OUTPUT FEEDBACK CONTROL OF LINEAR MULTIPASS PROCESSES

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ABSTRACT

An error actuated output feedback controller for a sub-class of linear multipass processes designated as 'differential unit memory' is defined. Further, the design of this controller for closed-loop stability is considered. In particular, a recently developed computationally feasible stability test is used to present some preliminary work on this problem.

1. Introduction

Multipass processes are a class of dynamic systems characterised [1] by a repetitive action with interaction between successive passes. Industrial examples include [1] *longwall coal cutting and metal rolling. Further, these* processes can [1] exhibit undesirable characteristics which require unique control action.

Previous work [2] has developed a rigorous stability theory for multipass processes using an abstract representation which includes the class of so-called [1] differential unit memory linear multipass processes as a special case. The members of this class are described by the state-space model.

$$\begin{aligned} &\dot{X}_{k+1}(t) = AX_{k+1}(t) + BU_{k+1}(t) + B_o Y_k(t), X_{k+1}(0) = 0 \\ &Y_{k+1}(t) = CX_{k+1}(t), 0 \le t \le \alpha, k \ge 0 \\ &X_k(t) \epsilon R^n, Y_k(t) \epsilon R^m, U_k(t) \epsilon R^l \end{aligned} \tag{1}$$

Here $Y_k(t)$ is the kth pass profile, $X_k(t)$ is the kth pass state vector, $U_k(t)$ is the kth pass control input and the pass length α is assumed finite.

The definition of, together with conditions for, stability of (1) can be found in [1]. Further, the testing of these conditions has been considered in [3]. This has resulted in a computationally feasible simulation based test. Note, however, that no work has yet been reported on the design of output feedback based control schemes. Consequently this paper defines a so-called current pass error actuated proportional output feedback controller for (1). Additionally, this recently developed simulation based test is used to present some preliminary work on its design for closed-loop stability.

2. Stability

The result of this section is based on the so-called [3] associated conventional linear system of (1) defined as

$$\begin{aligned} &\dot{X}(t) = AX(t) + B_o Y(t), X(O) = 0 \\ &W(t) = CX(t) \\ &or \quad W = LY \quad where \end{aligned} \tag{2}$$

$$(LY)(t) = C\int_o^t e^{A(t-\tau)} \quad B_o Y(\tau) d\tau \tag{3}$$

Hence, in effect, (2) has been obtained from (1) by setting B = 0, dropping the pass subscript and ignoring the pass length. Further, (2) is assumed to be controllable, observable and stable. In addition, it is assumed that its step response matrix $W^1(t) = C\int_o^t e^{At^1} B_o dt^1$ is available from appropriate *simulation studies.*

To proceed, let f be a scalar continuous function defined on any finite interval [0,t]. Then $N_t(f)$ denotes the norm of f on [0,t], i.e.

$$N_t(f) \triangleq |f(0^+)| + \sum_{j=1}^{k} |f(t_j) - f(t_{j-1})| + |f(t) - f(t_k)| \tag{4}$$

where $0 = t_o < t_1 < t_2 < \cdots$ are the local minima and maxima of f on $[0, +\infty]$ and k is the largest integer satisfying $t_k \le t$. For $t = +\infty$,

$$N_\infty(t) = \sup_{t \ge 0} N_t(f) = \lim_{t \to +\infty} \quad N_t(f) \tag{5}$$

whenever the limit exists. Note also that the computation of (4) and (5) is a simple exercise [4] from graphical display of f.

Suppose, therefore, that (5) is applied to each element in turn of $W^1(t)$ and denote the resulting matrix by $\|W^1{}_\infty\|p$. Then the following result, *for a proof see [3], constitutes a* sufficient condition for stability of (1) where r (.) denotes the spectral radius.

THEOREM 1: (1) is stable if

$$r\,(\|W^1{}_\infty\|p) < 1 \tag{6}$$

The testing of (6) for a given example is straightforward [3], consisting, essentially, of appropriate simulation studies to obtain W^1 and the subsequent computation of $r(\|W^1{}_\infty\|p)$. Hence this test is clearly computationally feasible and well suited to inclusion within a computer aided design package.

3. Output Feedback Control

One approach (for others see [3]) to altering the dynamic characteristics of (1) is to follow standard linear systems theory and employ current pass error actuated output feedback control. Thus a current pass error actuated proportional output feedback controller for (1) takes the form:

$$U_{k+l}(t) = Ke_{k+1}(t), 0 \le t \le \alpha, k \ge 0 \tag{7}$$

where K is an l x m real constant matrix, $e_{k+1}(t) = r_{k+1}(t) - Y_{k+1}(t)$ is the current pass error vector, and $r_{k+1}(t) \epsilon R^m$ represents desired behaviour on pass k+1. Suppose also that (7) is applied to (1). Then it follows immediately that the resulting closed-loop system is stable if theorem 1 holds for the linear operator defined by substituting A-BKC for A in (3).

Consider now the problem of designing (7) to stabilise (1). Then a fundamental question to be answered is when, and under what conditions, does such a stabilising control law exist. This is termed [3] the existence problem for (7) applied to (1) and its solution in the general case could prove a formidable task. For one special case, however, the following result provides a solution.

THEOREM: Suppose that $m = l$ and that the matrices A, B, B_o and C are given (after use of a state transformation if appropriate [3]) by $A = -A_o^{-1} A_1$, $B = A_o^{-1}$, $B_o = I_m$ and $C = I_m$ respectively, where A_o and A_1 are real constant matrices. Suppose also that $A_o^{-1} A_1$ has a diagonal canonical form and set

$$K = \rho A_o - A_1 \tag{8}$$

where ρ is a positive real scalar. Then $\| W_\infty^1 \|_\rho = \frac{1}{\rho} \; I_m$ and hence by theorem 1 the closed-loop system is stable for all choices of $\rho > 1$.

Note: Theorem 2 relates to the important practical case when the so-called derived conventional linear system, [3], of (1) has the structure of a multivariable first order lag.

4. Conclusions

A current pass error actuated proportional output feedback controller for differential unit memory linear multipass processes has been defined. Further, the design of this controller for closed-loop stability has been considered. In particular, a solution to this problem in one special case has been developed.

References

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