IAC-14.C1.1.1

STATION-KEEPING FOR QUASI-PERIODIC ORBITS

Marcel Duering
University of Strathclyde, Advanced Space Concepts Laboratory
email: marcel.duering@strath.ac.uk

Markus Landgraf
Mission Analysis Section, ESA/European Space Operations Centre
email: markus.landgraf@esa.int

Massimiliano Vasile
University of Strathclyde, Advanced Space Concepts Laboratory
email: massimiliano.vasile@strath.ac.uk

Abstract The shallow gravity gradient in the libration point regions enables manoeuvring at low Δv expenses, but implicates a sensitivity to small perturbations. A variety of bounded orbits can be determined around each libration point and station-keeping is required to maintain them for multiple revolutions. In this paper, a station-keeping algorithm based on the orbital lifetime expectancy is proposed for so-called quasi-periodic solutions. The method introduced is based on the identification of a manoeuvre maximising the lifetime of an orbit within defined boundaries. The manoeuvre direction and magnitude is finally optimised with a differential evolution algorithm. The novelty of the method presented here is the identification of the downstream centre manifold by the lifetime analysis to preserve the orbit with its properties forward in time. The study shows that the manoeuvre direction is directly correlated to stability information that is provided by the Floquet modal theory. Finally, numerical calculations were carried out for trajectories around the far-side libration point in the Earth-Moon system to show the effectiveness of this station-keeping approach. The robustness is proven by the introduction of errors and the evaluation of their impact.

I INTRODUCTION

Operational orbits near the co-linear libration points offer interesting opportunities and using them might enable many space mission scenarios. The balanced gravitational forces of the Earth and Moon build a prone dynamical environment resulting that small perturbation have a large effect on the spacecraft and their operational orbits. This behaviour, in particular, is beneficial for spacecraft projects that rely on regular orbital manoeuvres and transfers. Another advantage of the libration point regions is the location with respect to Moon and Earth. These orbits enable access to the surface of the Moon, and it is feasible to observe the Moon in a close distance without changing the gravity potential as it is required to land on the Moon. Furthermore, libration point and distant retrograde orbits require less Δv than low lunar orbits to maintain them. The libration point regions provide a variety of bounded orbits. The mentioned dynamical instabilities and the unstable behaviour of most of these orbits prevent the reference path from being followed precisely for several revolutions and frequent manoeuvres are demanded to remain on orbit. Strategies required to calculate those are discussed in this paper. Regardless of the approach, the fundamental objective of any station-keeping algorithm is to design manoeuvres that maintain a spacecraft orbit for some desired length of time. This is taken as starting point of this research and a methodology based on the lifetime parameter is applied and proven.

Station-keeping strategies have been studied for several dynamical frameworks and missions in the past. Methods using theoretical knowledge based on modal Floquet theory have been explored by [1]. With the help of modes the motion can be described by centre components, stable and unstable modes. The cancellation of the unstable components is used to maintain a spacecraft in a libration point orbit. A similar methodology is already applied in the past utilising a linearisation of the equation of motion [2]. For periodic orbits dominant manoeuvre directions can be determined by the eigenvectors if the monodromy matrix, see [3] and [8]. An optimal control approach is used to optimise the station-keeping manoeuvres over the

lifetime of the orbits including the fulfilment of constraints during the mission, see [6] and [7]. In addition, research focus on the usage of a reference path that is pre-calculated. In those cases control can be applied assuring that the spacecraft follows a desired orbit. The drawback of this method is that precise states on the reference orbit are required during the station-keeping. They have to be stored e.g. utilising truncated Fourier series. Furthermore, the integration of the variational equations is time consuming in the station-keeping process.

The paper starts with a short introduction to libration point orbits along with their invariant manifolds followed by the introduction of the station-keeping process. The proposed algorithm exploits the fact that the nature of the dynamics already provides the required information on the desired trajectory although the unstable components prohibits the propagation of long times spans. It consists of three main steps, the first is the determination of a manoeuvre magnitude and direction to maximise the orbital lifetime within some given boundaries. The arising optimisation problem is solved by a differential evolution algorithm, see [9]. The next step is the manoeuvre executing sending the spacecraft towards a trajectory that is ideally not part of neither the invariant stable nor unstable manifold. A forward propagation to the next xy-plane crossing finalises the station-keeping step. The previous two steps are repeated until the desired lifetime is assured. The station-keeping is applied once per revolution at the crossing of the xy-plane in positive direction as described above, but can be applied at arbitrary positions and number of revolutions to cope with perturbations or after a certain thrust is accumulated.

II BACKGROUND

The space around the Moon in particular if the distance to the Moon exceeds 60000 km provides a dynamical environment that is influenced by the gravity of multiple bodies. This distance can be determined by the mass ratio of the Earth and the Moon. The station-keeping is studied in a simple dynamical model to prove the concept, an advanced model fed by ephemeris data can be utilised in a later stage to more accurately predict station-keeping costs. In order to take the gravitational effect of the two bodies into account, the dynamics are modelled as circular restricted three-body problem (CR3BP). The model describes a vehicle moving under the gravitational forces of a primary and a secondary body. It is restricted in the sense that the particle is massless, and circular indicates that the motion of the secondary with respect to the primary is idealised as a circular orbit. The motion of the spacecraft is described in terms of rotating coordinates relative to the barycentre of the system primaries leading to equations of motion that are autonomous and the solutions are time-invariant. A so-called synodic reference frame is introduced with a rotating x-axis directed from the primary to the secondary body. The two primaries are stationary in this frame. The reference system rotates with a constant angular velocity about the barycentre at the same rotation rate as the secondary body rotates around the primary. The position of the centre of mass can be determined from initial conditions because of the constant angular velocity. The conservation of angular momentum allows a computation of the angular velocity from initial conditions. The second order differential equation of motion is introduced in the Euler-Lagrange form as

