Analysis of the Wave-Induced Vertical Bending Moment and Comparison with the Class Imposed Design Loads for 4250 TEU Container ship

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ABSTRACT: The long-term predictions of vertical wave bending moment are made for the extreme design values on ship. As part of the INCASS (Inspection Capabilities for Enhanced Ship Safety), this paper carried out a short-term estimation of wave loads for 4250 TEU container ship by the hydrodynamic analysis software of ANSYS-Aqwa based on three-dimensional linear potential flow theories. Based on the short-term prediction and the wave statistic of the North Atlantic Ocean, a long-term prediction of vertical wave bending moment is obtained. The results are required and processed to the Decision Support System (DSS), in order to assist to monitoring and risk analysis for ship structure and machinery the towards enhanced and efficient ship operations (Konstantinos, et al., 2015). The prediction values also provide a reference for the trend analysis of the past record signals (Ulrik Dam et al, 2015) for evaluation of longitudinal strength of container ship.
Keywords: long-term prediction, wave-induced, vertical bending moment, container ship

NOMENCLATURE

F_n	Froude number
H_{max}	maximum wave height
H_s	significant height
T_z	zero crossing period
ω_o	modal wave frequency
L	length between perpendicular
B	breadth moulded
T	draft at ballast arrive condition
μ	wave encounter direction
η_0	unit wave amplitude
ω_e	encounter frequency
κ	wave number
λ	wave length

1 INTRODUCTION

Longitudinal strength of the container ship has been a greater awareness to fatigue design criteria. Therefore, it is important to estimate the global loads such as the vertical bending moment for the integrity of the ship's structure. These expected long-term predictions of structural global loads being called design loads can be carried out a hydrodynamic analysis of the ships. The define wave-induced structural loads for the design of the ship structure is in most cases still based on empirical formulas from Classification Societies. However, presently there is a tendency to apply procedures based on direct calculations to define the design wave loads (Guedes et al., 2004).

The INCASS project aims to avoid ship accidents, promote maritime safety and protect the environment via structural and machinery monitoring, data gathering, reliability analysis and decision support (EC 2009). Monitoring is an important part that it is costly for ship companies with regard to either inspection or maintenance (Lazakis et al., 2016). Most new advanced ships have extensive data collection systems to be used for continuous monitoring of engine and hull performance, for voyage performance evaluation (Nielsen et al., 2011). From the initially sensors utilized for data acquisition, eventually, the DSS system uses the stored data in order to predict the probabilities of upcoming failures (Michala et al, 2016). The purpose of this paper is to estimate long-term values of VBM to investigate the influent of different significant wave height relations on the design values.

2 REVIEW

2.1 *Strip Theory*

Based on the concept of the slender body theory in aerodynamic, the applied hydrodynamic models are the Strip theories (Kouvin-Kroukovdky 1955), as the most appropriate tool, first developed for accurate prediction of the wave loads and ship responses to account for the forward speed. Due to the linear strip theories are computational simplicity and the generally satisfactory agreement with experiments, so far the theories have been widely accepted and used as the main tool for estimating the performance of a ship in waves, especially very straightforward in irregular waves (Wang, Z., 2000).

Based on a perturbational procedure, a non-linear quadratic strip theory formulated in the frequency domain predicts wave loads and ship responses in moderate seas (Jensen et al., 1996. Schlachter et al, 1989) used the non-linear strip theory of higher order differential (Soding, 1982). predicted vertical ship motions and wave loads. ITTC (1987) concluded that the slender body theory is not seem to possess advanced over the strip theory for the vertical motions of a ship at forward speed, whereas it seems to provide better predictions for sway and yaw motions.

