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1. Introduction

Many metallic alloys, particularly those with coarse initial microstructures, exhibit a phenomenon known as flow softening [1]. This involves a reduction in the flow stress of the material during plastic deformation. It tends to be restricted to processes e.g. forging, high temperature sheet forming, in which deformation is carried out at elevated temperatures. The use of high temperatures allows microstructural refinement or re-arrangement to occur e.g. by the occurrence of dynamic recrystallisation, which produces the softening effect.

At low temperatures microstructural refinement would lead to an increase in flow stress due to Hall-Petch considerations [2], however, at high temperatures the presence of large amounts of grain boundary generated, for example, by dynamic recrystallisation combined with rapid diffusion rates often produces an overall softening effect [3]. Mathematically it can be expressed as follows:

\[ \bar{\sigma} \sim d^{-k} \]

low temperatures \( k \leq 0 \)

high temperatures \( k \geq 0 \)
It has also been observed that transformations in microstructural morphology and/or crystallographic texture can generate softening effects [4]; much depends on the alloy type and its starting structure.

Such softening can have significant implications for the stability of metal flow. If a manufacturing process tends to produce some localization of flow - and most do - then this can rapidly become exacerbated and generate shear bands within the material and a widely differing microstructure across the section of the component. An example of this will be considered in the next section. The presence of a non-uniform microstructure can have major implications for the in-service performance of a component. An ability to accurately model such effects offers the possibility of optimizing process design to minimize such effects, or to use them to advantage.

This paper focuses on how such softening may be treated from a modelling perspective, how it might be controlled, or at least predicted; of significance here are both strain rate and thermal sensitivity of the material being formed. Of key importance is the accuracy of the material model used; examples will be given of how different material constitutive models may lead to accurate or inaccurate predictions.

2. Flow localization during ECAP

One of the most simple and typical examples of a flow localization problem arising from material softening can be observed during Equal Channel Angular Pressing (ECAP). Within this process the deformation is concentrated in a localized section of the workpiece and is quite intensive, with effective strains of about 1; see Fig. 1a. The main purpose of this process is to generate significant refinement of the material grain structure in the deformation zone. If the process takes place at low temperatures this lead to the material hardening.

The resistance of material towards deformation increases, higher loads are required to perform the extrusion, but from the viewpoint of the stability of material flow it is safe. In the case of high temperatures, which are used to reduce the deformation load, the situation may change. The decrease of the material resistance in the deformation zone can lead to the loss of stability within the metal flow. This can be expressed in different ways. This effect was nicely demonstrated and discussed in the work of DeLo and Semiatin [5]. An example of significant flow localization which finished with intensive shear lines across the material is shown in the Fig. 1b. There were also some attempts to simulate this effect numerically in the work of Balasundar et al [6].

However, the mechanism for such stability loss is still not clear and requires further investigation. To separate the effect of the deformation softening from the influence of other factors (e.g., temperature, strain rate) a number of artificial models of material with different fixed softening rates were constructed and used for numerical simulation of ECAP with the same geometry of extrusion channel as the one used in the work of DeLo et al. [5].

The notion of a softening rate is defined differently here than in [6]. In that work it was defined as “% flow softening = (max flow stress - steady or min. flow stress)/max. flow stress”. This definition is difficult to apply to materials with continuous softening for which a steady flow stress does not exist, and which exhibit no clear minimum value. Besides that, in real materials the softening rate can vary during the deformation (e.g. due to a change of the microstructural mechanism responsible for it, or through the saturation of some process). So, to be more universal, the specific normalized strain softening rate of the material here is defined similarly to the deformation hardening as [7]:

$$\gamma = \frac{1}{\sigma_{\text{max}}} \left[ \frac{\partial \sigma}{\partial \varepsilon} \right]_{T=\text{const}, \varepsilon=\text{const}}$$

(2)

Where $\sigma$ is effective stress, $\varepsilon$ - effective strain, $T$-temperature and $\varepsilon$ - effective strain rate.