$$\ddot{x} = 2\dot{x} + x - \frac{1-\mu}{r_1^3}(x+\mu) - \frac{\mu}{r_2^3}(x-1+\mu) \quad (1)$$

$$\ddot{y} = -2\dot{x} + y - \frac{1-\mu}{r_1^3}y - \frac{\mu}{r_2^3}y \quad (2)$$

$$\ddot{z} = -\frac{1-\mu}{r_1^3}z - \frac{\mu}{r_2^3}z \quad (3)$$

where $r_1^2 = (x-\mu)^2 + y^2 + z^2$ represents the distance from the spacecraft to the larger primary, and $r_2^2 = (x+1-\mu)^2 + y^2 + z^2$ to the larger primary. A known integral of motion in the circular restricted three-body problem is the Jacobi constant C which is directly related to the conserved energy given by

$$E = -\frac{C}{2} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \left\{\frac{1}{2}(x^2+y^2) + \frac{1-\mu}{r_1} - \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}\right\} \quad (4)$$

Later the Jacobi constant is used to classify periodic and quasi-periodic trajectories. Equation 3 defines the motion of the spacecraft in a normalised coordinates such that the gravitational parameter G is equal to one. The non-dimensional orbital period is normalised to 2π by factor t^*, which is the inverse of the mean motion of the primaries. The factor for distance quantities is the characteristic length l^*, which is the distance between the primaries. For weights the characteristic mass m^* is the total mass of the system. This study focus on the Earth/Moon three-body problem assuming a value for the mass ratio of $\mu = 3.0406 \cdot 10^{-6}$.

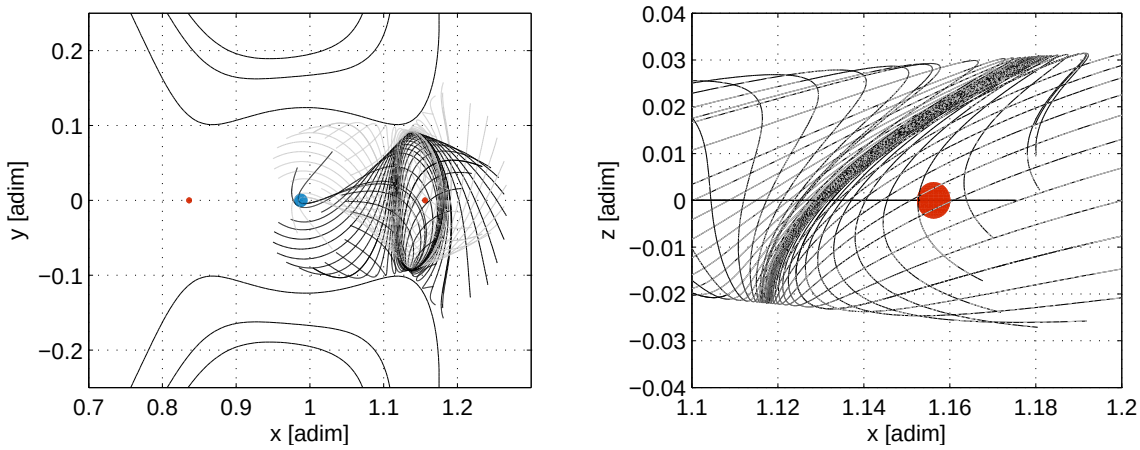

Figure 1: Periodic orbit around the far-side libration point with its stable and unstable hyperbolic manifold branches. Size of Moon plotted with a factor of 3. The red spheres indicate the libration points L_1 and L_2.

II.I Invariant manifold structure

In the framework of the restricted three-body problem, one-parameter periodic orbit families and two-parameter quasi-periodic orbit families are found, which are associated with the four-dimensional invariant centre manifold. There are two classes: the mentioned centre invariant and the hyperbolic invariant manifold, see [4]. These orbits are obtained numerically, the periodic orbits are solved by a two-point boundary value problem, whereas the quasi-periodic solutions require a calculation scheme based on the approximation by truncated Fourier series or equivalent methods, see [5]. Bounded orbits can be classified as stable and others as unstable through linearisation of the equations of motion about a libration point and bifurcation theory. The hyperbolic invariant manifold consists of a stable and an unstable set of trajectories associated to the centre invariant manifold. The focus in Fig. 1 is set to the proximity of the Moon with its two co-linear libration points L_1 and L_2. Both pictures indicate the manifold branches to the inner and outer regions, which can be divided by the zero velocity curves for a desired orbital energy. The zero velocity curve enables the introduction of forbidden regions, that cannot be reached at a given energy level. Fig. 4 indicates visually the escaping behaviour if the periodic orbit is propagated forward (black) and backward (grey) with a manoeuvre executed in the corresponding direction governed by the linearisation and study of the eigenvectors of the monodromy matrix. The zero velocity curves are plotted for three different values, the outer one corresponds to the energy of the halo orbit. Fig. 3 quantifies this deviation with the euclidean norm of the distance between the initial and returning positions. The solid line

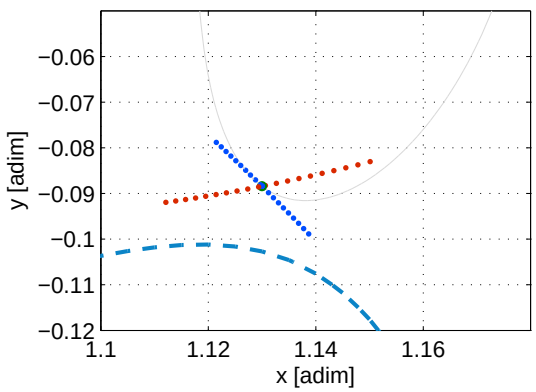

Figure 2: Periodic orbit around L_2 (grey). The red and violet dots represent the escaping behaviour here at the xy-plane crossing. The blue (dashed) line is the zero velocity curve at the energy level of the orbit.

