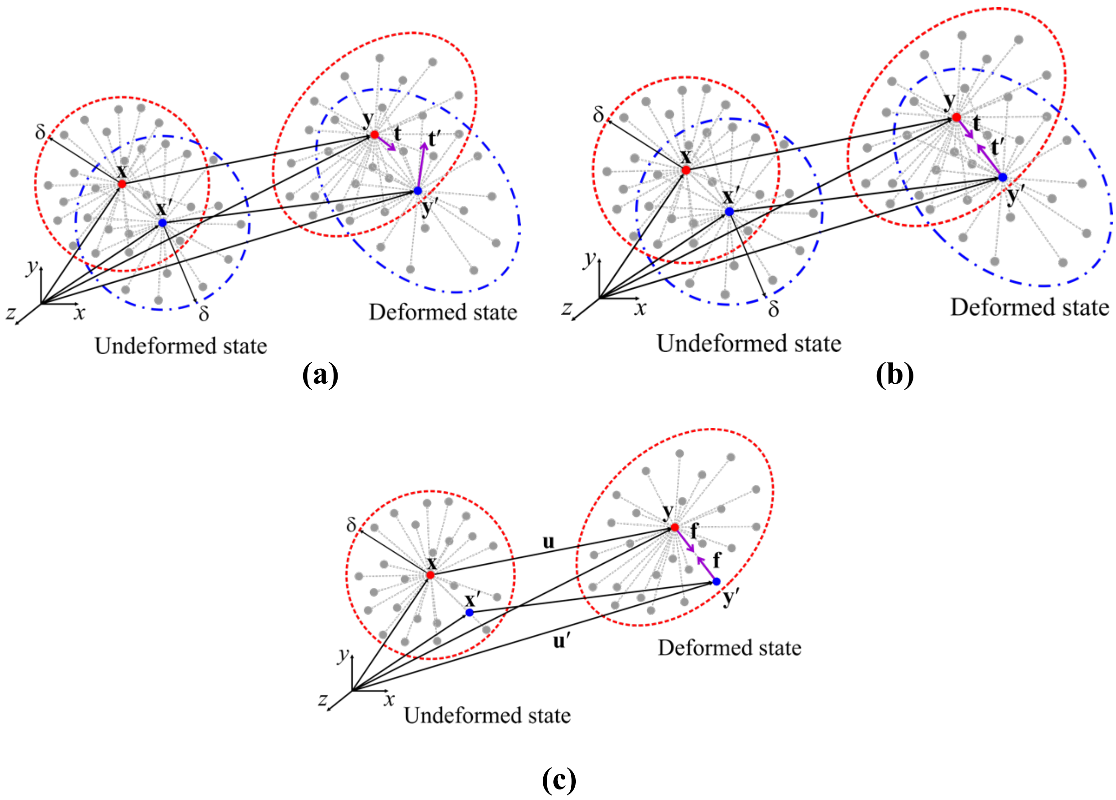

Fig. 3. Various peridynamic formulations; (a) non-ordinary state based peridynamics, b) ordinary state based peridynamics and (c) bond based peridynamics [5]

For the last 15 years, there has been significant research efforts on peridynamics. Currently, there are peridynamic formulations not only for elastic material behaviour, but also for plastic [8], viscoelastic [9] and viscoplastic [10] material behaviours. Moreover, it is not only limited for the analysis of isotropic materials that we commonly use such as steel, aluminium, etc. Various peridynamic formulations are proposed in the literature to analyse composite materials [11-17] which are also becoming a popular material in many industries due to their superior properties and low densities with respect to some other traditional materials. It is difficult to solve peridynamic equations analytically. Therefore, generally a "meshless" discretization scheme is utilized for numerical solutions (see Fig. 4).

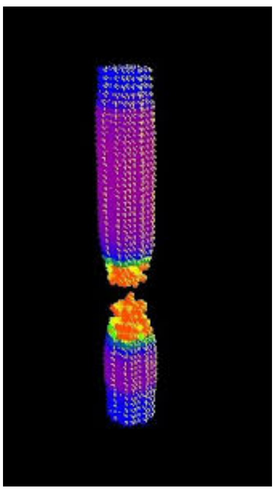

Fig. 4. A meshless discretization for a peridynamic model of a cylindrical bar under tension loading [18]

In order to analyse some sophisticated problems of today such as cracking behaviour of electronic packages, lightening strike damage, hydraulic fracturing, etc., it may not be sufficient to perform a single field analysis. Hence, it is essential to take into account the effect of multiple physical fields on each other. We name this type of analysis as "multiphysics analysis". Today, peridynamic formulations for many other physical fields are also available such as for heat transfer analysis, moisture diffusion analysis, porous flow analysis, etc. [19-26]. Hence, it is possible to perform "multiphysics analysis" in a single peridynamic framework which eliminates possible difficulties in linking different solution tools.

Conclusions

Peridynamics is a new non-local continuum mechanics formulation which is especially useful for failure prediction in materials and structures. It is not only applicable to be used at large scales, but also can be used at small scale analysis due to its "non-local" characteristic. Moreover, it is also suitable for "multiphysics analysis" since there are existing peridynamic formulations for many physical fields.

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