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Efficiency Improvement of Non-Uniformly-Aged PV Arrays

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The paper has not been presented at a conference or submitted elsewhere simultaneously.
Abstract—The utilization of solar energy by photovoltaic (PV) systems have received much research and development (R&D) attention across the globe. In the past decades, a large number of PV array have been installed. Since the installed PV arrays often operate in harsh environments, non-uniform aging can occur and impact adversely on the performance of PV systems, especially in the middle and late periods of their service life. Due to the high cost of replacing aged PV modules by new modules, it is appealing to improve energy efficiency of aged PV systems. For this purpose, this paper presents a PV module reconfiguration strategy to achieve the maximum power generation from non-uniformly aged PV arrays without significant investment. The proposed reconfiguration strategy is based on the cell-unit structure of PV modules, the operating voltage limit of grid-connected converter, and the resulted bucket-effect of the maximum short circuit current. The objectives are to analyze all the potential reorganization options of the PV modules, find the maximum power point and express it in a proposition. This proposition is further developed into an easily implementable algorithm to calculate the maximum power generation and the corresponding reconfiguration of the PV modules. The immediate benefits from this reconfiguration are the increased total power output and maximum power point voltage information for global maximum power point tracking (MPPT). A PV array simulation model is used to illustrate the proposed method under three different cases. Furthermore, a 3×3 array experimental rig is built to verify the effectiveness of the proposed method.

Index Terms—Maximum power tracking, non-uniform aging, offline reconfiguration, output characteristics, photovoltaics (PVs), solar energy.

I. INTRODUCTION

Solar energy utilization has received much attention across the globe over the last decades [1]-[6]. Currently, photovoltaic (PV) power devices are gaining in popularity in the global renewable energy market, primarily owing to the reducing manufacture costs of PV panels and continuous improvement in power conversion technologies [7]-[8]. In practice, high energy conversion efficiency and long effective service time can help reducing capital and operating costs and thus are highly desired.

With the improvement of materials technologies, monocrystalline silicon and multicrystalline silicon now can be economically produced in large quantities. However, their energy conversion efficiency from solar to electricity is still low. Typical efficiency for monocrystalline silicon solar cells is around 20% while it is 18% for multicrystalline silicon solar cells [9]. On the power electronics side, high-performance switching devices (e.g. silicon carbon (SiC), super junction MOSFETs) and new converter topologies (e.g. multi-level DC-AC and resonant DC-DC converters) can improve energy conversion efficiency [10]-[12]. This part of
energy conversion efficiency can reach as high as 95% [12]. However, these figures are typically for nominal and healthy operation of PV cells while in reality they are subject to various faults and aging conditions, which reduce the lifetime of the PV cells and their operational efficiency. For these faulted or aged PV systems, an easy approach to improve energy efficiency is to replace aged PV modules by brand new ones. However, this is not economically acceptable to most of the PV system owners. This paper aims to propose a reconfiguration strategy for faulted or aged PV systems so that the maximum power generation can be improved by simply rearranging the positions of the PV modules. This reconfiguration strategy is derived from the bucket effect of the maximum short circuit current of PV strings, therefore, the basic structure and working principles of a PV system need to be introduced.

There are four levels of components to form a PV system. Namely, PV cell-unit, PV module, PV string and PV array, as illustrated in Fig. 1. In order to restrict hotspots in the PV module, a bypass diode is connected in parallel with PV cells; such a structure is named a cell-unit (including m PV cells). In the PV system considered, assume that n cell-units are connected in series to form a PV module to raise the output voltage, and s PV modules are connected in series to construct a PV string. A number of PV strings are connected with diodes and then in parallel to become a PV array. The diodes can stop the current flow between strings, which is harmful.