2.2 *Long term prediction of bending moment*

The St. Denis-Pierson method is originated to predict the ship motions at other conditions by the contributions of various wave frequencies to response motions in regular or random seas (St. Denis and Pierson, 1953). The short term response of wave induced motions and loads in frequency-domain are obtained as the product of an input wave spectrum by a transfer function. It is discussed the mainly choice of wave scatter diagram and calculation of the transfer functions, on the definition of the design requirement for wave induced vertical bending moments(Soares et al., 1990). Variability in the wave-induced extreme response for marine structure is important due to the statistical uncertainty for sea states in various service periods. (Moan, Gao and Ayala-Uraga, 2005).

The adequacy of the Pierson-Moskowitz spectrum for long-term formulations of wave-induced load effects has been tested for ships of different characteristics (Soares et al., 1990). IACS has issued the standard wave data (IACS Recommendation 34) which is recommended to use a return period of at 20 years, ship's service life $T_0=20$ year, corresponding to about 10-8 probability of exceedance (p.o.e) per cycle. Lee et al. (2011a, 2011b) report the non-linear hydroelastic analysis of a container ship (336m), by comparing the calculated vertical bending moments with results from model tests with flexible model in head seas.

2.3 *Long term prediction methodology*

The approach for calculating the long-term prediction of ship responses has been established for several years. In this approach it is necessary to have the probability density function of a response, conditional on certain physical conditions being met.
These response characteristic are commonly referred to as response amplitude operators (RAOs) and are proportional to amplitude. The transfer function or modal receptance which relates input forces to output response:

$$\lfloor X_{jm} \rfloor = H \lfloor F_{jm} \rfloor (i=1,6) \tag{1}$$

Where

$$H = \left\{ -\omega_e^2 (M_{matrix} + M_{mass}) - i\omega_e C + K_{hys} \right\}^{-1} \tag{2}$$

Where M_{matrix} is 6×6 structural mass matrix, C are the 6×6 hydrodynamic added mass and damping matrices including the hydrodynamic interaction coupling terms between different structures, K_{hys} is the assembled hydrostatic stiffness matrix; ω_e is the encounter frequency.

In regular waves, the solutions to the equation of transfer functions of ship attitudes (Lloyd et al.,1989) have the form:

$$X_i = X_{i0} \sin(\omega_e t + \delta_i) m (i=1,6) \tag{3}$$

Where X_i are three linear motion displacements of the center of gravity (surge, sway and heave) and three angular rotations about the axes (roll, pitch and yaw); X_{io} is motion amplitudes which is proportional to the wave amplitude η; ω_o is the encounter frequency and δ_i is phase angle of ship attitude leading the wave depression from 0 radian. The response spectrum is adopted the Pierson-Moskowitz spectrum, and the spectral ordinate at a frequency ω (rad/s) is given by (DNV-RP-C205 3.5.5.1).

$$S_w(\omega) = 4\pi^3 \frac{H_s^2}{T_z} \frac{1}{\omega^5} \exp(-\frac{16\pi^3}{T_z^4} \frac{1}{\omega^4}) \tag{4}$$

The frequency domain dynamic analysis outputs the significant amplitudes of forces and responses

$$R_s = 2\sqrt{m_0} \tag{5}$$

Where $m_0 = \int_0^\infty S_w(\omega)$ in which $S_w(\omega)$ is a force or response spectral density, Pierson-Moskowitz spectrum.

With sea states scatter diagram of North Atlantic (IACS Rec.34) from the calculation of the short-term probability occurrence of VBM. The long-term extremes can be estimated by considering only a few short term sea states. In this approach, each short-term probabilities distribution of the vertical bending moment corresponding to each sea state parameters, H_s & T_z. The response spectrum $S_R(\omega)$ is obtained from the input wave spectrum $S_w(\omega)$ and the transfer function $|H_{Rw}(\omega)|^2$, all considered to be one-sided:

$$S_R(\omega) = |H_{Rw}(\omega)|^2 S_\omega(\omega) \tag{6}$$

From the amount results of experiments and analysis, the short-term response of ship motion is yielded to Rayleigh distribution, which the probability of density and probability distribution function is:

$$f(x) = \frac{x}{\sigma^2} \exp[-(\frac{x^2}{2\sigma^2})] \tag{7}$$

$$F(x) = 1 - \exp[-(\frac{x^2}{2\sigma^2})] \tag{8}$$

Where σ is the standard normal variable of the x. With the calculation of the significant amplitude M_s of vertical bending moment during 3hours by Aqwa, the evaluated short-term distribution is able to get from the equation (DNV No.30.6.1 4.5.7, 1992):

$$X = [-0.5 M_s^2 \cdot \ln\{-\frac{T_r}{D} \ln(-\upsilon_m(x)D)\}]^{1/2} \tag{9}$$

A two-parameter Weibull distribution may then be fitted to the long-term force amplitude distribution of equation rules (DNV-RP-C205 3.6.2.1)

$$F_L(x) = 1 - \exp[-(\frac{x}{a})^\beta] \tag{10}$$

Where α and β is the scale and slope parameters.
The long-term exceeding probability for a level M_1 of vertical bending moment is (DNV No.30.6.1 4.5.7, 1992):

$$q(M \geq M_1) = \exp[-(\frac{M_1 - M_{mean}}{\sigma^2})^2] \tag{11}$$

Where M_1 is the long-term extreme value; M_{mean} and σ are the mean value and standard deviation of the vertical bending moment during in the irregular short-term sea state.
Based on the equation (DNV No.34.1 3.5.2, 2013) chosen as:

$$\xi_{[m]} = \frac{long \quad term \quad respomse[Nm]}{transfer \quad function \quad peak[\frac{Nm}{m}]} \tag{12}$$

The design regular wave amplitude can be obtained.
The maximum wave height H_b is given (DNV-RP-H103 2.2.4.1, 2011)

$$\frac{H_b}{\lambda} = 0.142 \frac{2\pi d}{\lambda} \tag{13}$$

Where λ is the wave length corresponding to wave depth d. And in deep water the breaking wave limit corresponds to a maximum steepness $S_{max} = H_b/\lambda$ =1/7.

The average wave steepness for short irregular sea states are defined as (DNV-RP-C205 3.5.4):

$$S_s = \frac{2\pi}{g} \frac{H_s}{T_z^2} \tag{14}$$

The limiting values of S_s may, in absence of order reliable sources, be taken as

$$S_s = 1/10 \, for T_Z \leq 6s \tag{15}$$

$$S_s = 1/15 \, for T_Z \geq 12s \tag{16}$$

And interpolated linearly between the boundaries.

3 CASE STUDY

3.1 *Hydrodynamic modelling in regular wave*

The FE code used for the analysis is Maxsurf-Stability (Bentley Systems, 2014) and ANSYS-Aqwa (ANSYS® Aqwa, 2010). The former is used software to calculate the stability of the hull form and latter is widely used finite element code for Hydrodynamic analyses. The geometry model drawing in the Maxsurf-modeler (Bentley Systems, 2014) and stability calculation of the hull form is in the Maxsurf-stability. Then the hull form meshing operation, environment parameters, including wave direction and wave frequency is carried out using the software of Aqwa, which is shown in Fig. 1. This is consistent interface for creating, submitting, and evaluating the hydrodynamic results from the hull form from this Hydrodynamic software.
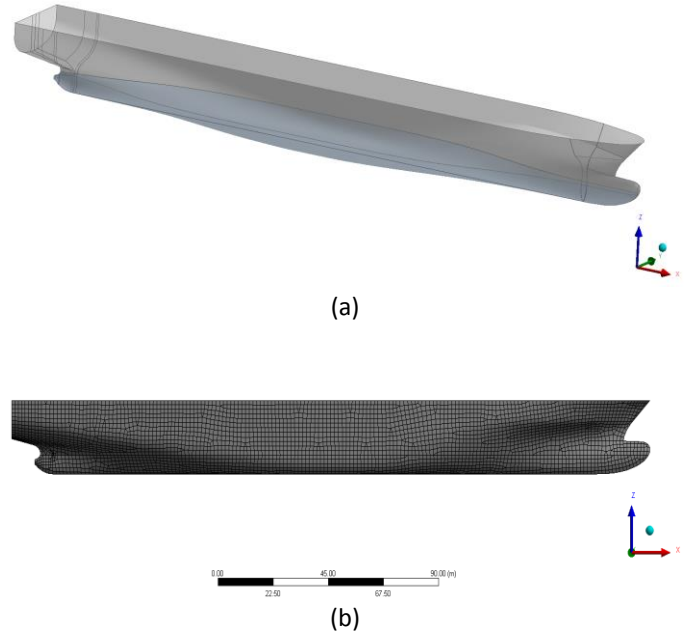

