

Multiple Shift QR Decomposition for Polynomial Matrices

Fraser K. Coutts*, Jamie Corr*, Keith Thompson*, Stephan Weiss*, Ian K. Proudler*,†,
and John G. McWhirter‡

* Department of Electronic & Electrical Engineering, University of Strathclyde, Glasgow, Scotland

* ,† School of Electrical, Electronics & Systems Engineering, Loughborough Univ., Loughborough, UK

‡ School of Engineering, Cardiff University, Cardiff, Wales, UK

e-mail: {fraser.coutts, stephan.weiss}@strath.ac.uk

ABSTRACT

Polynomial matrix representations can elegantly express broadband multichannel problems. The use of such formulations can be found in multichannel factorisation [1], broadband MIMO precoding and equalisation [2], polyphase analysis and synthesis matrices for filter banks [3], broadband angle of arrival estimation [4], broadband beamforming [5], optimal subband coding [6], and channel coding [7] to name but a few. For polynomial matrix problems that require an extension of the the QR decomposition (QRD), a polynomial matrix QR decomposition (PQRD) has been defined in [8, 9]. The PQRD uses finite impulse response (FIR) paraunitary matrices [10] to approximately upper-triangularise a polynomial matrix.

Algorithms to compute the PQRD include the original PQRD by steps (PQRD-BS) algorithm [8] and its successor, the PQRD by columns (PQRD-BC) algorithm [9]. Both of these algorithms employ an iterative approach to approximately upper-triangularise a polynomial matrix, stopping when some suitable threshold is reached. In both algorithms, the elements beneath the diagonal in each column of the input polynomial matrix are made approximately zero in succession, moving from left to right across the matrix. This is achieved through the use of a sequence of row shift and Givens rotation pairs, which are applied at each iteration using the location and value of the largest element in the search space of PQRD-BS and PQRD-BC. The largest element in the search space is zeroed by this process, and its energy is transferred to the upper-triangle of the matrix. The search space of PQRD-BS contains only the elements of one polynomial beneath the diagonal in the column of interest, and the search space of PQRD-BC contains all elements beneath the diagonal in the column of interest.

Applying a multiple shift approach to the sequential matrix diagonalisation (SMD) polynomial eigenvalue decomposition (PEVD) algorithms for parahermitian matrices has previously proven beneficial [11, 12]. At each iteration, a sequence of shifts is applied to maximise the transfer of of-diagonal energy onto the diagonal. This paper describes a similar shifting approach applied to the PQRD-BC algorithm to create a multiple shift PQRD-BC (MS-PQRD-BS) algorithm. By applying a number of row shifts at each iteration of MS-PQRD-BS, the largest element in each row beneath the diagonal in a single column can be transferred to the zerolag at once. A series of Givens rotations can then be applied to approximately zero each shifted element in the column of interest. This process can be repeated for each column of the matrix until a similar stopping criterion to PQRD-BC is reached.

We demonstrate that using the developed shifting approach, increased performance relative to PQRD-BC can be achieved without significant accuracy loss. Fig. 1 underlines this increase in upper-triangularisation performance when using the proposed MS-PQRD-BC algorithm versus the standard PQRD-BC algorithm for a 9×5 polynomial matrix $\mathbf{A}(z)$ of order 60. The metric $E_{\text{norm}}^{(i)}$ divides the energy below the diagonal at the i th iteration by the total energy of $\mathbf{A}(z)$. Simulations were performed over an ensemble of 10^3 instantiations of $\mathbf{A}(z)$, which was constructed as a random paraunitary matrix using the source model in [13].

The full paper will provide insight into the algorithm and investigate metrics for both accuracy and complexity over an ensemble of general rectangular polynomial matrices. We will derive a suitable source model that yields non-paraunitary polynomial matrices, i.e. matrices where the conditioning can be problematic and therefore be realistic for applications such as broadband MIMO channels.