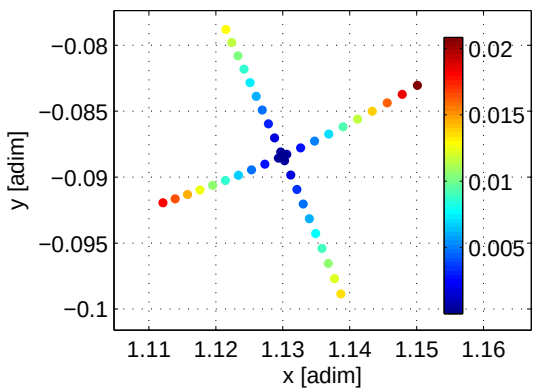

Figure 3: Deviation from nominal orbit plotted for number of downstream xy-plane crossings.

is governed by forward the dashed one by backwards propagation.

III STATION-KEEPING METHODOLOGY

The previously described dynamically sensitive behaviour of most libration point orbits prohibits the propagation of multiple revolutions. A technique is required to guide the trajectory by introduced manoeuvre to compensate for perturbations either introduced by gravitational forces or numerical errors. The objective of the algorithm introduced in the following is to identify the a manoeuvre direction and magnitudein such a way that the orbit properties are maintained and the orbital lifetime is maximised. The station-keeping methodology is explained in the next section followed by the application to periodic and quasi-periodic trajectories. For a periodic orbit the identifying parameter is the Jacobian, for quasi-periodic orbit there is an additional parameter, either the frequency ω_2 or the rotational number ρ. Before the station-keeping procedure is explained, the focus is set on the basic idea of the algorithm, which is the determination and implementation of manoeuvres that increase the lifetime of an orbit.

III.I Determination of the orbital lifetime and manoeuvre directions

Manoeuvres for station-keeping purposes are introduced by evaluating the time a trajectory evolves in certain boundaries around a libration point. Assuming that the initial state is already within these limits and on the xy-plane in the synodic reference frame, the state can be perturbed in such a way that the orbital lifetime changes. The objective is now to find a manoeuvre that the lifetime within

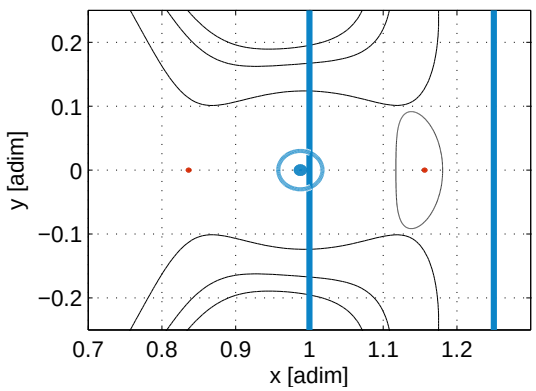

Figure 4: Boundary definitions (blue) required for the orbital lifetime definition process. For equations, see 7.

the geometric limits is maximised not leaving a defined region. For orbits with larger amplitudes, when the limits include the secondary body it is required to prevent the spacecraft falling into the primary. For this reason the region around the secondary body is added as an exclusion zone, see Eq. 7. The forbidden region defined by the zero velocity curves enables a introduction of the limits that are purely defined on the x-component of the state vector. In order to determine the correction velocity, the trajectory is propagated forward from the initial state or from the previous correction point. The numerical integration stops when the trajectory passes through the boundaries set as

$$x < x_{L2} + \delta_1 \tag{5}$$

$$x > x_{L2} - \delta_1 \tag{6}$$

$$\delta_2 < \sqrt{(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2} \tag{7}$$

where δ_1 is the maximal amplitude in x-direction of the orbit and $\delta_2 = 10^{-4}$. Restricting the station-keeping manoeuvre to the in-plane direction and allowing it only at the crossing of the x-y plane reduces the number of parameters to be optimised. In this case $z = 0$ and $\dot{z}$ is evaluated as

$$\dot{z}^2 = v^2 - \dot{x}^2 - \dot{z}^2 \tag{8}$$

The starting point for the evaluation of the orbital lifetime are state vectors on the xy-plane crossing in positive direction. The initial states are propagated forward until a boundary condition is reached. The time until this event is defined as T_m which is maximised by a differential evolution algorithm. In order to extend the lifetime of an orbit, either the velocity, the position or both can be modified. The implementation of a velocity change is straight forward. Changing the position would leave a discontinuity in the trajectory. To cope with this problem a position change is implemented with two velocity changes, the first one instantly to vary the position of a point downstream of the trajectory, where the second manoeuvre is applied. The re-directing manoeuvre guiding escaping trajectory towards longer lifetime, non-escaping solutions is now used to design a station-keeping algorithm.