(a)

(b)

Figure 1. (a) Hull form in ANSYS-Aqwa; (b) Hull mesh with quadrilateral panels.

The vessel selected as a base case for the 4250 TEU Container Vessel. The main particular of the vessel and model meshing numbers of nodes and elements are summarised in Table 1. The mesh of the hull is made with a maximum size of elements of 1.5m for analysis of smaller frequencies, which is very high number of quadrilateral panels.

Table 1. Main particulars of the container ship

Dimension	Values	Units
Length of all(L.O.A)	260	m
Length between perp., L.P.P	244.8	m
Breadth, B (MLD)	32.25	m
Depth, D(MLD)	19.3	m
ballast arrive draft, T	6.345	m
Deadweight, DWT	16534.8	ton
meshing nodes	8438	--
meshing elements	8344	--

(b)

Figure 2. Ship response Induced by regular wave: (a) Heave. (b) Pitch.

It is seen that, the variance of wave direction μ from equal to 90º and 180º has large affection on the ship motion, i.e. pitch angle and roll angle has maximum values correspondingly, and for corresponding motions has different peak frequency.

3.2 *RAOs in regular waves*

To obtain the hydrodynamic response of a large – volume structure in waves, the most common numerical tools are three dimensional panel methods which is based on fluid potential theory. ANSYS-Aqwa (ANSYS® Aqwa, 2010) solves a set of linear algebraic equation to obtain the harmonic response of the body to regular wave.

The two-dimensional computed values for heave, pitch and roll motion of the no forward-speed of the ship in unit wave amplitude (1m) is presented in the Fig. 2, where the wave directions μ are from 0 ° to -180 ° (heading wave is equal to 0º), and the wave frequencies are from 0.06 to 0.2, 0.38 to 0.5stepped by 0.02, then from 0.2 to 4, 0.5 to 1.9 stepped by 0.1. Fig. 3 shows the non-dimensional amplitude response, for heave motions, the amplitude are made non-dimensional by dividing by the wave amplitude (unit wave amplitude η_0 (1m)); for angular motions, it is divided by the wave slope amplitude $\kappa\eta_0$. From the graphs, in very long waves (low frequencies) the ship contours the waves and relative motion are very small, because of the ship motion being the same as the wave motion. On the contrary, ship is essential stationary due to short waves (high frequencies).
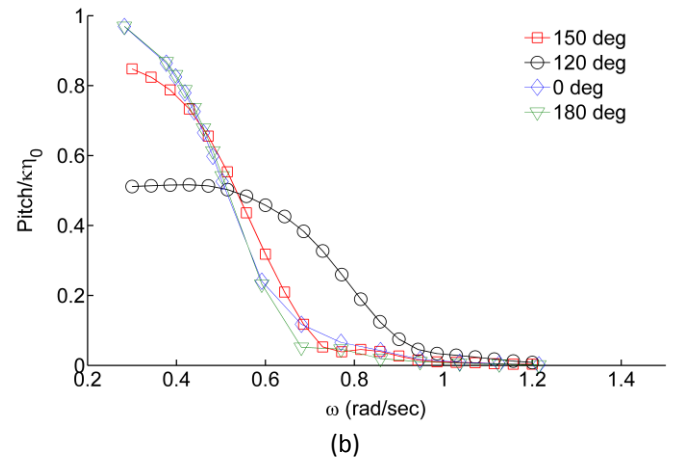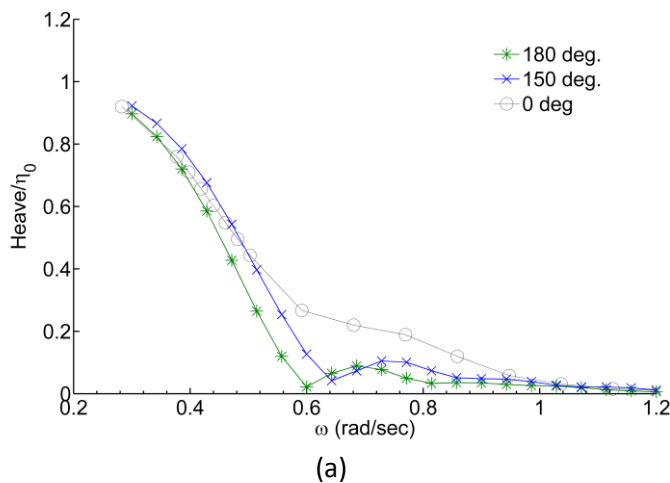
3.3 *Vertical bending moment RAOs in regular wave*

