Electromechanical forces acting on bio-membranes in external electric fields

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Abstract: Membranes of microorganisms stressed with electric field can be deformed and ruptured due to unbalanced electro-mechanical forces. The paper provides an analytical analysis of the forces acting on bio-membranes in liquid and gaseous environment. This model can help in optimisation and further development of novel field and plasma based decontamination methods.

Keywords: Bio-membranes, electric field, electro-mechanical stress

1. Introduction

There is a significant demand in novel methods of biological decontamination including electric field-based and non-thermal plasma-based methods. In the case of non-thermal plasma inactivation microorganisms are subjected to a number of physical and chemical stresses including actions of chemically active species produced in plasma and electric field, [1]. It is known that bio-membranes of microorganisms stressed with the electric field can be stretched and ruptured due to development of electro-mechanical stresses acting on the membrane, [2, 3]. This electroporation effect is used for non-thermal inactivation of microorganisms by the electric field. The present paper is focused on the analysis of the electro-mechanical forces acting on a spherical bio-membrane placed in the external uniform electric field, E_o.

2. Electromechanical forces acting on bio-membranes

If the membrane is considered as a spherical dielectric shell with dielectric permittivity of ε_m surrounded by a fluid with dielectric permittivity of ε_ℓ , the external electric field generates uncancelled Maxwell stresses within the membrane, these stresses in turn produce a net mechanical force which tends to stretch and deform the membrane. As a result the membrane can be ruptured which could lead to the death of microorganisms.

The normal, f_r, and tangential, f_θ, components of the electro-mechanicals force exerted on the interface between the external surface of the membrane and surrounding fluid can be found using the following expressions:

$$f_r = \frac{1}{2}\varepsilon_0\varepsilon_\ell\left(E_r^2 - E_\theta^2\right)$$
$$f_\theta = \varepsilon_0\varepsilon_\ell\, E_r\, E_\theta \qquad (1)$$

where E_{rm} and $E_{\theta m}$ are the normal and tangential components of the field in the membrane.

In the case when microorganism is in liquid (normally in water or water-based solution) the resulting electromechanical stress is directed from the liquid towards the membrane as the membrane has lower dielectric permittivity as compared with water/water-based solution.

The total force which acts on the membrane is the difference between the electromechanical forces exerted on the internal and external surfaces of the membrane. In order to calculate this force, its normal and tangential components should be evaluated. The normal and tangential component of the field across the external membrane interface are:

$$E_r = 3E_0\cos\theta - 2\frac{3E_0\left(1-\gamma^3\right)}{2\gamma^3(K-1)+K+2}\cos\theta$$

$$E_\theta = -\frac{3E_0\left(1-\gamma^3\right)}{2\gamma^3(K-1)+K+2}\sin\theta \qquad (2)$$

where E_0 is external electric field, γ is the ratio of the internal and external radii of the membrane and K is the ratio of the relative permittivities of the membrane and the fluid, $K = \varepsilon_m / \varepsilon_f$.

Integration over the membrane's surface provides the expressions for *z*-components (tangential and normal) of the electromechanical force:

$$F_\theta = \varepsilon_\ell\varepsilon_0\oint E_r E_\theta \sin\theta\, ds$$
$$F_r = \frac{1}{2}\varepsilon_0\varepsilon_\ell\oint \left(E_r^2 - E_\theta^2\right)\cos\theta\, ds \qquad (3)$$