Numerical tests which have been carried out have shown that behaviour similar to that described by DeLo and Semiatin (Fig.1b) can be obtained for a fixed softening rate of 20% (per unit of effective strain; $\gamma=0.2$); see Fig.2a, Here, and subsequently all simulations were carried out assuming a rigid plastic model. In the effective stress map (Fig.2b) the zones of low stress can clearly be seen. We have screened out the effect of temperature and strain rate - this is direct result of the material softening characteristics. This explains the nature of the shear bands, but does not give an understanding of why these bands have a particular periodicity and why they may be inhibited at lower softening rates.

To get the answer to this question a few physical modelling experiments were conducted. To simulate a localized zone of softening surrounded by a much stiffer material, some steel cubes with one internal oblique layer of lead were prepared; Fig.3a. The original cubes, made of AISI 1018 steel with...
a yield stress of about 300 MPa, were cut into two parts along a 45 degree oblique plane. The surfaces of the cut faces were specially treated to obtain the required bonding between steel and lead. Then a 5 mm thickness layer of lead with a yield stress of 19 MPa was cast in between the two steel parts.

In the specimens thus prepared the lead layer comes out to the two opposite faces of the cube. Two different compressive tests were conducted in such way, that in test A the faces with the lead contacted the tools whilst for test B they were free; Fig.3a and Fig.3d. Both types of tests were also numerically simulated.

These simple experiments have shown that if the soft layer comes into contact with the tool faces whilst having hard material on both sides (test A), it remains constrained (Fig.3a) and hence any tendency for shear is inhibited. The plastic deformation of such a specimen only starts when the main supporting harder material reaches its yield stress at some point. The only contribution of the soft layer in this case is the redistribution of the stresses in the main material. As a result, yielding, even with negligible friction, starts non-uniformly across the volume of the sample; mainly at the edges close to the lead layer (Fig.3b). However, this is sufficient to transfer pure compression into the shear zone. Most probably a similar mechanism leads to a stability loss due to microstructure refinement or adiabatic heating causing localised soft zones in such materials as e.g. α+β titanium alloys (Fig.3c).

A different situation is observed when the lead layer comes out to the free faces (Test B, Fig.3d). Plastic deformation starts immediately as the effective stresses in the lead layer reaches the yield point. The two steel parts begin to “slide” about each other due to shear in the lead, while remaining elastic themselves. (Fig. 3e). But this process reaches saturation when these steel parts touch the tools. Then the situation in the Test B scenario becomes similar to that described above for Test A. The soft layer appears to get arrested and deformation in it stops (Fig.3f). This mechanism looks to be the reason for the periodicity of the flow localisation in the experiments of DeLo et al. The soft layer is getting formed about the diagonal of the corner of the channel. Then, due to the pressing its end comes out to the free surface. After that deformation takes place according to the scenario of test B - deformation is localized in this layer; the left and right parts of the material slide about each other until the soft layer will get locked between them. Then sliding in this zone stops and new soft layer starts to be formed. The cycle then repeats.

3. The effect of the softening rate as well as strain rate and temperature sensitivity on the flow localization

Another question, which it is also useful to investigate, is the role of the softening rate in the processes of flow localization and possibilities of controlling unstable metal flow.

The role of the softening rate was studied on the basis of three numerical experiments with artificial materials having fixed softening rates of 20%, 10% and 5% (γ=0.2, 0.1 and 0.05). The results of these simulations are shown in Figure 4.

