Abstract: This paper examines the economic costs of population ageing. It is argued that population ageing leads to lower rates of economic growth, which makes it more difficult to generate the revenue needed to accommodate the growing numbers in the older age groups. An overlapping generations computable general equilibrium model (OLG-CGE) is constructed in order to quantify such effects. The model is calibrated for Scotland, a country that is expected to age considerably in the future. Simulations carried out, under what are believable demographic assumptions, suggest considerably lower rates of economic growth and large welfare losses associated with population ageing in the future. An analysis is carried out that attempts to estimate the rate of “technological change” (captured for example by changes in labour productivity) needed to avoid these welfare losses.

JEL Classification: J11

Keywords: OLG-CGE, technological change, population ageing, Scotland

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1. Introduction

There is no universally agreed upon definition of what constitutes “population ageing”. However, central to most definitions is an increasing share of the total population in the “older” age groups and a decreasing share in the “younger” age groups. In addition, population ageing is usually associated with low, zero or negative rates of growth of the population of labour force age. The predominant view is that an ageing population supports lower economic growth because the labour force age group gets “squeezed”, leading to low (or negative) rates of labour force growth. In addition, population ageing causes a shift away from investment to consumption since government expenditure tends to increase to meet the increased demand for pensions and other old age-related benefits. The populations of most higher-income countries are ageing rapidly, and there is some doubt that standard of living increases can be sustained.

This paper examines the impact of population ageing on output growth using a computable general equilibrium (CGE) model (see Burfisher, 2011; Dixon and Jorgenson, 2012; Fossati and Wiegard, 2001; Hosoe, Gasawa and Hashimoto, 2010). It is therefore not surprising to find that CGE models have been used to evaluate the impact of population ageing (see Bommier and Lee, 2003; Boersch-Supan, Ludwig and Winter, 2006; Fougère and Mérette, 1999; Fougère, Mercenier and Mérette, 2007; Fougère et al., 2004; Giesecke and G.A. Meagher, 2009; Mérette and Georges, 2010). The type of CGE model that is best suited to demographic research has an overlapping generations (OLG) structure, introduced by Auerbach and Kotlikoff (1987). In this paper such a model is calibrated for Scotland. The Scottish context is
of interest since rapid population ageing is expected in the immediate future. In addition, population decline and labour force decline are not unlikely.

It is common in CGE modelling to assume some long-run “average” rate of exogenous technological change or productivity growth. It is important to point out that there is no productivity growth in the model. That is, productivity is assumed not to change in the simulation period. While this may not be realistic, it is assumed in order to highlight the economic impact of changing population size and age structure, which is the main aim of the analysis. It is worth noting that the model is a simulation model and not a forecasting model. We believe that assuming no productivity growth helps isolate the effect of the “demographic shock” of population ageing. It is also worth noting that any long-run assumption about productivity growth would be arbitrary since we have little idea of what future technological changes or human capital enhancements would generate productivity increases, especially in an ageing context.

This does not mean that technological change is unimportant. In fact, just the opposite is the case. The model in this configuration can be used to estimate the “cost” of population ageing in terms of output growth loss. With these estimates, it is straightforward to calculate the rate of technological change (in terms of total factor productivity growth) needed to compensate for the output growth loss caused by population. This in turn can be compared to historical estimates in order to obtain a feel for the “scale” of technological change needed.

The remainder of this paper is organised as follows. Section 2 outlines the structure of the model. Section 3 discusses how the model is calibrated and gives the data sources. The main findings are presented in Section 4. A brief conclusion follow in Section 5.
2. Model

The model presented in this section is designed to analyse the long-term economic and labour market implications of population ageing in Scotland in a context of its tight fiscal relationship with the rest of the UK (RUK). Scotland is modelled as a small open economy. The RUK is not explicitly modelled. It is present in the model mainly to close the government budget constraint and the current account of Scotland. Below we outline the main features of the production, household and government sectors. We also describe the demographic structure of the model. Demographic change is considered as an exogenous shock. It is the difference in these shocks that drives the simulations results.

2.1 Demographic Structure

The population is divided into 21 generations or age groups (i.e., 0-4, 5-9, 10-14, 15-19, …, 100-104). Population projections represent an exogenous shock. In other words, demographic variables such as fertility, mortality (life expectancy) and net-migration are assumed to be exogenous. This is a simplifying assumption given that such variables are likely endogenous and affected by, for example, differences in economic growth. Every cohort is described by two indices. The first is “t”, which denotes time. The second is “g”, which denotes a specific generation or age group.