After integration over the spherical surface of the membrane, the following expressions for the normal and tangential components of the electromechanical force acting along the axis which coincides with the direction of the external uniform field, E_0, have been obtained:

$$F_r = 9\varepsilon_0\varepsilon_\ell R_{ext}^2 \frac{9\pi}{4} E_0 \frac{K^2\left(2\gamma^3+1\right)^2-\left(1-\gamma^3\right)^2}{\left(2\gamma^3(K-1)+K+2\right)^2}$$

$$F_\theta = 2\varepsilon_0\varepsilon_\ell R_{ext}^2 \frac{9\pi}{4} E_0 \frac{K\left(2\gamma^3+1\right)^2\left(1-\gamma^3\right)}{\left(2\gamma^3(K-1)+K+2\right)^2} \quad (4)$$

where R_{ext} is the external radius of the membrane. In all calculations conducted in this paper $\gamma = R_{int}/R_{ext} = 0.999$ (R_{in} is the internal radius of the membrane). These equations can be used for evaluation of the relative contributions of both, normal and tangential components to the total electromechanical force acting along the field. If the external fluid is water ($\varepsilon_f = 80$) then the tangential force found to be only 8% or less of the normal force components; in the case of gaseous external environment ($\varepsilon_f = 1$) the tangential force is 0.1% of the normal force components. Therefore, these tangential components of the field can be neglected in the present evaluation of the total electromechanical force acting on the spherical microbiological membrane.

The total force acting on the biological membrane in the external electric field is the difference between the forces exerted by the field on the internal and external surfaces of the membrane. Thus, this total electromechanical force which in z-direction can be obtained using the following equation:

$$F_z = \oint \left(f_r(R_1) - f_r(R_2)\right)\cos\theta ds \quad (5)$$

The tangential components of the force are not included in this equation as they make relatively small contribution to the total force as estimated above. After integration, the following expression has been obtained for the force which is acting in the direction of the field:

$$F_z = 1.5\pi\varepsilon_0\varepsilon_m R_{ext}^2 A^2\left(9\gamma^2 - \left(1+2\gamma^3\right)^2(1-K)\right) \quad (6)$$

where $A = \dfrac{3E_0}{\left(2\gamma(K-1)+K+2\right)}$. (7)

As the same force is acting on the negative z-direction, so no net translation force exists on the membrane. F_z can be estimated for two different cases: in the case when microorganism is surrounded by liquid (water, ε_f=80), and when the surrounding fluid is gas (air for example, ε_f=1).

In the first case when microorganism is surrounded by water, the two components of the net force in Equation 5 have different signs as water outside the membrane and cytoplasm inside membrane have significantly higher permittivities (ε_f=80) as compared with dielectric permittivity of the membrane itself (ε_m=2). The membrane has non-zero thickness, therefore its external radius is greater than its internal radius, $R_2 > R_1$. This results in the different surface charge densities on the internal and external membrane surfaces. The electromechanical force across the interface between two dielectric materials is directed from material with low permittivity toward the material with higher permittivity. Therefore, in the case of the external fluid the membrane will experience both compression and deformation due to the forces exerted on the internal and external surfaces of the membrane.

In the case of gaseous environment, the situation will be different. The force exerted on the external interface will be directed outwards of the membrane as the air has lower dielectric permittivity, $\varepsilon_f = 1$. Therefore, the magnitude of the total force, F_z will be different for these two cases. In the case of water environment F_z is given by:

$$F_z\big|_{water} = 0.1671\pi\varepsilon_0\varepsilon_m A^2 R_{ext}^2 \quad (8)$$

and in the case of gaseous environment this force is

$$F_z\big|_{gas} = 11.98\pi\varepsilon_0\varepsilon_m A^2 R_{ext}^2 \quad (9)$$

Therefore, the membrane will experience a significantly higher stress in the case if microorganisms will be surrounded by air (air-born microorganisms) as compared with the case when microorganisms will be stressed by the external electric field in liquid medium.

3. Conclusions

The model proposed in the present paper can be used for analysis of the electromechanical stresses developed across bio-membranes of spherical airborne and waterborne microorganisms. It is important to understand the interaction between bio-membranes and the electric field, there is still a significant gap in the understanding of modifications in biological membranes caused by the ionic flux and electric field. This investigation will help in optimisation and further development of practical systems and technological applications such as non-thermal electric field and plasma inactivation of microorganisms [4], electrostatic removal of fine and ultrafine particles and bio-aerosols from air [5].

4. References

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