A variety of algorithms are tested to achieve the identification of the next centre manifold, for this purpose an initial state vector on a periodic orbit is chosen as targeting point on the xy-plane. The centre manifold is described as the phase space that is neither affected by the attraction of the stable manifold nor the repulsion of the unstable one. The following three cases are considered:

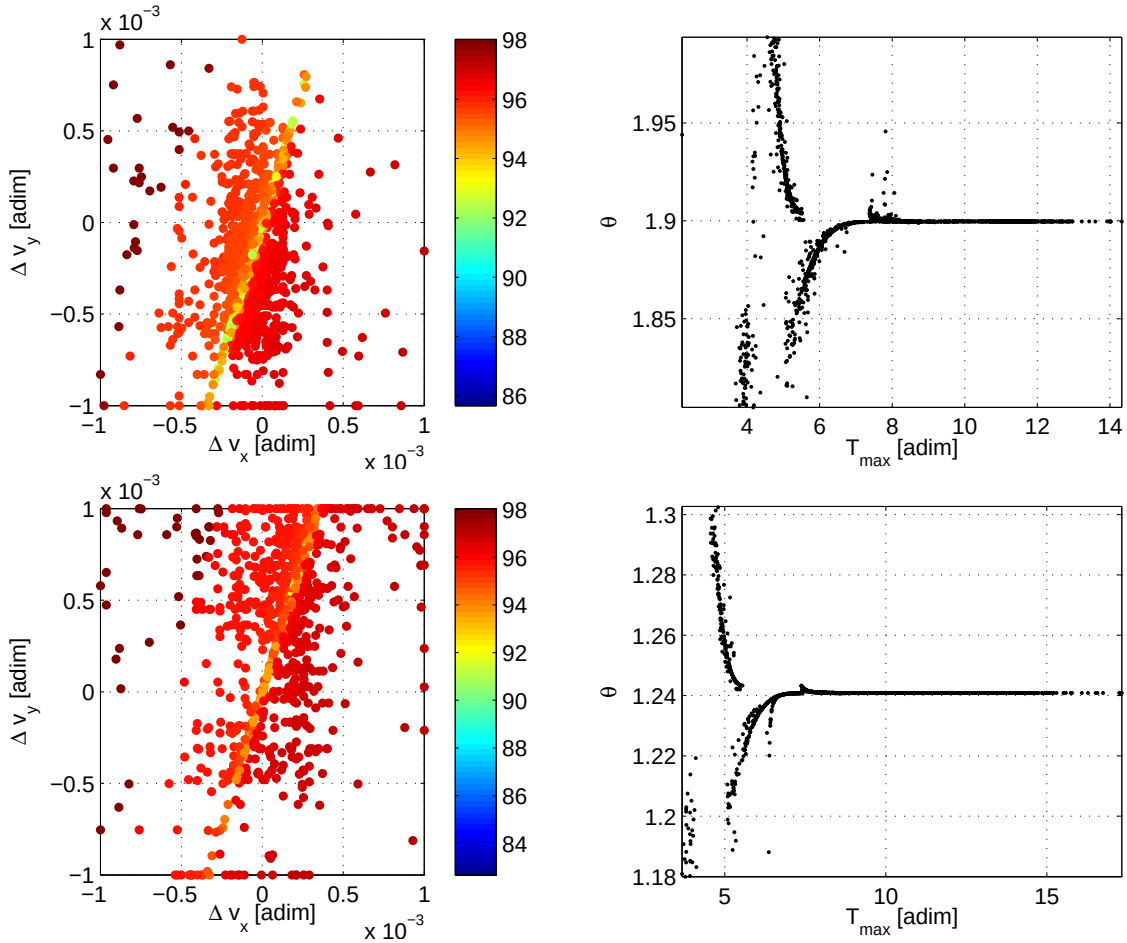

Figure 5: Results of the optimisation process. Optimal manoeuvre and its direction downstream (forward) (top). For the upstream (backward propagation) (bottom).

1. Fix position vector x, optimal Δv applied at xy-plane crossing to maximise the orbital lifetime T_m downstream. The fixed position vector prohibits a movement along the stable manifold direction.

2. Fixed position vector x, v chosen to maximize the mean lifetime $mean(T_m^+, T_m^-)$ both upstream and downstream. This only leads to feasible solutions if the velocity vector is accurate.

3. Adaptation of x and v aiming a maximal orbital lifetime T_m downstream. In this case the position can walk along the stable manifold increasing the lifetime on the one side but moving away from the desired solution.

III.II Station-keeping strategy

The determination of the next downstream centre manifold is significant for a station-keeping algorithm that on the one side should keep the orbital properties of a particular orbit and on the other side is not relying on a reference trajectory. Targeting the next downstream centre manifold implies that no stable or unstable component is added and the spacecraft evolves still on the 'same' orbit. From the previously mentioned strategies, the first one has been proven as the most effective ap-

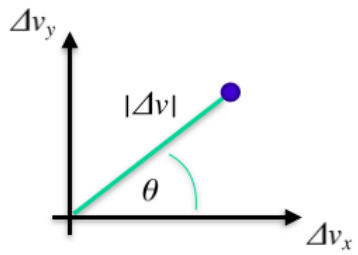

Figure 6: Definition of the lifetime extension manoeuvre on the velocity plane. The manoeuvre direction is defined as θ, $|\Delta v|$ the magnitude.

proach for the identification of the centre manifold. The extension of the lifetime causes the return of the spacecraft to the vicinity of the original orbit around the libration point. During the optimisation process a constant Jacobian is assured by adjusting the Δv_z component, see Eq. 8. This enables full steering capabilities during the optimisation. This leaves two optimisation parameters, which are Δv_x and Δv_y. Alternatively the direction θ and magnitude $|\Delta v|$ can be defined. Two optimisation runs will show the correlation between these two parameters, one conducted for a forward and the other one for a backward propagation, indicating the stable and unstable (dominant) directions. The size of the population is set to $N = 20$ and 200 generations are calculated. The differential evolution algorithm has two parameter subject