The size of the cohort, “Pop”, belonging to generation “g+k” in period “t” is given by two laws of motion:

\begin{align}
\text{Pop}_{t,g+k} &= \begin{cases} 
\text{Pop}_{t-1,g+k+5} f_{t-1} & \text{for } k = 0 \\
\text{Pop}_{t-1,g+k-1} (s_{t-1,g+k-1} + mr_{t-1,g+k-1}) & \text{for } k \in [1,20]
\end{cases}
\end{align}

The first law of motion simply implies that the number of children born at time “t” (age group g+k = g, i.e. age group 0-4) is equal to the size of the first adult
age group \((g+k+5=g+5)\) at time “\(t-1\)” multiplied by the “fertility rate”, “\(fr\)”, in that period. If every couple on average has two children, the fertility rate is approximately equal to 1 and the size of the youngest generation \(g\) at time \(t\) is approximately equal to the size of the first adult generation “\(g+5\)”, one year in the past.

The second law of motion gives the size of any age group “\(g+k\)” beyond the first generation, \(g\), as the size of this generation a year ago multiplied by the sum of age specific conditional survival rate, “\(sr\)”, and net migration rate, “\(mr\)”, at time “\(t-1\)”.

In this model survival and net migration rates vary across time and age. For the final generation the age group 100-104 \((k=20)\), the conditional survival rate is zero. This means that for the oldest age group at the end of the period, everyone dies with certainty.

With respect to the three main demographic variables—fertility, mortality and net-migration—the model allows them to change over time. Demographic change is assumed to be exogenous. Changes in population size, age-structure and sex-structure are driven by a set of precise assumption relating to future level of fertility, mortality and net-migration. Therefore the modelling of changing demography is integral part of the model. The methodology followed is analogous to “building in” a cohort-component population projection structure to the model. This feature makes it ideal for studying the impact of a variety of what can be termed “demographic shocks”, such as different rates of population ageing.

2.1 Production Sector

A representative firm produces at time “\(t\)” a single good using a Cobb-Douglas technology. The firm hires labour and rents physical capital. The production may be written:
\[ Y_t = AK_t^\alpha L_t^{1-\alpha} \]

where “\( Y \)” is output, “\( K \)” is physical capital, “\( L \)” is effective units of labour, “\( A \)” is a scaling factor and “\( \alpha \)” is the share of physical capital in value added. A firm is assumed to be perfectly competitive and factor demands follow from profit maximization:

\[ re_t = \alpha A \left( \frac{K_t}{L_t} \right)^{\alpha-1} \]

\[ w_t = (1-\alpha) A \left( \frac{K_t}{L_t} \right)^\alpha \]

where “\( re \)” is the rental rate of capital and “\( w \)” is the wage rate.

### 2.2 Household Behaviour

Household behaviour in the model is captured by 21 representative households in an Allais-Samuelson overlapping generations structure representing each of the age groups (as described above). Individuals enter the labour market at the age of 20, retire (on average) at age 65, and die at the latest by age 104. Younger generations (i.e. 0-4, 5-9, 10-14 and 15-19) are fully dependent on their parents and play no active role in the model. However, they do influence the age dependent components of public expenditure such as health and education. An exogenous age/time-variable survival rate determines life expectancy.
Adult generations (i.e. age groups 20-24, 25-29, …, 100-104) optimise their consumption/saving patterns. A household’s optimization problem consists of choosing a profile of consumption over the life cycle by maximizing a CES type inter-temporal utility function that is subject to lifetime budget constraint. Inter-temporal preferences of an individual born at time $t$ are given by:

$$U = \frac{1}{1 - \theta} \sum_{k=0}^{20} \left\{ \frac{1}{1 + \rho} \right\}^{k} u_{sr_{t+k,g+k}} \left( C_{t+k,g+k} \right)^{1-\theta} \quad 0 < \theta < 1$$

where “$C$” denotes consumption, “$\rho$” is the pure rate of time preference and “$\theta$” is the inverse of the constant inter-temporal elasticity of substitution. Future consumption is discounted by unconditional survival rate, “$usr$”, which is the probability of survival up to the age “$g+k$” and may be written:

$$usr_{t+k,g+k} = \Pi_{m=0}^{k} sr_{t+m,g+m}$$

where “$sr_{t+m,g+m}$” denotes the age/time-variable conditional survival rate between periods “$t+m$” and “$t+m+1$” and between ages “$g+m$” and “$g+m+1$”

In is important to note that a “period” in the model corresponds to five years and a unit increment in the index, “$k$”, represents both the next period, “$t+k$”, and, for this individual, a shift to the next age group, “$g+k$”.