In order to take into account the forward speed influence, the vertical bending moments varied with different forward speed are shown in Fig.3. In the present market situation, more economical voyage speed of containership is about 21 knots, compared to earlier top speeds of 25 knots or more. It is compared with the four segmented 6250 TEU container ship model used for WILS I model tests. The vertical wave bending moment (VBM) curves of the midship in frequencies-domain comparison with the experimental and predictions results (Lee et al., 2011a), are also shown in Fig. 3.
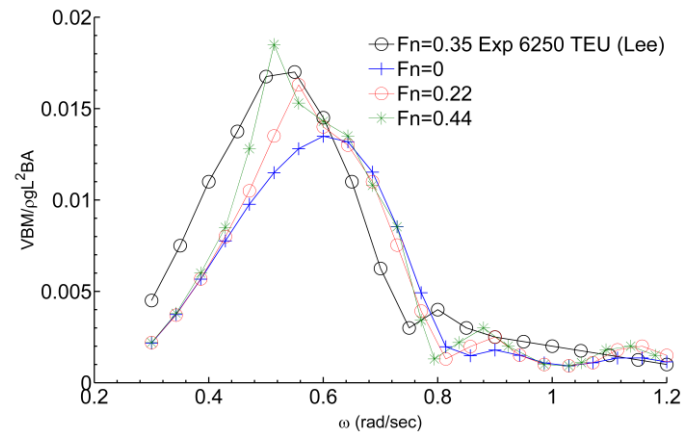

(a)

Figure 3. Vertical bending moment varied with different forward speed and comparison with experiment value(Lee et al., 2011a)

Where the symbol 'A' in graph means the wave amplitude η_0 . From the Fig 3, the numerical simulation method is validated by the comparison the VBM dimensionless with the results from the predicted

analysis and model tests (Lee et al., 2011a). The maximum VBM in regular heading wave, M_w=0.255 (GNm) (per meter of wave amplitude), when the Fn=0.