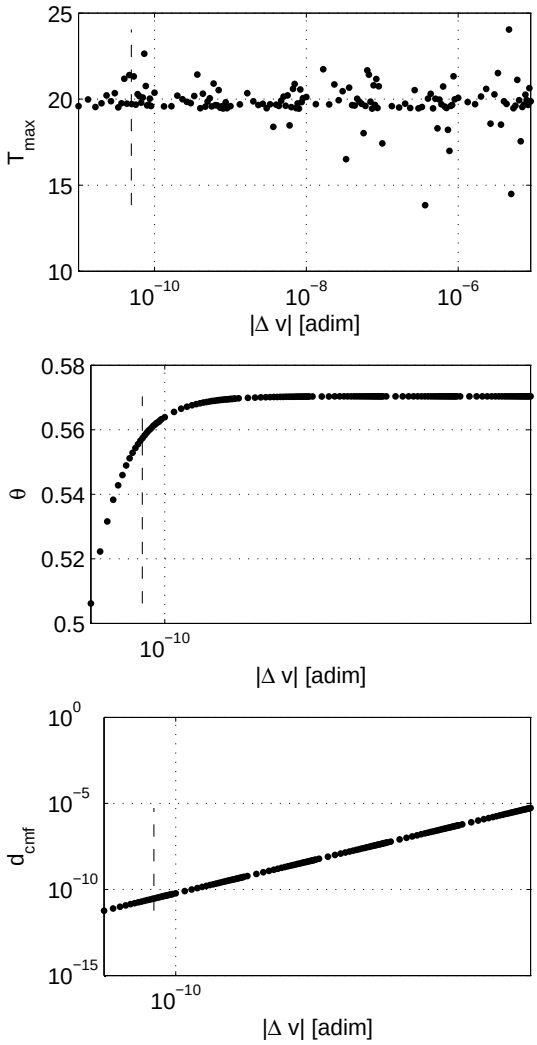

Figure 7: Correlation between the maximal orbital lifetime T_{max}, the direction θ and the magnitude of the manoeuvre, here for a precise initial state (unperturbed).

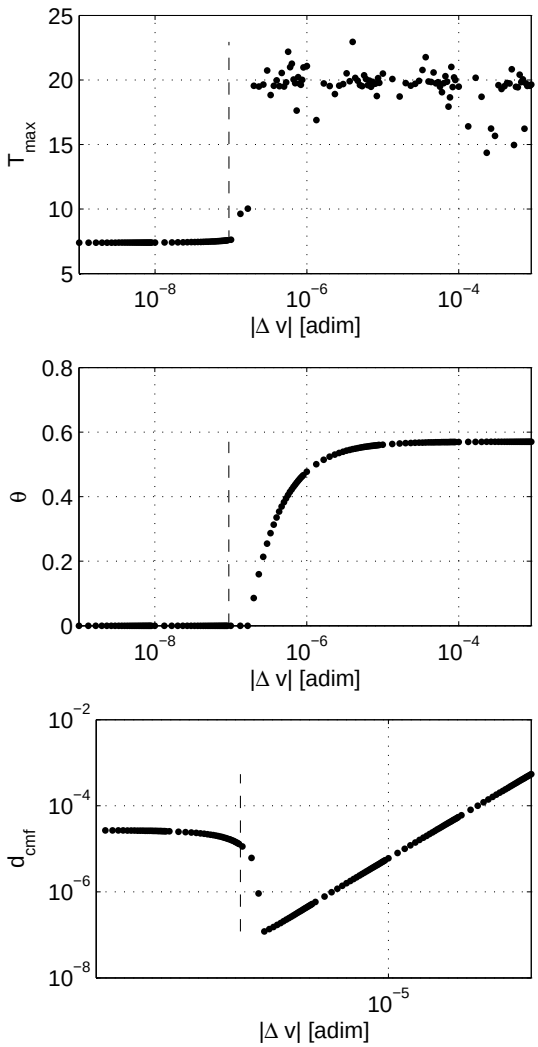

Figure 8: Correlation between the maximal orbital lifetime T_{max}, the direction θ and the magnitude of the manoeuvre, here for a state with an error that is gained by propagating the state from study in Fig. 5 for two revolutions downstream. (perturbed).

to adaptation, the differential weight $F = [0, 2]$ and $CR = [0, 1]$ representing the crossover probability. The tuning parameters CR and F are set to $[0.5, 0.8]$ for our purpose.

Fig. 5 show all evaluated members during the differential evolution process. The Δv_x-Δv_y plots show the correlation between the manoeuvre and the orbital lifetime. The best 10% of the solution in Fig. 5 (right) are taken to produce the T_{max} - θ plots, shown the manoeuvre direction. Significant in Fig. 5 (left) is the chaotic behaviour for small orbital lifetimes. With increasing T_{max} the graphs bifurcate into a dominant solution.

The relation between manoeuvre directions, magnitudes and their corresponding orbital lifetimes T_{max} are further studied as they represent

a major step towards the design of the station-keeping algorithm. Two initial state vectors, one that belongs to a periodic orbit, which is the outcome of the solved two-point boundary value problem, and the second one afflicted with errors introduced by the propagation of two revolutions downstream are used. Fig. 7 shows the outcome of the optimisation for the unperturbed, whereas Fig. 8 for the perturbed case. For visualisation purposes the lifetime, manoeuvre direction, and the associated error on the next xy-plane crossing are evaluated over a wide range manoeuvre magnitudes. No optimisation is required for the results in Fig. 7 and Fig. 8. The optimisation results are marked with a vertical dashed lines. There is a precise error measurement available for periodic orbit as the state on the xy-plane maintains on every return. The euclidean distance d_{cmf} between the initial state an each return is shown in the $|\Delta v|$ - d_{cmf} plots. Remarkable is the smooth behaviour for the error function d_{cmf} and manoeuvre direction θ, whereas the orbital lifetime indicates a chaotic pattern. A reason for this is the fact that the location where the trajectory escapes moves along the boundary planes. In some cases the is also the switching between the two boundary planes visible.

