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Numerical Simulation of Triaxial tests to determine the Drucker-Prager Parameters of Silicon

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Abstract: **Finite element simulation of material response behavior under deformation entails identification of constitutive model parameters to truly expound the material behavior. Silicon is found to be hard and brittle and in the absence of experimental data, it is difficult to obtain constitutive model parameters to simulate the material deformation. In this paper numerical simulation of triaxial compression and triaxial tension tests are performed at different confining pressures and a method was adopted to determine the Drucker Prager parameters of silicon. The method involves extracting the data from stress-strain plots of triaxial compression and tension tests to calibrate the ultimate yield surface and then plotting the data in the meridional (p-t) stress plane.**

Keywords- Drucker prager, triaxial test, silicon, finite element

I. INTRODUCTION

Silicon has great importance in opto-electronics, semiconductor, MEMS, space and defense industries due to its great electro-mechanical properties, great high temperature strength and low thermal expansion. Silicon is the second most abundant element after oxygen and marks 28% of earth's crust by mass. It has been widely employed as semiconductor in computer peripherals, camera technology, and in micro-electronic industries. However silicon with these enviable characteristics is correspondingly difficult to machine material and brittle fracture is an impediment to high surface quality during machining. High compressive strength and high stiffness of silicon make it hard to explicate its behavior under loading conditions. Numerical simulation has widely been adopted to understand the material response behavior under different conditions in order to avoid costly experimental techniques and trials. Drucker Prager model along with its optimized models are successfully been employed to simulate the material response behavior of pressure-dependent soil, rocks and concrete **[1-2].** Experimental uniaxial and triaxial tests are required to obtain the constitutive parameters of materials for different versions of Drucker Prager model. No experimental triaxial compression and tension data is available for silicon in the author's knowledge. Yield's strength of silicon is also a contentious property as it is been reported from 350MPA to 7GPa **[3-4].** In the absence of experimental data, parameter optimization techniques can be used to obtain these parameters. However the resultant parameters from optimization techniques are highly dependent on the initial guess. Another approach is to perform finite element simulation of experimental work to acquire required material parameters.

Drucker Prager model

Since the von Mises yield criterion imply the dependence of material yielding solely on second deviatoric stress tensor J_2 and is independent of the first stress invariant I_1, the yielding sensitivity to hydrostatic stress tensor is not incorporated for pressure-sensitive materials. Drucker and Prager in 1952**[5]** proposed a model to address the effect of mean (hydrostatic) stress for pressure sensitive materials which von Mises yield criterion failed to address. The proposition acknowledged as Drucker-Prager (DP) model (also known as extended von Mises model).

Drucker Prager (DP) model explicate the material response behavior of granular-like soils, rocks and other alike pressure-dependent constitutive materials. The response behavior of pressure-dependent materials can be expressed in terms of strength increase with increasing pressure. Compressive strength of silicon is higher than its tensile strength **[3]** and under certain hydrostatic stress, the material is found to behave in ductile mode rather than brittle fracture **[6].** This behavior clearly predicts increase in strength of silicon under loading conditions. In order to implement DP model to simulate deformation behavior of silicon, compressive crushing of concrete can be replaced by compressive plasticity of silicon and tensile dilatancy of concrete will be ignored **[7]**.

DP theory in principle is also a modified form of Mohr-Coulomb's theory. The Drucker-Prager (DP) yield criterion is expressed as:

$$f(I_1, J_2) = \alpha I_1 + \sqrt{J_2} - d = 0 \qquad (1)$$

Where I_1 is the first invariant of stress tensor and J_2 is the second invariant of the deviatoric stress tensor. α is the pressure sensitivity coefficient and d is known as the cohesion of the material. In DP model, the yield surface is the function of pressure and J_2.