3.4 *Comparison with design VBM requirements*

The determination of Hydrodynamic loads is based on long term distribution of motion that the ship will experience during its operation life. As the problem presented above, in this case, this part would research the design wave. Normally it is assumed that maximum load will result in also the maximized stress VBM response, so then the design wave height is the key parameter for long-term analysis, with the Pierson- Moskowitz spectrum, which has two parameters, significant wave height Hs and mean Zero-Crossing period TZ. In this Case, it is adopted the probability of occurrence of the sea states scatter diagram of North Atlantic (IACS Rec.34). By extracting the significant bending moment value of M_{sig} from each distributed seaway-spectrum (correspondingly- Hs & TZ), it is obtained the probability occurrence of M_{sig}. Through the module of wave spectra of Pierson-Moskowitz spectrum in software ANSYS-Aqwa (ANSYS® Aqwa, 2010), parameters values and type of wave spectrum is defined, the probability distribution of the M_{sig} in each distributed seaway-spectrum is shown in Fig.4.
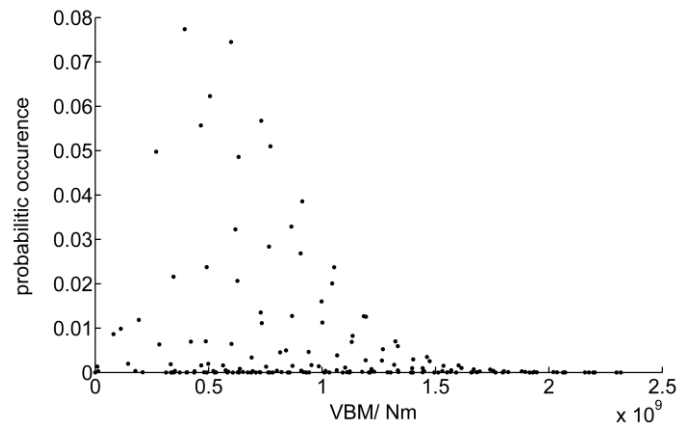

Figure 4. Probability of short-term occurrence of VBM

The final long-term result is determinate by the graph above and bonded with the Rayleigh distribution. The operation life is normally taken as 20 years, considered to correspond to a maximum wave response of 10-8 probability of exceedance in the North Atlantic (Nikolaidis E., and P. Kaplan. 1991). Due to the distribution of the Msig is by equation (7), calculating the scale and shape parameters with the data from Fig.4, the scale and slope parameters of α and β is 108 and 0.84. From the equation (10), in the exceedance level of 10-8 probability, the long-term prediction of the probability occurrence of the Mw at mid ship position is 2.065 × 109 Nm. The extreme long-term bending moment as requirement of DNV classification rules (DNV-Rules for Classification of Ships Pt.3 Ch.1 Sec.5 B100, 2016) is 2 × 109 Nm.

The comparison of numerical simulation values of VBM values in regular wave with DNV (Det Norske Veritas) Classification Rules is shown in Table 2.

Table 2. Comparison with the VBM Values in midship

Item	DNV Classification Rules VBM $10^6 \times$kNm	Numerical Values VBM $10^6 \times$kNm
Still Water	2.28	2.255
Wave Induced	2	2.065
Total	4.28	4.32

From the table 2, the long-term prediction by DNV method is compared with the mimum design vertical bending moment of classification rules, the results is fitting well.

Table 5. Design values of random variables

Parameter	values
Design significant wave height (m)	14.5
Peak wave period (sec)	10.47
Zero-cross period (sec)	9.5
Wave steepness S_S	0.103
Extreme wave bending moment (GNm)	2.065
Design regular wave amplitude (m)	7.8
Design regular wave height (m)	15.6

Current design criteria generally consider significant wave heights less than 11 meters, based on the analysis above, this criterion is inadequate and consideration should be given to design for significant wave height 15.6 meters in the 20-year lifetime.

3.5 *Design of Significant Wave Height Hs and Zero-cross Period T_z*

With the calculation of the equation (8), the design regular wave amplitude is equal to 11 m. And design regular wave height is 22 m; this value is corresponding to the range of 20-30m of the research (Smith, 2007) for large vessel. For analysing the design significant wave height, this paper choses the most dangerous zone 16 in World Scatter Diagrams of Nautic Zones, which the two-parameter Weibull parameters and Log-Normal distribution parameters for HS and TZ (Υ_s=0), showed in Table 2.