![Componential structure of the PV array.](image)

In order to increase the PV device’s effective service time, fault diagnosis and remedial measures are two
important approaches. Since PV panels often operate in outdoor harsh environments, potential hazards from
dust, bird-dropping, partial shading and cell aging will affect the power generation performance [13]-[15].
Therefore, detection techniques such as thermal cameras [16]-[21], earth-capacitance measurement (ECM)
[22], time-domain reflectometry (TDR) [23], and voltage/current sensors are widely applied to identify the
irregularity of PV cells. Upon a fault, a non-uniform temperature distribution can build up on the PV array
and a thermal camera can help to locate the faulty PV module. The ECM can locate the disconnection of any
PV modules and the TDR can estimate the degradation of PV arrays. Nonetheless, both ECM and TDR are
expensive and only be employed as offline fault diagnosis tools [22][23]. References [24] and [25] propose a
PV array fault detection method by comparing simulated and measured output powers of PV arrays based on
the environment data. Paper [26] analyzes the dynamic current-voltage characteristics to achieve fault
diagnosis. In paper [27], machine learning techniques are employed for PV fault detection by measuring PV
array voltage, current, irradiance and temperature.

After a fault is diagnosed, certain remedial measures need to take place. In-situ reconfiguration is an
effective solution [28]-[34]. But this can only work if a large number of relays are used and the state of
health (SoH) information of every PV module is available all the time. These two conditions cause higher
system costs and also make the system controls complicated. In-situ reconfiguration also needs powerful
CPUs to calculate optimal solutions quickly enough, which increases system costs. Paper [34] proposes a
diffusion charge redistribution method to achieve the maximum power. By taking advantage of the intrinsic
diffusion capacitance of the solar cells, the number of power devices used is reduced to simplify the system.
Nonetheless, the majority of PV arrays are not equipped with online reconfiguration equipment due to the
high cost.

So far the PV fault diagnosis and online reconfiguration technology are still under development. In the
middle and late lifetime of the PV arrays, aging, especially non-uniform aging, is a severe phenomenon that
significantly decreases PV system efficiency [35]-[37]. In the literature, a cost effective technique to
improve the energy efficiency of the aged PV arrays is still lacking. This paper attempts to fill the gap by
developing an offline reconfiguration strategy for the “middle-aged and elderly” PV arrays so as to
maximize the solar power generation.

II. MATHEMATICAL MODELING

Non-uniform aging is a common problem in PVs which can be caused by lasting dust, shading, or water
corrosion over a long period of time [13][14]. Usually, there are typical scenarios to cause aging differences.
Scenario 1: Due to the harsh environment, hail or stone can break the glass of PV modules. New modules
are usually needed to replace broken modules. Scenario 2: Modules in typical positions also suffer
differently from aging influences. Modules in a terminal side of a PV array have more dust than other modules. Furthermore, modules in the front string also have more dust than others. Scenario 3: Modules with large aging differences also present possibly due to quality reasons amongst others. In all those conditions, the PV array will have a wide range of aging conditions.

A. Model of Healthy PV Cells

The electrical characteristics of PVs are influenced by both temperature and illumination. The electrical model of the PV cell is expressed by [2].

\[
I = I_L - I_o \exp(\frac{q \cdot V}{T_m}) - 1
\]

(1)

\[\varepsilon = \frac{q}{N_s \cdot K \cdot A}\]

(2)

\[
I_L = \frac{G}{G_{ref}}[I_{L_{ref}} + k_i (T_m - T_{ref})]
\]

(3)

\[
I_o = I_{o_{ref}} \cdot \exp\left[\frac{q \cdot E_{sc}}{N_s \cdot A \cdot K \cdot T_{ref}} \cdot \left(\frac{1}{T_m} - \frac{1}{T_{ref}}\right)\right]
\]

(4)

where \(I\) is the PV module output current, \(I_L\) is the photon current, \(q\) is the quantity of electric charge, \(A\) is the diode characteristic factor, \(K\) is the Boltzmann constant, \(I_o\) is the saturated current, \(T_m\) is the PV module temperature, \(G\) is the irradiance, \(V\) is the output voltage, \(G_{ref}\) is the reference irradiance level (1000 W/m²), \(I_{L_{ref}}, I_{o_{ref}}\) are the reference values for \(I_L\) and \(I_o\), \(k_i\) is the current-temperature coefficient provided by the PV manufacturer. \(T_{ref}\) is the reference temperature, \(N_s\) is the number of series-connected cells, \(T_m\) is the PV module temperature. \(\varepsilon\) is a constant depending on \(q, N_s, K, A\), and is calculated by the following equation:

\[
I_{sc_{ref}} - I_{m_{pp_{ref}}} = \frac{I_{sc_{ref}}}{\exp(\frac{q \cdot V_{m_{pp_{ref}}}}{T_{ref}}) - 1}
\]

(5)

where \(I_{m_{pp_{ref}}}, I_{sc_{ref}}, V_{m_{pp_{ref}}}\) and \(V_{oc_{ref}}\) are the maximum power point (MPP) current, short-circuit current, MPP voltage and open-circuit voltage at a reference condition defined by the relevant standard.