The household is not altruistic. It does not leave intentional bequests to children. However, it leaves unintentional bequests due to uncertainty of life duration. The unintentional bequests are distributed through a perfect annuity market, as
described theoretically by Yaari (1965). This idea was implemented in an OLG context by Boersch-Supan et al. (2006).

Given the assumption of a perfect annuity market, the household’s dynamic budget constraint takes the following form:

(7)  
\[ HA_{t+1,g} = \frac{1}{sr_{t,g}} \times \left[ Y_{t,g}^L \left( 1 - \tau^L - \text{Ctr}_t \right) + Pens_{t,g} + TRF_{t,g} + \left( 1 + \left( 1 - \tau^K \right) Ri_t \right) HA_{t,g} - C_{t,g} \right] \]

where “\( Ri \)” is the rate of return on physical assets, “\( \tau^K \)” is the effective tax rate on capital, “\( \tau^L \)” is the effective tax rate on labour, “\( \text{Ctr} \)” is the contribution to the public pension system, “\( Y^L \)” is labour income and “\( Pens \)” is pension benefits. The intuition behind the term “\( 1/sr \)” is that the assets of those who die during the period “\( t \)” are distributed equally between their peers. Therefore, if the survival rate at time “\( t \)” in age group “\( g \)” is less than one, then at time “\( t+1 \)” everyone in their group has more assets. That is, they all receive an unintentional bequest through the perfect annuity market.

Labour income is defined as:

(8)  
\[ Y_{t,g}^L = w_t EP_g LS_g \]

where “\( LS \)” is the exogenous supply of labour. It is assumed that labour income is a function of the individual’s age-specific productivity. In turn, it is assumed that these age-specific productivity differences are captured in age-earnings profiles. These profiles, “\( EP \)”, are quadratic functions of age:
with parametric values estimated from micro-data (as discussed below). Retirees’ pension benefits are assumed to be the same across all generations and stay constant in real terms.

Differentiating the household utility function with respect to its lifetime budget constraint yields the following first-order condition for consumption, commonly known as Euler’s equation:

\[ C_{t+1,g+1} = \frac{[1 + (1 - \tau_{t+1}) R_{t+1}]}{(1 + \rho)} C_{t,g} \]

It is important to note that survival probabilities are present in both the utility function and the budget constraint. Therefore, they cancel each other out and are not present in the Euler’s equation.

2.3 Investment and Asset Returns

Migrants in any period are assumed to own the same level of assets the domestic population of the same age. This implies that when net-migration is positive, migrants’ assets add to the stock of capital. Therefore the motion law of capital stock, “\( K_{stock} \)”, takes into account depreciation and assets of newly arrived migrants:

\[ K_{stock_{t+1}} = Inv_t + (1 - \delta) K_{stock_t} + \sum_g HA_{t+1,g+1} NM_{t+1,g+1} \]
where “\(Inv\)” represents investment, “\(\delta\)” is the depreciation rate of capital, “\(HA\)” is the level of household assets and “\(NM\)” is the level of net-migration.

Financial markets are fully integrated implying that financial capital is undifferentiated so that interest rate parity holds. Let “\(R_i\)” be the rate of return on physical assets. It can be defined as the rental rate minus the depreciation rate:

\[
1 + R_i = re_i + (1 - \delta)
\]

### 2.4 Government Sector

Currently Scotland has only limited tax-raising powers. For the majority of spending, the Scottish Government receives a so-called “block grant” from the UK Government. This grant is calculated based on the “Barnett Formula”, which takes into account both the contributions of each region to tax revenue and the demand for services. In our model, the revenue side of the government budget transfer on a decomposition of the block grant calculated by the Scottish Government. Income and consumption tax revenues are differentiated, as well as income from government assets. The difference between government revenues and government expenditures – net fiscal balance – in the model closes the government budget constraint. We call this variable UK transfer, “\(UKTRF\)”, because in recent years this balance has been negative, i.e. Scotland received a larger grant from the UK Government than was its contribution to the centralised budget. Consequently government budget constraint is defined as:

\[
\sum_g \text{Pop}_{t,g} \left[ (\tau^i_c + C_{i}\omega_{i,g} L_{i,g}) + \tau^{c,g}_c \left( P^{c}_{i,g} C_{i,g} \right) + R_{i,G} A_{i} + UKTRF_{i} \right] = \text{Gov}_{i} + \text{Gov}_{E_{i}} + \text{Gov}_{H_{i}} + \sum_g \text{Pop}_{t,g} \left( \text{TRF}_{i,g} + \text{Pens}_{i,g} \right)
\]
where “$\ell^C$” is the effective tax rate on consumption, “$GA$” is value of government assets, “$GovE$” is public expenditures on education, “$GovH$” is public expenditures on health care and “$Gov$” is public expenditures on other sectors (e.g. transport). The left-hand side of this equation shows tax revenues from different sources, the interest income from government assets and the transfers received from the UK government. The right hand side of the equation refers to government expenditures and transfers to households. Note that the representative household of generation “$g$” at time “$t$” represents a specific cohort of size “$Pop_{t,g}$”. The size of each cohort must be taken into account when computing total tax revenues and transfers to households in a specific period of time. Note that the pension program is part of the overall government budget.

Public expenditures on health and education are age-dependent. They are fixed per person of a specific age. More specifically, “$ASHEPC_g$” is age-specific health expenditure per-person and “$ASEEPC_g$” is age-specific education expenditure per-person. Therefore, total public expenditure in these categories depends not only on the size of the population but also on its age structure:

\[
GovH_t = \sum_g Pop_{t,g} ASHEPC_g
\]

(14)

\[
GovE_t = \sum_g Pop_{t,g} ASEEPC_g
\]

(15)

Other types of public expenditures, “$GEPC$”, are assumed to be age-invariant. That is, they are fixed per-person and hence total expenditure, “$Gov$”, depends only on the size of the total population, “$TPop$”.

11
2.5 Market and Aggregation Conditions

The model assumes that all markets are perfectly competitive. The equilibrium condition for the goods market is that Scotland’s output, together with return on foreign assets, “FA”, and transfers from RUK, must be equal to total demand originating from consumption, investment and government spending:

\[ Y_t + Ri_t FA_t + UKTRF_t = \sum_{g} Pop_{t,g} C_{t,g} + Inv_t + Gov_t + GovH_t + GovE_t \]

The demand for labour is equal to the supply:

\[ L_t = \sum_{g} Pop_{t,g} LS_g EP_g \]

and the stock of capital accumulated in period “t” is equal to the demand expressed by a firm:

\[ K_{stock_t} = K_t \]

The capital market is assumed to be in equilibrium. The total stock of private wealth, “HA”, and government assets, “GA”, accumulated at the end of period “t” must be equal to the value of the total stock of capital and foreign assets at the end of period “t”:

\[ Gov_t = TPop_t GEPC \]
Note that the current account can be derived from this model as the difference between national savings and domestic investment:

(21) \[ CA_t = \left( \sum_{g} \text{Pop}_{t+1,g+1} \text{HA}_{t+1,g+1} - \sum_{g} \text{Pop}_{t,g+1} \text{HA}_{t,g+1} \right) - \left( \text{Kstock}_{t+1} - \text{Kstock}_t \right) \]

Alternatively, the current account is either given as the trade balance plus the interest revenues from net foreign asset holdings, or as the difference between nominal GNP (i.e. GDP including interest revenues on net foreign assets) and domestic absorption.

3. Calibration

The model is calibrated using 2006 data for Scotland (where available). The 2006 year is chosen to avoid the effects of the financial crisis, which had a strong negative impact on the performance of the Scottish economy and government finances. The data for demographic shock is taken from the “official” population projections carried out by the Office of National Statistics (discussed further below). Population projections are used for calibration of fertility, survival and migration rates used in the model.

The macro side of the model is calibrated base on aggregated 2006 social accounting matrix (SAM). Data on public finances and GDP are taken from 2006-07 Government Expenditure and Revenue Scotland Report (GERS) (Scottish Government, 2008). The estimate used assumes that North Sea revenues are
distributed on a geographical share basis. Effective wage income and consumption tax rates are calculated from the corresponding government revenue category and calibrated tax base i.e. total employment income and aggregate consumption. The total amount of pensions and other transfers is taken from the *Department of Work and Pensions, Benefit Expenditure by Country, Region and Parliamentary Constituency*. Based on this information the effective pension contribution rate and the average size of pension benefits are calculated. For effective pension contribution rate calculation it is assumed that the same contribution rate is paid on all wage income. For average size of pension benefits the total amount of pension benefits is divided by the total number of people of pension age. For simplicity it is assumed that both males and females start receiving pension benefits at age 65.

The source of the labour market data is the *Quarterly Labour Force Survey* (QLFS). To avoid single observation biases data for three quarters is used (i.e. Q1:2008, Q1:2009 and Q1:2010). From these pooled data, parameters of the age-specific productivity (earnings) profiles are estimated. These data are also used to calculate age-specific labour force participation rates. For age-specific productivity profiles, Mincer age-earnings regressions are estimated (Mincer, 1958).