It can be observed that when the softening rate is decreased from 20% (Fig.4a.) to 10% (Fig.4b.), though some periodicity can be seen in the shape of the humps on the upper surface of workpiece as well in the strain pattern, the nature of the metal flow becomes completely different. This can be noted from the shape of the flow lines shown in the figures. In the case of 20% softening there is evident flow localization. Localized shear bands separate the parts of almost undeformed material very similarly to the way observed in the physical modelling discussed in the previous section. But this is possible only when the gradient of stresses in the neighbouring zones is quite significant, i.e. for large softening rates. When this difference decreases, deformation becomes more uniform and so strongly marked flow localization does not happen. Instead of this we can see continuous longitudinal waves along all of the deformed part; this can be regarded as a loss in flow stability. As can be seen in Figure 4c, if the softening rate reduces further, to about 5%, the longitudinal flow lines become almost straight, this indicates that the flow has been stabilised.

These observations provide useful information for the development of hot forging processes. The rate of softening expected at the particular temperatures and strain rates can be accessed from uniaxial tests and high softening rates that might initiate unstable metal flow can be avoided.

However, it is not always possible to avoid the presence of significant metal softening in manufacturing processes. So the question arises, whether there are any possibilities to control softening for the minimisation of its negative effects. Of course the proper answer to this question requires separate

![Fig.3. Physical modelling of the mechanism of flow localization due to softening](image)

![Fig.4. Numerical simulation of the role played by softening in metal flow](image)
systematic investigation. Here only one small trial attempt is offered in this direction - to illustrate the interplay of the softening and strain rate sensitivity.

![Fig.5. Softening at 20% with fixed strain rate sensitivity of m=0.4: a) rheological data, b) results of simulation with the resulting stress-strain curves, c) stress field in deformation zone, d) strain rate field in the deformation zone](image)

For this purpose, one more artificial material model was developed, also having 20% (γ=0.2) strain softening, but with the strain rate sensitivity fixed at m=0.4. The stress-strain diagrams of this material are shown in Fig. 5a. The results of the numerical simulations shown in Fig. 5b demonstrate significant improvement of the metal flow as compared to the cases considered previously (Fig.2 and Fig 4a.) The origin of this improvement becomes clear from the resultant stress-strain diagrams of the material points (Fig. 5b).

Though at each constant strain rate the flow stress decreases significantly, due to the variation of the strain rate across the specimen and especially in the main deformation zone, the net effective stress in the majority of the material points either increases or demonstrates restricted softening during active deformation and reduces mainly at the unloading stage. This mechanism of strain rate hardening is illustrated with figures Fig.5c and Fig.5d. The increase of the effective stresses in the main deformation zone, shown in Fig.5c originates from the significant increase of the strain rate taking place in this zone (Fig.5d). A similar effect can be expected due to temperature reduction.

These results look quite promising, though as was noted before a lot more study is required in this direction. And now we would like to focus on another problem related to the softening - the problem of accounting for it in numerical simulations.

4. The accounting of the softening effect in FEM simulations

Flow softening can create certain problems in terms of predicting its effects in FEM simulations. The experimental data fed to the software is normally obtained from either tensile or compression tests (very rarely from torsion) and is limited to the values of strains for which the deformation remains uniform. For compression tests this is normally about 0.7 (everywhere in the manuscript - logarithmic true strain is assumed), for tensile tests this value can be slightly more or less depending on the ductility and strain rate sensitivity of the material. However, during simulation of a chain of manufacturing operations e.g. complex forging, the accumulated plastic strain can reach the value of 3 or even 5. In this case the available experimental data is often extrapolated automatically by the software. This can be done in two ways; either by assuming that the stress reaches a constant value or by extrapolating a constant slope; see Fig.6. Neither of these variants of extrapolation is ideal, but in the case of hardening it is mainly a problem of accuracy, while in the case of softening it may lead to computational failure or erroneous prediction, as will be illustrated with the example below. The reason for this is rather simple. The assumption of a constant value stress extrapolation may lead to unrealistically high stresses being predicted by the FE model, while the constant slope extrapolation can lead to zero or even negative values of stress being predicted which will terminate the simulation.

![Fig.6 Different modes of extrapolation used to calculate stresses at strains beyond those obtained by experiment](image)

As an example of this problem, the simulation of metal flow in the hot forging of a titanium disk can be analyzed. This analysis will be focused only on the influence of the assumed rheological properties, so the specific material and the exact forging conditions are not key factors.