B. Terminal Characteristics of Aged Cells

When a PV cell is subject to aging, a direct indication is its lower output power than normal. Due to the p-n junction characteristics of the PV cell, its open-circuit voltage only changes slightly while the short-circuit current changes dramatically. According to references [38][39], the degradation of short-circuit current is about 10%, while the degradation open-circuit voltage is 2% in average after one year operation, which means the short circuit has a dominated influence. From [36], the short current has close change rate with power loss. Reference [40] also gives the conclusion that short current has dominated influence while the open circuit voltage with negligible change after a 1.5 year aging experiment. Therefore, in this paper, we
take use of the short-circuit current to evaluate the aging condition of PV cells; and use the same open
circuit voltage to approximate aging conditions of PV cells.

Fig. 2 presents a cell unit with m non-uniformly aged PV cells, where $I_{sc1}$, $I_{sc2}$, $I_{sc3}$ … $I_{scm}$ are the short-circuit current for cells 1, 2, 3 … m, respectively. There are three ranges in the current-voltage output characteristics. In Range 1, the maximum current is the minimum of all cells current ($I_{sc1}$, $I_{sc2}$, $I_{sc3}$… $I_{scm}$) and all the cells generate electricity. Range 2 is a transitional interval. Its equivalent circuit is presented in Fig. 3 and its terminal output voltage is given in Eq. (6). Due to a voltage drop on $R_e$, the output voltage of the cell-unit is lower than a healthy cell-unit.

$$\sum_{i=1}^{m-1} V_{cell,i} - i_{12} \cdot R_e = V_{cu} \quad (6)$$

where $V_{cell}$ is the output voltage of the PV cell, $R_e$ is the equivalent resistance of aged PV cell, and $V_{cu}$ is the output voltage of the cell-unit.

As $i_{12}$ increases, $V_{cu}$ decreases to zero. The current switches from Range 2 to Range 3. In Range 3, the cell-unit is bypassed by a diode, and the corresponding terminal voltage is -0.5 V (i.e. diode voltage drop). In Ranges 1 and 2, the current passing the cell-unit is $i_{12} = i_m$, where $i_m$ is the PV module current. In Range 3, the current passing the bypass diode is $i_3$, which is equal to $i_m$.

From the analysis of Range 1-3, it can be found that the non-uniform aging of PV cells limits the power generation capacity of cell-units. This is termed the “bucket effect”. 
C. Model of Non-Uniformly Aged Cells

A PV array can age differently at the cell-unit, module and string levels.

For a cell-unit with $m$ series-connected PV cells, the relationship between the output current $i_{cu}$ and the terminal output voltage $V_{cu}$ depends on the PV’s operating points. To facilitate discussion on the three ranges, it is assumed that the magnitude of the short-circuit currents for $n$ cells is

$$I_{sc_{i_1}} \leq I_{sc_{i_2}} \leq \cdots \leq I_{sc_{i_m}}$$

(8)

Define $i_{cell}$ as the actual current passing the PV cells. When the current $i_{cell}$ starts to increase from 0 to $I_{sc_{i_1}}$, all the cells generate electricity. When $i_{cell}$ exceeds $I_{sc_{i_1}}$ but less than $I_{sc_{i_2}}$, cell $i_1$ cannot generate electricity: it is either bypassed or turned into a resistor because of the bucket effect. As a result, the relationship of $I_{cu}$ and $V_{cu}$ is summarized as follows.

1) If $i_{cell} \leq I_{sc_{i_1}}$, the unit-cell operates in Range 1.

$$i_{cu} = i_{cell} \leq I_{sc_{i_1}}$$

(9)

$$V_{cu} = m V_{cell}$$

(10)

where $V_{cell}$ is equal to the voltage of every cell.