The methodology of this part of the algorithm is explained in the previous. In the following the performance of the algorithms is tested. Perturbed

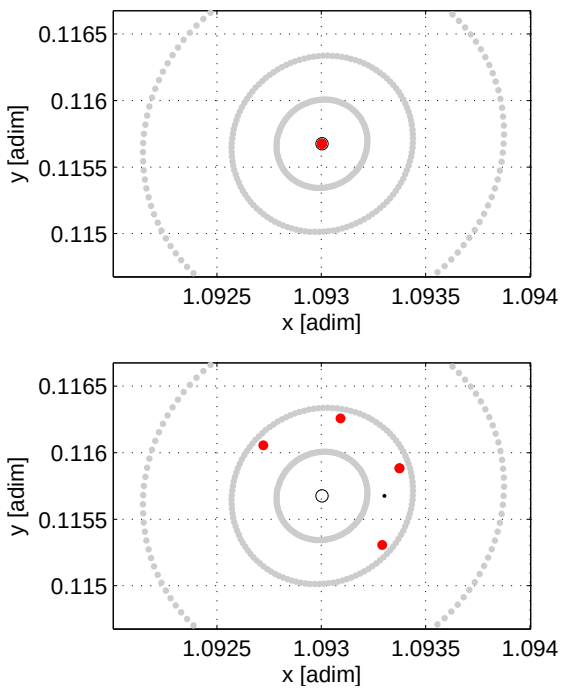

Figure 9: Return map on the xy-plane for an initial state on a periodic orbit. Unperturbed state (top), perturbed state (bottom).

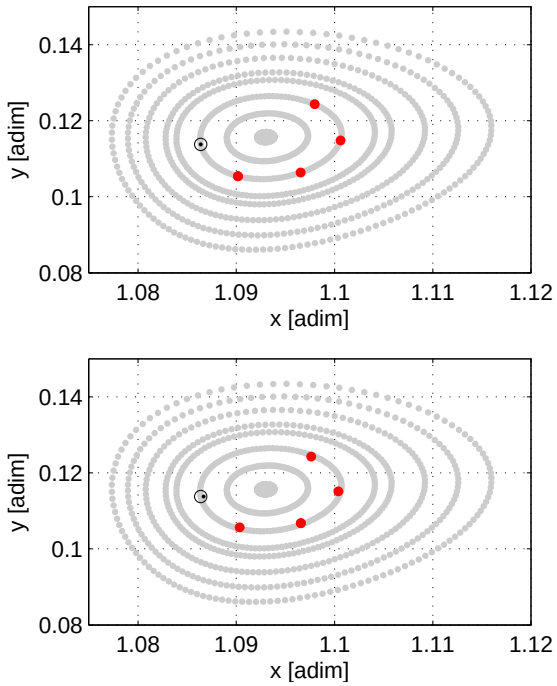

Figure 10: Return map on the xy-plane for an initial state on a quasi-periodic orbit. Unperturbed state (top), perturbed state (bottom).

and unperturbed state vectors provide the entry point, the optimal manoeuvre is determined and executed the trajectory is propagated to the next xy-plane crossing (full revolution). This procedure is repeated until the required number of station-keeping manoeuvres for the trajectory lifetime is determined. Fig. 9 and Fig. 10 show the xy-plane map with several returns for a initial state on a periodic and quasi-periodic orbit. The grey (dotted) rings present the centre manifold structure around the initial state vector, which is indicated as black marker. The representation of the centre manifold by those curves is widely used and enables the visualisation on the plane with $z = 0$. The black dot that is for the unperturbed case in the centre of the marker. The red markers are the returning points after each station-keeping manoeuvre. Once a return point is identified it is used as starting condition for the next manoeuvre evaluation.

IV PRELIMINARY RESULTS

Among the existing bounded libration point orbits station-keeping costs are computed for periodic and quasi-periodic cases following the same approach. In general, the station-keeping algorithm is applicable to all libration point orbits that possess an invariant hyperbolic manifold structure. The numerical algorithm is used to continue and

extend the orbital lifetime of orbits. Applying the algorithm in a dynamical regime modelled by the circular restricted three-body problem accounting neither for uncertainties nor a navigation budget leads to very small manoeuvres that simply compensate numerical errors introduced by the propagation.

IV.I Station-keeping for periodic orbits

The previously explained station-keeping algorithm is applied to a quasi-periodic trajectory. The maintenance effort in terms of Δv is determined for halo orbits with an orbital period between 13.3 to 14.8 days. About 10-15 station-keeping manoeuvres are evaluated leading to an orbital lifetime of about half a year. The maximal manoeuvre magnitude is limited during the optimisation. For a better assessment of the manoeuvre magnitude and directions the station-keeping are applied every second crossing of the xy-plane.

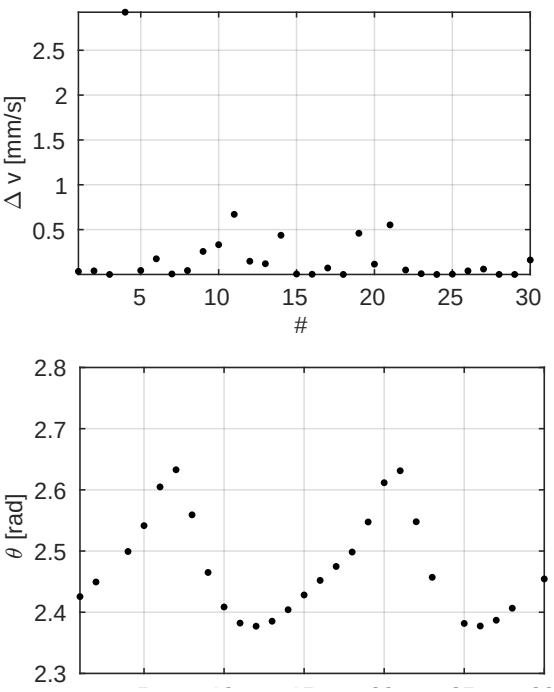

Figure 11: Station-keeping manoeuvres for a periodic orbit, magnitude (top), direction (bottom).