Since the finite element simulation was carried out in ABAQUS, the DP model representation will be followed as presented in this FEA software. The pressure-dependent linear DP yield function in Abaqus **[8]** is expressed in three stress invariants and inscribed as

$$f = t - ptan\beta - c = 0 \qquad (2)$$

Where p is the equivalent pressure stress and c is the material parameter known as the cohesion of the material. The term tanβ represents the yielding sensitivity to hydrostatic pressure and β itself is the slope of the linear yield surface in meridional p-t stress plane and also known as friction angle of the material. The parameter t is deviatoric effective stress and expressed as

$$t = \frac{1}{2} q \left[1 + \frac{1}{k} - \left(1 - \frac{1}{k}\right) \left(\frac{f}{q}\right)^3 \right] \qquad (3)$$

and for uniaxial compression

$$C = \left(1 - \frac{1}{3}\tan\beta\right)\sigma_c \qquad (4)$$

Where K is the ratio of yield stress in the triaxial tension to triaxial compression, q is von Mises equivalent stress and f *is* the third invariant of deviatoric stress.
The evolution of equivalent plastic stain can be expounded using flow rule during deformation and provides the plastic strain relevance to stress components. Flow rule is stated in terms of plastic strain rate in the form of following equation

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \qquad (5)$$

In Abaqus, the flow potential is written in the form as

$$g = t - ptan\psi \qquad (6)$$

Where g is the flow potential and Ψ is dilation angle in the p-t plane.
The dilation angle Ψ relates to the volumetric strain during plastic deformation and it remains constant during plastic yielding. For $\Psi=0$ corresponds no volumetric strain, $\Psi>0$ shows volume increase and $\Psi<0$ signify reduction in volume. Silicon exhibit volume reductions of 20-25% **[9]** under loading when endures pressure induced phase transformation correspond to negative dilation angle.

Triaxial Compression and tension tests

There are various triaxial tests conducted for geological materials however very few are available for metals and ceramics. Sandia National Laboratories **[10]** performed uniaxial and triaxial tests under various loading conditions and confining pressures to understand the behaviour of SiC-N in the area of hypervelocity penetration of metal clad armour.
In the triaxial test, strength of the material is found to increase with the increasing confining pressure. The height to diameter ratio, confining pressure and loading rate significantly influence the stress-strain behaviour of the material. Drucker Prager model is frequently adopted for rocks and concrete for which crack propagation is prevented under confining pressure in triaxial testing.

It is an established fact that during the machining of Silicon, the hydrostatic pressure is the governing factor result in increase in strength of material and entails ductile deformation rather than brittle fracture. The brittleness of Silicon also disappears under hydrostatic pressure and material shows the ductile behavior. Axial strength of silicon increases significantly with the increasing lateral confining pressure and above certain pressure result in change of brittle behavior into ductile behavior of silicon.

In linear Drucker-Prager model, angle of friction β, flow stress ratio K, and dilation angle Ψ are the target parameters to be identified. Triaxial compression and tension tests at different levels of confining pressures are required to obtain these parameters. The guideline of the methodology to determine the parameters is provided in **[8].**

SIMULATION OF TRIAXIAL TEST

In triaxial test, a general approach is to use cylindrical specimens for balanced pressure. In this simulation, for simplicity, the geometric configuration of triaxial compression test simulation is a 2D axisymmetric part between top and bottom rigid platens. The bottom platen is fixed while the top platen can move in the direction of loading. The specimen is meshed with CAX4R element with reduced integration for axisymmetric stress analysis. Ductile-brittle transition in silicon is primarily due to hydrostatic stresses and temperature doesn't reach to the extent to cause thermal softening. Therefore thermal part of the simulation was not performed.

The height to diameter ratio of 2 is chosen as recommended for triaxial tests in order to reduce the geometry effect on the shear strength of material **[11]**. Fig.1 shows the schematic of 2D axisymmetric finite element model used.