2) If $i_{cell} > I_{sc_{i_1}}$, the cell-unit operates in Range 3.

$$V_{cu} = -0.5 \text{ V}$$

(11)

$$i_{cell} = 0$$

(12)

$$i_{cu} = i_{diode}$$

(13)

where $i_{diode}$ is the bypass current flowing through the diode.

The PV cells can work in Range 2 if there exists an integer $k < m$ satisfying the conditions:

$$I_{sc_{i_k}} < i_{cell} \leq I_{sc_{i_{k+1}}}$$

$$(m-k)V_{cell} - i_{cell} \sum_{j=1}^{k} R_{ej} \geq 0$$

(14)

When the cell-unit operates in Range 2, $i_{cu} = i_{cell}$ and

$$V_{cu} = (m-k)V_{cell} - i_{cell} \sum_{j=1}^{k} R_{ej}$$

(15)

where $R_{ej}$ is the equivalent resistance of the $j$th cell.

Usually, Ranges 1 and 3 are the steady-state operational conditions while Range 2 is a short transitional range between the two and can often be ignored.

A PV string consists of $s$ PV modules, with the terminal voltage $V_{string}$ and current $i_{string}$. Let the terminal voltage, current and maximum current from the $k$th PV module be $V_{module_{k}}$, $i_{module_{k}}$, and $i_{max_{module_{k}}}$, respectively. The following relationship can be established.

$$i_{string} = i_{module_{1}} = i_{module_{2}} = \cdots = i_{module_{s}}$$

(16)

$$V_{string} = V_{module_{1}} + V_{module_{2}} + \cdots + V_{module_{s}}$$

(17)
Similarly, the bucket effect indicates that the maximum current in the PV string is limited by the minimum $i_{\text{module},k}^{\text{max}}$ of those non-bypassed modules. That is, $i_{\text{string}} \leq i_{\text{module},k}^{\text{max}}, 1 \leq k \leq s$, and the $k$th module is not bypassed.

In practice, the cell-units within a PV module may be aged differently and thus have different maximum short-circuit currents. This case is called the “general non-uniform aging” in the paper. A simpler case for non-uniformly aged PV modules is that all cell-units in the same PV module are aged uniformly so that the whole PV module can be characterized with a single maximum short-circuit current of any cell-unit. This is termed the simplified non-uniform aging in this paper.

A PV array consists of $p$ parallel-connected PV strings; its terminal voltage and current are denoted by $V_{\text{array}}$ and $i_{\text{array}}$, respectively. Let the terminal voltage and current for the $j$th PV string be $V_{\text{string},j}$ and $i_{\text{string},j}$, respectively. Therefore:

$$i_{\text{array}} = i_{\text{string},1} + i_{\text{string},2} + \cdots + i_{\text{string},p} \quad (18)$$
$$V_{\text{array}} = V_{\text{string},1} = V_{\text{string},2} = \cdots = V_{\text{string},p} \quad (19)$$

The power output from the PV array is the sum of $p$ strings, and is also limited by the bucket effect. That is, the maximum power output from the simplified non-uniform aging PV array can be written as $\sum_{j=1}^{p} \min\{p_{j,k}^{\text{max}}: 1 \leq k \leq s, \text{ and the } (j,k)\text{th module is un-bypassed}\}$, where $p_{j,k}^{\text{max}}$ is the maximum power output from the un-bypassed PV module at the position $(j,k)$ ($k$th module in the $j$th string) of the PV array. Define $i_{\text{module},j,k}$ as the maximum short-circuit current in the $(j,k)$ module; and $q$ as the number of PV modules which generate electricity in the $j$th string. Thus, $(s - q)$ PV modules are bypassed by diodes in the $j$th string. Then the maximum power $p_{j,k}^{\text{max}}$ is calculated as

$$p_{j,k}^{\text{max}} = qV_{\text{module}}i_{j}^{q} \quad (20)$$

where $V_{\text{module}}$ is the MPP voltage supplied by a PV module, and $i_{j}^{q}$ is the $q$th largest short-circuit current within the set $\{i_{\text{module},j,1}, i_{\text{module},j,2}, \ldots, i_{\text{module},j,s}\}$. For a normal PV module consisting of 3 cell-units, $V_{\text{module}} = 3V_{\text{cu}}$, and $V_{\text{cu}}$ is the MPP voltage a PV cell-unit can provide.