Table 3. Weibull parameters and Log-Normal distribution parameters for H_S and T_Z ($\Upsilon_s=0$)

Area	α_s	β_s	a_1	a_2	b_1	b_2
16	3.42	1.56	1.243	0.126	0.0898	-0.0528

By the Weibull distribution equation (7), it can be obtained that the 20-year return period estimate significant wave height Hs20 is 15.6 m, also theTZ20 is 11.4 sec; and the 100-year return period estimate is Hs100 is 16.5 m, also theTZ100=10.5 sec, which is shown in table 4.

From the short-term extreme vertical bending moment analysis, the design severe storm distribution within the sea state of the scatter diagram, major contribution are 15.5 - 16.5 (m) of significant wave height and 9.5 – 12.5 (sec) of zero-cross period.

Table 4. Design of H_S and T_Z design in different return period

Return period	H_S (m)	T_Z (sec)	VBM GMm
20-year return	15.6	11.4	2.07
100-year return	16.5	10.5	2.30

Based on frequency domain method, with the numerical and probability analysis data, the prediction values of long-term bending moment during the 100 year return period is about 15% larger than the rule minimum requirement in most dangerous zones 16. In that case, different researcher compared the M_{design}/M_{rule} in different ship parameters, the Container ship with parameter of C_B/LBT (10-5-10-6) has the rate of M_{design}/M_{rule} is from 1.39-1.5 (Guedes et al., 1991). Guedes Soares et al. presented results for an FPSO hypothetically moored in European Area 8 and there would be an estimated long-term sagging bending moment of 30% (linear prediction) greater value than the rules. Also in the S175 calculations in time domain in storm conditions presents that minimum values are not adequate for such situations.

About the design significant wave height, many classification rules have define the wave parameter: IACS, and DNV adopt the value not more than the 10.75 with the length between 300 m and 350 m; LR (Pt 4, Ch 8, Sec 15.3) adopts the value not more than 11.65 m with the length of ship less that the 500 m. Due to these vertical bending moment values of classification rules of are referred to a 10-4 probability, and the extreme bending moment is 1.7 time larger that these values which are refer to a 10-8 probability, in that case, we can have the extreme significant wave height value: 18.275 m and 19.805 m, correspondingly to the former classification and LR (Lloyd's Register). On the contrary, Adam Weintrit et al (Weintrit and Neumann, 2011) advised

the significant wave height is large than the 0.04 ship's length, in this paper, the H_s =9.8m.

Common Structural Rules developing the rules for purpose of effective and safe operation to last for period of 25 years, withstand North Atlantic ocean condition for 25 years, the significant wave height is equal 16.0 m, which corresponds to 31.0 m individual wave height (Han, G.Y., 2004). This calculation result is corresponding to the paper results.

Assessment of operation limit n in view of sea state forecasts, these systematic studies can be transformed into operational criteria expressed as limiting Hs versus ship length, and since the operational criteria about the radio of limiting to the design significant wave height is approximate 0.7. As a result, the limitation value is about 16m (Moan et al, 2006).

4 CONCLUSION

The overall research of this paper is as follow:

(a) Hydrostatic performance of hull form analysis about the mass, buoyage, bending moment and shear force in still water under light-ballast condition is close the load manual;

(b) In unit wave height and different single regular wave direction and frequency, the linear response amplitude operator (RAOs) of VBM, heave, pitch and roll are assess in frequency domain. Comparing the peak frequency value corresponding to the VBM values of experiment and numerical simulation verify the analysis.

(c) The long-term distribution of the VBM in midship is obtained by the calculation of short-term distribution of VBM corresponding to the joint distribution of significant wave height and zero-cross period. The prediction values of long-term bending moment is similar to the rule minimum requirement in most dangerous zones 16. When the 100-year return period is adopted, the Mdesign/Mrule is about 1.15.

(d) Value of Hs=15.6m is in the range of the extreme wave height and it can be the design long-term values corresponding to the return period of 20 years, and satisfied the requirements from the classification rules and other research results.

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