There are no Scotland specific estimates of the age structure of government spending on health and education. Data of this type for Scotland and the UK are under construction and were not available at the time of writing. However, there is some evidence that suggests that the age-specific structure of government expenditure has a similar shape in high-income countries. Lee and Mason (2011) report National Transfer Accounts for many countries and demonstrate that age-specific profiles of government spending on health and education are similar in high-income countries. The specific estimates used in this paper are from the Canadian National Transfer Accounts (see Zhang and Mérette, 2011). The majority of education spending occurs
between the ages 5-9 and 20-24. Health spending grows slowly until the age of 55-59 when it starts increasing much faster and accelerates after age 75-79. There is no reason to believe that the “shape” of the Scottish profile would be radically different. Capital share of the output ($\alpha$) is set to 0.3. The (5-year) intertemporal elasticity of substitution ($1/\gamma$) is set to 1.5.

There are three main steps in the calibration procedure. The first step consists of using the information on output, capital and labour demands and the first-order conditions of the firm problem to calibrate the scaling parameter for the productivity function, plus wage and rental rates.

The second step is the most challenging involving equations pertaining to the household’s optimisation problem, the equilibrium conditions in the assets and goods markets to calibrate the rate of time preference and government expenditures on sectors other than health and education (Gov). In other words, the (5-year) rate of time preference is solved endogenously in the calibration procedure in order to generate realistic consumption profiles and capital ownership profiles per age group, for which no data are easily available. Capital ownership profiles must also satisfy the equilibrium condition on the asset market. Public expenditures on other sectors (Gov) is endogenously determined to close the budget constraint of the government and ensures the equilibrium on the goods market. Note that the rate of time preference and the inter-temporal elasticity of substitution together determine the slope of the consumption profiles across age groups in the calibration of the model (when the population is assumed to be stable). This is also the slope of the consumption profile of an individual across his lifetime in the simulated model in the absence of demographic shocks or economic growth.

The third and final step uses the calibration results of the first three steps to verify the model is able to replicate the observed data corresponding to the initial
equilibrium. Only when the initial equilibrium is perfectly replicated with the calibration solution can the model be used to evaluate the consequences of demographic shocks associated with population ageing.

4. Findings

The baseline demographic scenario is the “official” 2010-based principal population projection for Scotland (National Records of Scotland, 2011). This projection is summarised in Figure 1 which shows the growth rates of key age groups. According to this projection, by 2106 total population will increase by 22%, the pension age population (65+) will increase by 127%; the children/youth population (<20) will stay constant and the working age population (20-64) will stagnate during this period with an increase of 1%. It is clear from this projection that significant population ageing is expected.

Figure 2 reports simulated changes in output until 2106. Figure 3 shows simulated changed in output per-person. The simulation suggests a reduction in output per-person above 15% over the next 100 years. This corresponds to a considerable welfare loss “caused” by population ageing.

With the model it is possible to use the results ex-post to approximate how much total factor productivity growth is needed to compensate for the output loss called by population ageing. The model suggests that productivity growth of about 15% is needed over the next 100 years just to keep output per-person constant. In other words, productivity growth averaging around 0.14% per year is needed.

This estimated rate of future growth is well below recent trend productivity growth in the UK. Data from the EU KLEMS project suggest that productivity growth over the past two decades has averaged 2% per years (see O’Mahony and Timmer,
Therefore, the analysis carried out in this paper suggests that about 7% of (recent) historically observed productivity growth is required to counteract the negative effect of population ageing on output per person. This is a significant change and suggests that a “technological leap” will be needed that leads to a large increase in productivity. Of course, the key unanswered question is what will cause such a big change?

5. Concluding Comment

This paper developed an overlapping generations (OLG) computable general equilibrium (CGE) model in order to evaluate the macro-economic impacts of population ageing in Scotland. The model is particularly well suited to this task since its OLG structure explicitly allows for the incorporation of ageing effects related to age-specific labour force participation, age-specific productivity differences and age-specific government expenditures. Population ageing is associated with lower output per-person suggesting that it is welfare reducing. It was demonstrated that the technical change needed to counteract this loss in welfare is considerable.
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Figure 1
Projected Change in Scottish Population by Age Groups, 2011-2106

Figure 2
Simulated Changes in Output, Scotland, 20011-2106

Figure 3
Simulated Changes in Output Per-person, 2011-2106