### III. Detection of PV Aging

Aged modules have two typical characteristics: abnormal temperature and terminal electricity characteristics [3]. Accordingly, the detection of PV aging relies on the effective identification of one of the two characteristics.
A. Thermal Cameras

When the PV array is operational, a part of effective solar energy on the PV panel is transferred into electricity while the rest is transferred into heat. Assume that the temperature difference between PV cells and cover glass is neglected; cell temperature is uniform in a healthy module; and there is no thermal propagation across PV cells. Then the energy balance can be established as [5]:

\[ S = V \cdot I + H_{pv} A_m (T_m - T_a) \]  
\[ S = G \cdot A_m \]  

where \( S \) is the effective solar absorbed flux, \( T_a \) is the ambient temperature, \( H_{pv} \) is an overall heat exchange coefficient in relation to the total surface area of the module, \( A_m \) is the PV module area.

Eqs. (1) and (21) form a parameter-based model with key parameters \( I \), \( V \), \( T_m \), \( S \), \( H_{pv} \) and \( T_a \). Fig. 4 illustrates a multi-physical link of the PV array in the parameter-based model, where \( E \) represents the electrical output power of the PV cell. The electrical model is mainly influenced by effective solar energy \( S \) and module temperature \( T_m \), as illustrated in Eqs. (3) and (22). The thermal characteristic is mainly influenced by electrical power and effective solar energy, as shown in Eq. (21). The temperature \( T_m \) and the total effective solar energy \( S \) are linked by the electro-thermal characteristics. For a given \( S \), the module temperature depends on the electrical power of the PV module. The parameters \( T_m \), \( I \) and \( V \) can be retrieved using thermography, current, and voltage sensors, respectively.

![Diagram](image)

**Fig. 4 Energy conversion within the PV array [3].**

When operating in Range 1, all the cells contribute to electricity generation and it is impossible to distinguish an aged module from the PV string. When operating at range 3, the most aged cells cannot generate electricity at the string current working point so that a short circuit occurs at the cell-unit by its bypass diode. Therefore, At Range 3, the aged cell-unit is open-circuited and there is no solar energy transferred into electricity. This leads to a higher temperature in the aged cell-unit than the healthy ones. By changing the working points of PV array, all aged cell-units can be located from their thermal images.
B. Time Domain Reflectometry (TDR)

TDR is another aging detection method. In TDR, a signal is injected into the transmission line, and the signal will be distorted when mismatch occurs [22, 23]. Like a radar, the TDR method analyzes the input signal and output signal, as shown in Fig. 5; the aging condition can be estimated according to the signal degradation. Note that illumination can influence the impedance of PV cell; therefore, TDR can only be used in the night.

Both of the thermal camera and TDR equipment can be used temporarily to obtain the aging information of PV array. This is to say, there is no need to permanently install the reconfiguration equipment while temporary renting will be sufficient to obtain the PV aging information. This saves hardware investment.

IV. OPTIMAL PV MODULE CONFIGURATION

After an aged module is detected, a remedial measure can be employed to rearrange the faulted PV modules, prior to the replacement of the faulted modules which increases capital costs.

A. Theoretical Analysis

From Eq. (20), it is noted that the maximum power output of a PV array depends on the maximum short-circuit current of each PV modules. Therefore, it is possible to rearrange aged PV modules in a PV array in order to maximize the power output. Now, reorganizing all the maximum short circuit current values from the highest to the lowest:

$$\beta_1 > \beta_2 > \cdots > \beta_{ps}$$  \hspace{1cm} (23)

where $\beta_1$ is the highest current and $\beta_{ps}$ the lowest from maximum short circuit currents $\{i_{module,j,k} : j = 1, \ldots, p; k = 1, \ldots, s\}$.