A typical initial ingot geometry, as-forged workpiece and final machined disk are shown in Fig.7a. In this type of forging there are a few types of common defects that may often be observed; Fig.7b. These are: laps or surface defects in the zone marked A, under-filling in zones B and C, internal defects at D and shear bands along directions EF and GH. However, process simulations conducted with the normal approach outlined above (Fig. 6) fail to show the majority of these defects; Fig.7c (it is assumed that all boundary and loading conditions are set up correctly and physically verified). The main reason for this lack of accuracy lies in inadequate material property data. As will be noted from Fig.8a, the strains in the major part of the workpiece, including the critical zones, are quite large, reaching values of 2-3. This is considerable higher that the strain values for which accurate data exists (Fig. 7c). In this case, as discussed above, many commercial software packages will automatically extrapolate...
to obtain the missing data. For the result in Fig. 7c the data was extrapolated at constant values of stress (the same as those at $\varepsilon = 0.5$). From the mechanical point of view such a condition would correspond to an almost ideal viscous liquid flow, which is highly unlikely under any normal (non-isothermal) forging condition.

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The attempt to extrapolate experimental curves linearly does not give good results either. For large values of strain the curves may intersect (which can create an apparent negative strain rate sensitivity) or generate flow stress values below zero.

A better approach for accounting for deformation softening of materials can be done by using the internal state variable principle proposed by Yu.N.Rabotnov [8]. In this example a two-phase $\alpha + \beta$ titanium alloy (Ti-6Al-4V) with an initial coarse grained transformed beta microstructure is assumed. For this type of the material the main reason for softening is related to the grain refinement as the coarse structure breaks up and progressively becomes more spheroidised. This determines the choice of the main internal variable (as a first approach we can work with only one internal variable - for many cases it appears to be sufficient for capturing the main effects). The structural parameter $D$, reflecting the state of the microstructural refinement, is included in the macroscopic constitutive equation (3). The parameters $A$, $m$, $Q$ and $k$ in this equation are dynamic ones and depend on temperature and strain rate, more details see in [9]. For elevated temperatures (>0.3Tm) the value of the parameter $k$ often becomes positive (inverse Hall-Petch effect). So, if the value of the variable $D$ decreases, the flow stress will also reduce.

To close the mathematical model one more equation describing the kinetics of the structural parameter has to be added. For the forging simulated in this example the elapsed time of the process is quite small, so all the effects related to the grain growth can be neglected. The description of the grain refinement can be simplified by considering only its integral features in terms of its phenomenology (i.e. without going into details of the microscopic mechanisms involved in it). The assumption here is that the refinement is driven by the strain energy and may be mathematically expressed in the form of the equation (4).

$$\bar{\sigma} = A \bar{\varepsilon}^m \cdot \exp \left( \frac{Q}{RT} \right) \left( \frac{D}{D_0} \right)^k$$  \hspace{1cm} (3)

$$D = B \cdot \left( D_1 + D_2 \exp \left( -C \int \bar{\sigma} \cdot \dot{\varepsilon} \cdot dt \right) \right)$$  \hspace{1cm} (4)

Here $\bar{\sigma}$ is effective stress, $T$-temperature and $\dot{\varepsilon}$ – effective strain rate, $t$-time, $R$-gas constant, $A$, $m$, $Q$ and $k$ – dynamic constants, $B$, $C$, $D_1$ and $D_2$ – fixed constants.