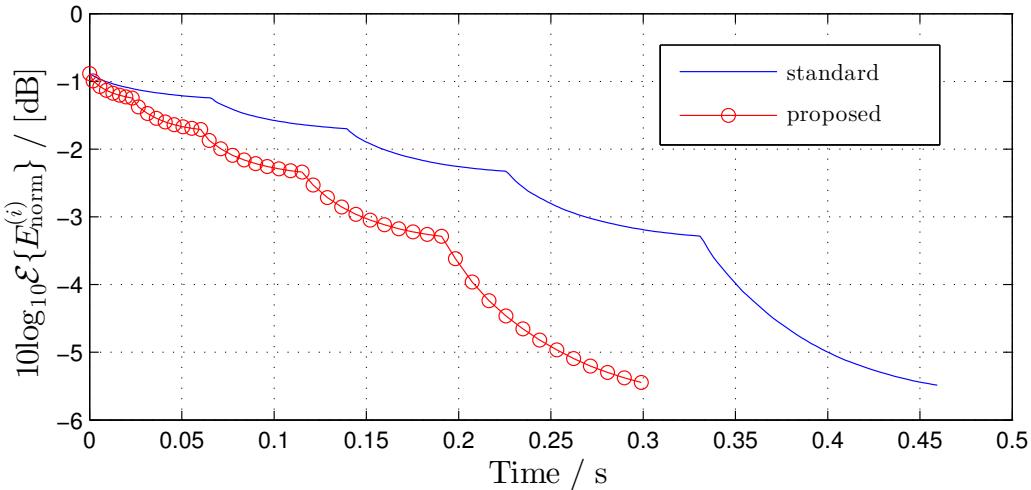


Fig. 1: Upper-triangularisation metric vs. algorithm execution time for the proposed and standard implementations for the example of 9×5 polynomial matrix of order 60.

References

- [1] Z. Wang, J. G. McWhirter, and S. Weiss. Multichannel spectral factorization algorithm using polynomial matrix eigenvalue decomposition. In *Asilomar SSC*, Pacific Grove, CA, Nov. 2015.
- [2] C. H. Ta and S. Weiss. A design of precoding and equalisation for broadband MIMO systems. In *Asilomar SSC*, pp. 1616–1620, Pacific Grove, CA, Nov. 2007.
- [3] P. P. Vaidyanathan. *Multirate Systems and Filter Banks*. Prentice Hall, Englewood Cliffs, 1993.
- [4] M. Alrmah, S. Weiss, and S. Lambotharan. An extension of the MUSIC algorithm to broadband scenarios using polynomial eigenvalue decomposition. In *EUSIPCO*, pp. 629–633, Barcelona, Spain, Aug. 2011.
- [5] S. Weiss, S. Bendoukha, A. Alzin, F. Coutts, I. Proudler, and J. Chambers. MVDR broadband beam-forming using polynomial matrix techniques. In *EUSIPCO*, pp. 839–843, Nice, France, Sep. 2015.
- [6] S. Redif, J. McWhirter, and S. Weiss. Design of FIR paraunitary filter banks for subband coding using a polynomial eigenvalue decomposition. *IEEE Trans. SP*, 59(11):5253–5264, Nov. 2011.
- [7] S. Weiss, S. Redif, T. Cooper, C. Liu, P. D. Baxter, and J. G. McWhirter. Paraunitary Oversampled Filter Bank Design for Channel Coding. *EURASIP J. Applied SP*, 2006.
- [8] J. A. Foster, J. G. McWhirter, and J. A. Chambers. An algorithm for computing the QR decomposition of a polynomial matrix. In *Proc. 15th Int. Conf. Digital Signal Process.*, pp. 71–74, Cardiff, Wales, July 2007.
- [9] J. A. Foster, J. G. McWhirter, and J. A. Chambers. A novel algorithm for calculating the QR decomposition of a polynomial matrix. In *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, pp. 3177–3180, Taipei, Taiwan, April 2009.
- [10] S. Icart and P. Comon. Some properties of Laurent polynomial matrices. In *IMA Maths in Signal Proc.*, Birmingham, UK, Dec. 2012.
- [11] J. Corr, K. Thompson, S. Weiss, J. McWhirter, S. Redif, and I. Proudler. Multiple shift maximum element sequential matrix diagonalisation for parahermitian matrices. In *IEEE SSP*, pp. 312–315, Gold Coast, Australia, June 2014.
- [12] Z. Wang, J. G. McWhirter, J. Corr, and S. Weiss. Multiple shift second order sequential best rotation algorithm for polynomial matrix EVD. In *European Signal Processing Conference*, pages 844–848, Nice, France, September 2015.
- [13] S. Redif, S. Weiss, and J. McWhirter. Sequential matrix diagonalization algorithms for polynomial EVD of parahermitian matrices. *IEEE Trans. SP*, 63(1):81–89, Jan. 2015.