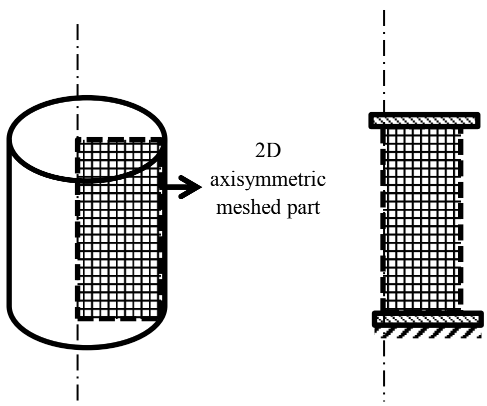
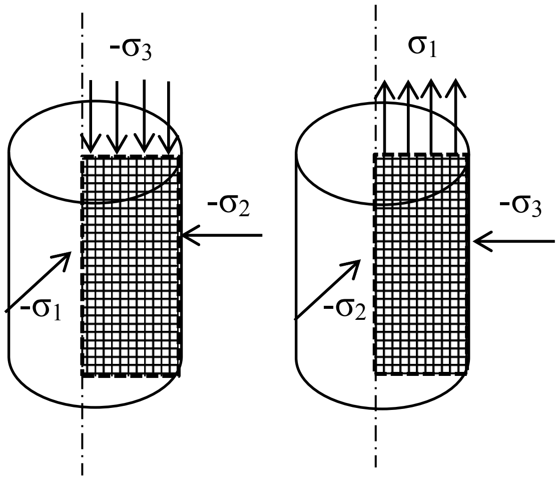


Fig. 1- 2D axisymmetric model represent cylindrical specimen

An elastic modulus of 146GPa, poison's ratio 0.27 and density 2329 kg/m^3 was used. The constitutive behavior of silicon is modeled with Johnson's Cook (J-C)



(a)-Triaxial compression (b)-Triaxial tension

Fig. 2- Triaxial compression and tension simulation

plasticity model.

In simulation, non-uniform mesh causes stress concentration in areas which doesn't undergo similar concentration in that area during experimentation. Therefore the mesh size was kept constant for the whole axisymmetric part.
The displacement in axial and lateral directions is recorded in order to calibrate the volume change.

The axisymmetric part tested was subjected to 200,400, 1200, 2000, 3000, 4000 and 6000 (MPA) lateral confining pressures followed by an axial loading of 7500MPA. The test specimen undergoes constant fixed pressure stress throughout the axial loading. In the triaxial compression test, stress σ_3 represent the axial stress and confining pressure is represented by σ_1 and σ_2. Fig.2 represents the pressure and loading conditions for triaxial compression and triaxial tension tests.

RESULTS AND ANALYSIS

The results obtained from the numerical simulation of triaxial compression and tension tests are analyzed and plotted to obtain the values of linear Drucker Prager model. Stress-strain plot with different confining pressures is presented in Fig.3. It is clearly observed from the figure that increase in the confining pressure resulted in increase of elastic yield limit of the part and ultimately increased yield strength of material. At the confining pressure of 6GPa, the plastic deformation of the specimen disappeared and only the elastic limit was observed. The yield points on the onset of plastic deformation from each stress-strain curves were chosen in order to calibrate the ultimate yield surface.

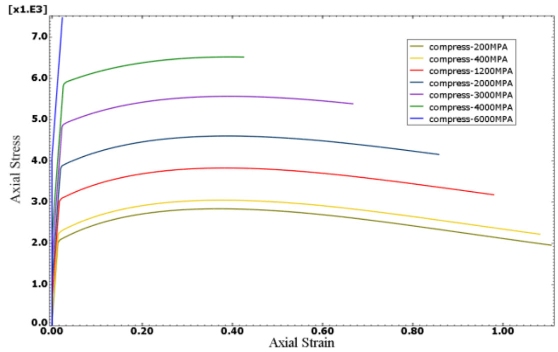


Fig. 3- Axial Stress-strain plot with different confining pressures

The hydrostatic stress is expressed in the form

$$Hydrostatic\ Stress = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Since pressure is negative of hydrostatic stress, the pressure, P can be written as:

$$Pressure = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

For triaxial compression,

$$q = \sigma_1 - \sigma_3 \text{ and } t = q$$

Figure 4 is the yield surface calibration in the p-t plane from the triaxial compression results.
From the compression result data, linear regression line was drawn. The values of β, and cohesion for the linear Drucker Prager model were obtained from the compression data linear trend line. The angle β is the angle made by the triaxial compression data line with the horizontal p axis and is calculated 44°with cohesion of 576MPa.The cohesion in metals and ceramics is different than usually measured for soils and therefore the cohesion obtained for silicon should be further investigated. The value of dilation angle was calculated 28.77° using flow potential equation. The dilation angle is dependent on the internal friction angle and is always less than the internal friction angle.

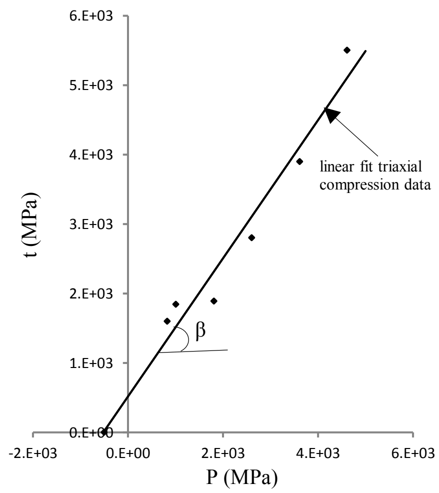


Fig.4 Data fitting triaxial compression

For triaxial tension test, the relationship of t with q is influenced by flow stress ratio

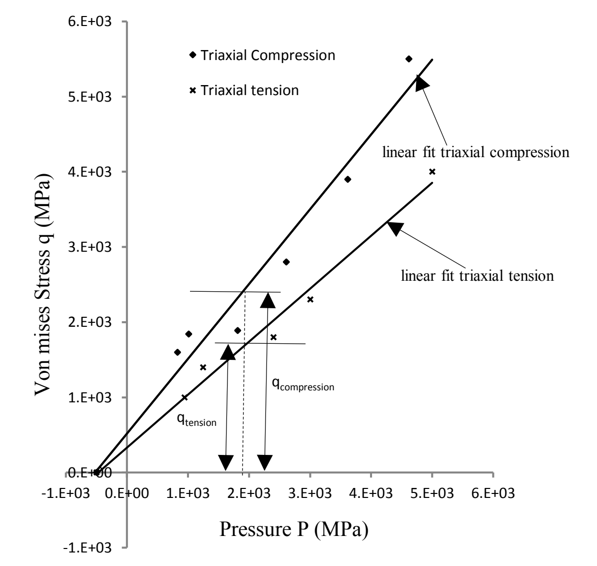


Fig. 5 Data fitting triaxial compression and tension

$$t = \frac{q}{K}$$

In order to find value of K, stress-strain data from triaxial compression and tension tests data was plotted on p-q plane. Fig 5 is the plot of linear regression of triaxial compression and tension stress-strain data plotted on P-q plane. The flow stress ratio can be calculated from

$$K = \frac{q_{tension}}{q_{compression}}$$

Since in Abaqus the value of K has to follow the condition $0.78<K<1$, its value will be taken at pressure where K obey the condition. The value of K at pressure of 1910MPa is found to be 0.81.

Conclusion

Numerical simulations of triaxial compression and tension tests were conducted on 2D axisymmetric model in order to obtain the linear Drucker prager model parameters of silicon. The values of internal friction angle, flow stress ratio, cohesion and dilatancy angle were calculated by analyzing the result data. The obtained parameters can also be used to other parameters of exponent form of Drucker Prager model in Abaqus. Increase in confining pressure was found to increase the yield strength of silicon.

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