For the periodic case the station-keeping error can be easily evaluated as the trajectory return to the same point on the xy-plane. The euclidean distance between the returns serve as error function. The order of magnitude of the error function in this case is 10^{-9}. The optimisation shows that all the station-keeping manoeuvre are conducted in the same direction (or 180 deg shifted). Additional the vectors are align with the direction of the stable mode eigenvectors for the periodic case. This

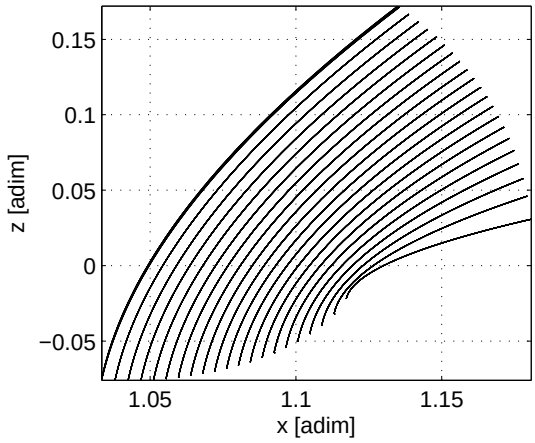

Figure 12: Resulting trajectories for the halo orbit family, mean propagation time of orbits is 180 days.

is not surprising station-keeping methods incorporating Floquet analysis utilise manoeuvres that are aligned with the unstable mode aiming for a cancellation of the unstable component. The manoeuvre history is shown in Fig. 11. The resulting halo orbits are plotted in Fig. 12.

IV.II Station-keeping for quasi-periodic orbits

The station-keeping method for quasi-periodic orbits is straight forward. The periodic and quasi-periodic case differ only in the return map on the xy-plane, which is now described as invariant curve instead of a point for the periodic case. This means that the optimal-station keeping direction vary depending on the return direction on the invariant curve, see Fig. 13 as comparison. For this pur-

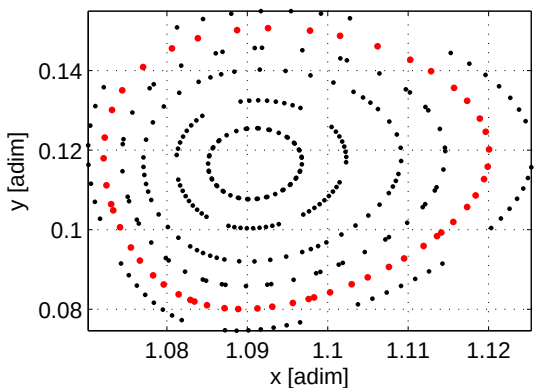

Figure 13: Station-keeping results visualised on the return map (xy-plane) for a family of quasi-periodic orbits rising from a halo orbit (quasi-halo type).

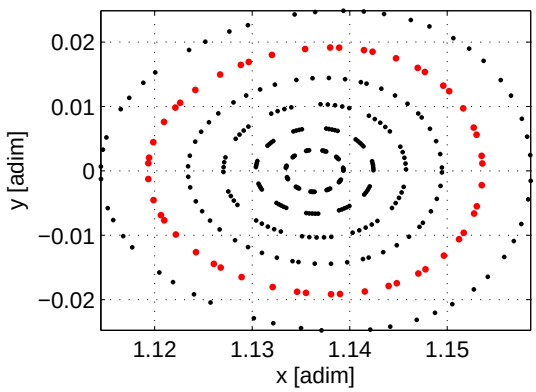

Figure 14: Station-keeping results visualised on the return map on the xy-plane for a family of quasi-periodic orbits rising from a vertical lyapunov orbit (lissajous type).

pose 50 crossing are evaluated to resolve the invariant curve. The station-keeping is applied to two families of quasi-periodic orbits with an energy of $E = -C/2 = -1.548$. Quasi-periodic orbit families share the orbital energy but differ in the second parameter which is the rotation number ρ. The manoeuvre magnitude is below $0.1mm/s$ for all cases having only the purpose to cancel out

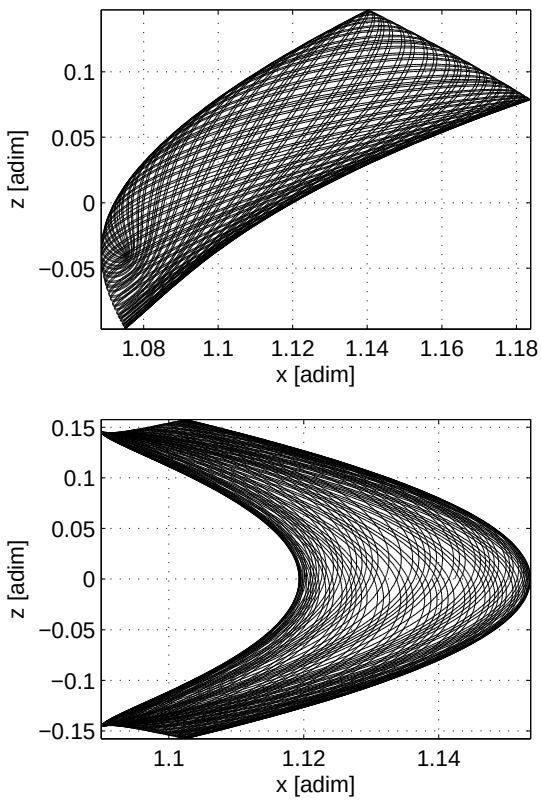

Figure 15: Quasi-periodic trajectory with an orbital lifetime extended to 180 days.

the numerically introduced error. Fig. 13 and Fig. 14 show the locations of the crossings on the xy-plane. For two cases the corresponding trajectories are plotted in Fig. 15, the section representation is highlighted in red in Fig. 13 and Fig. 14.