This model was programmed as a user function into the commercial metal forming software QForm. The results of the simulation of the same problem conducted on the basis of this model are shown in the Fig. 7d. It can be seen that the predicted metal flow patterns are considerably modified and a number of potential defects become apparent. The origin of all of them is the flow localization caused by material softening. The flow lines in zone A indicate the possibility of lap formation; this is an area where flow localization creates a line of instability that breaks through onto the workpiece surface; Fig.8b. Parts B and C remain under-filled. This is a result of these areas effectively becoming isolated (dead zones) due to the metal flow being localized elsewhere in the forging. Other consequences of deformation softening would also be that there would be major differences in microstructure across the section and, in extreme cases the formation of internal voids within intense shear zones.

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![Fig 7. Simulation of metal flow in the hot forging of a titanium disk: a) manufacturing sequence; b) the location of typical defects; c) simulation using experimental stress-strain curves directly; d) simulation using proposed internal variable model](image)

![Fig 8. Prediction of the plastic strain distribution in the FEM simulation a) using experimental stress-strain curves directly; b) using proposed internal variable model](image)
In principle, the internal variable, used in the proposed constitutive model, does not necessarily have to have a direct correlation to some specific element of the microstructure e.g. grain size. It is sufficient that it serves the purpose of improving the accuracy of the phenomenological constitutive model as shown above. However, in order to be able to guide the choice of heat treatment for a final part, or for informing estimations about in service performance it is useful if the parameter D can be linked to microstructure directly.

Titanium alloys are quite complicated and have a wide range of morphological forms [10]. As a result the parameter D cannot be easily mapped against average grain size, especially taking into account that for coarse Widmanstatten morphologies, the notion of an average grain size has little meaning.

To resolve this difficulty, a different type of “classification” approach can be proposed here. The idea of it is as follows. A representative set of the morphological states of the material can be divided into a number of “classes” [10]; see Fig 9. Each class can be associated with a particular set of microstructural characteristics, e.g. the percentage β, primary and secondary α-phase, the size and percentage of globular α-phase, length distribution and thickness of α-laths, the size of α-colonies, primary β-grains etc. To these classes one continuous numerical scale can be assigned, changing in the range from $D_{\text{min}}$ to $D_{\text{max}}$. This scale can be experimentally calibrated in such a way that the structural internal variable D used in the proposed model will indicate the class of the microstructure at some point in the workpiece. In this case, being plotted as a field in the FEM software, it can provide the through-process assessment description of the microstructural transformation. An example of this is shown in the Fig. 10. Used in this way, the internal variable D can be considered as an “effective microstructural unit”. As its value varies continuously, the parameter D can indicate not only the class of microstructure, but also the level of deviation from the ideal value relating to some class. The assessment value of any of the above mentioned microstructural characteristics can be then found as an interpolation of the typical values of the neighbouring classes. However, the practical implementation of this approach requires serious experimental investigation to underpin it.

5. Conclusion

- Deformation softening occurring during the hot-working of different alloys can significantly affect the character of the deformation process. This may express in a variety of forms e.g. as an increase in the non-uniformity of strain or in severe microstructural variability across a workpiece. All of this leads to difficulties in the design and simulation of such processes, which require special attention in choosing an adequate constitutive model for the material and solving the boundary-value problem.

- Virtual experiments in the numerical simulation of the ECAP process with strain softening materials confer the ability to assess the effect of different factors (e.g. strain rate and temperature sensitivity) on the risk of the emergence and the character of different types of stability loss in the deformation process. The significant role of the specific softening rate in the development of the flow localization and the generation of shear bands was observed.

- Simple experiments using specially prepared model samples assist in illustrating different scenarios for the behaviour of the localized zone of softened material in ECAP process. It was demonstrated that periodicity of flow localisation can be observed in these experiments at some orientations of the softening layer.

- A constitutive model with a single internal variable was proposed for the characterisation of microstructure evolution in the simulation of hotworking processes where deformation softening is occurring. The efficiency of this approach was demonstrated with an example of the hot forging of an axisymmetric disk-shaped part.

- Further practical validation of this approach is required, however, it is proposed that through further development of the current methodology, it should become more possible to improve metal forming processes to minimise the negative effects of deformation softening and perhaps utilise it to advantage.
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