IV.III Evaluation of manoeuvre directions

The purpose of the previously introduced station-keeping approach is to maintain an orbit for an arbitrary lifetime preserving orbital properties. The results, in particular, the obtained manoeuvre direction can be compared with results from a Floquet mode analysis. Subspaces can be determined that correspond either to unstable or stable parts of the motions. Fig. 16 highlights the identified dominant manoeuvre direction for the halo orbit family with parameters as in Fig. 12. The optimisation results in this study for the halo orbit family are shown for an escape towards the stable (red, bold) and unstable (blue, bold) hyperbolic manifold. The results plotted with thin markers correspond to the in-plane (x and y components of the position vector) determined by this subspaces. The discrepancies between the Floquet mode approach and the station-keeping algorithm proposed in this paper increase with growing orbital amplitudes. The explanation for this is the linearisation that is required for the Floquet approach. With growing amplitudes, here in Fig. 16 increasing Jacobian C, non-linearities are introduced.

Another outcome of this study is the dominant manoeuvre direction that pushes the spacecraft towards the invariant stable or unstable manifold branches. These trajectories are required for planning e.g. transfer and rendez-vous scenarios in space mission design. Fig. 7 and Fig. 8 show the

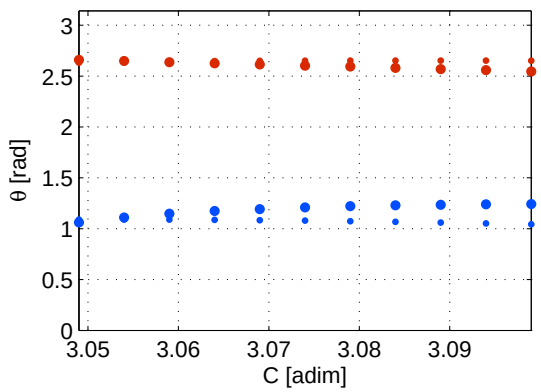

Figure 16: Comparison of optimal manoeuvre directions with results obtained from Floquet subspaces, evaluated for the halo orbit family for an escape towards the stable (red) and unstable (blue) hyperbolic manifold.

transition between the region with a manoeuvre magnitude at an order of 10^{-6} with a direction increasing to 0.58 and the region where the direction stabilises to a fix values, but higher manoeuvre magnitude. For the evaluation of this dominant directions, the manoeuvre magnitude is set to 10^{-3}. Forward (unstable) or backward (stable) propagation is required in the lifetime assessment.

V SUMMARY AND CONCLUDING REMARKS

The proposed station-keeping algorithm is based on an orbital lifetime analysis paired with a differential evolution algorithm for optimisation exploiting the natural interaction between the centre and hyperbolic invariant manifolds. It has been demonstrated that the manoeuvre determination by lifetime measurements along with its optimisation designing station-keeping manoeuvre is a robust method, fast in implementation and flexible in its application to a variety of orbits. For a periodic orbit the results of the proposed algorithm are comparable to the modal control that can be derived from the Floquet modes. This is correct for orbits with small amplitudes. Non-linearities are introduced with growing amplitudes and optimisation approach performs better than the Floquet procedure. Another advantage is that there is no intermediate linearisation required. A detailed study is required to accounting for navigation and manoeuvring uncertainties. This station-keeping approach is easily transferable into a non-autonomous dynamical model fed by ephemeris data. Furthermore, is can be used to generate families of orbits by introducing a simple family continuation method.

REFERENCES

[1] Ariadna Farrés and Àngel Jorba. Station keeping of a solar sail around a Halo orbit. *Acta Astronautica*, 94(1):527–539, January 2014.

[2] Martin Hechler. Herschel, Planck and Gaia orbit design. *ESA Report*, 2002.

[3] KC Howell and HJ Pernicka. Station-keeping method for libration point trajectories. *Journal of Guidance, Control, and Dynamics*, 16(1):151–159, January 1993.

[4] Angel Jorba and Josep Masdemont. Dynamics in the center manifold of the collinear points of the restricted three body problem. *Physica D: Nonlinear Phenomena*, 132(1):189–213, 1999.

[5] E Kolemen, NJ Kasdin, and P Gurfil. Multiple Poincaré sections method for finding the quasiperiodic orbits of the restricted three body problem. *Celestial Mechanics and Dynamical Astronomy*, 2011.

[6] MW Lo, R Serban, L Petzold, WS Koon, SD Ross, J Marsden, and R Wilson. Halo Orbit Mission Correction Maneuvers Using Optimal Control. *Automatica*, 38(4):571–583, April 2000.

[7] Thomas Pavlak and Kathleen Howell. Strategy for Optimal, Long-Term Stationkeeping of Libration Point Orbits in the Earth-Moon System. *AIAA/AAS Astrodynamics Specialist Conference*, pages 1–16, August 2012.

[8] C Simo, G Gomez, J Llibre, R Martinez, and J Rodriguez. On the optimal station keeping control of halo orbits. *Acta Astronautica*, 15(6-7):391–397, June 1987.

[9] Rainer Storn and Kenneth Price. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization*, 11(4):341–359, 1997.