When the PV array power generation is maximal, the number of working PV modules should be equal to that of un-bypassed modules in all strings. This number (denoted by $a$) may vary between 1 and $s$. Thus the output voltage of the PV string equalizes this number multiplied by $V_{module}$. 
**Proposition:** The maximum power output from a simplified non-uniform aging PV array is \( \max\{p_1^{\text{max}}, p_2^{\text{max}}, p_3^{\text{max}}, \ldots, p_s^{\text{max}}\} \), where \( p_1^{\text{max}}, p_2^{\text{max}}, p_3^{\text{max}}, \ldots, p_s^{\text{max}} \) are determined by:

\[
p_1^{\text{max}} = (\beta_1 + \beta_2 + \beta_3 + \cdots + \beta_{(p-1)} + \beta_p)V_{\text{module}},
\]

\[
p_2^{\text{max}} = 2(\beta_2 + \beta_4 + \beta_6 + \cdots + \beta_{2(p-1)} + \beta_{2p})V_{\text{module}},
\]

\[
\vdots
\]

\[
p_{s-1}^{\text{max}} = (s-1)(\beta_{s-1} + \beta_{2(s-1)} + \beta_{3(s-1)} + \cdots + \beta_{(p-1)(s-1)} + \beta_{p(s-1)})V_{\text{module}},
\]

\[
p_s^{\text{max}} = s(\beta_s + \beta_{2s} + \beta_{3s} + \cdots + \beta_{(p-1)s} + \beta_{ps})V_{\text{module}}.
\]

Consider a \( 2 \times 3 \) PV array for example. This array has two strings and each string has 3 PV modules \( (p = 2, s = 3) \). The module maximum short-circuit currents are 0.9 pu, 0.8 pu, 0.2 pu; 0.4 pu, 0.5 pu, 0.7 pu, respectively. If each string has only one operational module, the maximum powers from the first and second strings are 0.9\( V_{\text{module}} \) and 0.7\( V_{\text{module}} \), respectively. The total power output is 1.6 \( V_{\text{module}} \). If each string has two operational modules, the maximum power is (0.8+0.8) \( V_{\text{module}} \) from the first string and (0.5+0.5) \( V_{\text{module}} \) =1\( V_{\text{module}} \) from the second string due to the bucket effect. The total power output is 2.6\( V_{\text{module}} \). If all modules are operational, the maximum power is (0.2+0.2+0.2) \( V_{\text{module}} \) = 0.6\( V_{\text{module}} \) for the first string, and (0.4+0.4+0.4) \( V_{\text{module}} \) = 1.2\( V_{\text{module}} \) for the second string. The total maximum power output is 1.8\( V_{\text{module}} \). From this analysis, the maximum possible power generation is equal to the \( \max\{1.6V_{\text{module}}, 2.6V_{\text{module}}, 1.8V_{\text{module}}\} = 2.6 V_{\text{module}} \). Now re-arrange these maximum short-circuit currents as follows:

\[
\beta_1 = 0.9 > \beta_2 = 0.8 > \beta_3 = 0.7 > \beta_4 = 0.5 > \beta_5 = 0.4 > \beta_6 = 0.2
\]

The power generation of the rearranged PV modules can be maximized when there are 1, 2 or 3 modules generating electricity in each string (\( \alpha \) is unknown).

If \( \alpha = 1 \), there is one module generating electricity in a string, then the rearrangement (0.9 pu, 0.7 pu, 0.4 pu; 0.8 pu, 0.5 pu, 0.2 pu) can ensure the maximum power generation. The corresponding maximum power is \( \beta_1 V_{\text{module}} \) from the first string and \( \beta_2 V_{\text{module}} \) from the second string, thus the total power output is \( (\beta_1 + \beta_2) V_{\text{module}} = 1.7 V_{\text{module}} \). This explains Eq. (24).

If \( \alpha = 2 \), there are two modules generating electricity in each string, the rearrangement (0.9 pu, 0.8 pu, 0.4 pu; 0.7 pu, 0.5 pu, 0.2 pu) will produce the maximum power. The maximum power is calculated by \( (\beta_2 + \beta_3) V_{\text{module}} = 2\beta_2 V_{\text{module}} \) for the first string and \( (\beta_4 + \beta_5) V_{\text{module}} = 2\beta_4 V_{\text{module}} \) for the second string. The total power is \( 2(\beta_2 + \beta_3) V_{\text{module}} = 2.6 V_{\text{module}} \). This explains Eq. (25).

If \( \alpha = 3 \), all the three modules in a string generate electricity, and the rearrangement (0.9 pu, 0.8 pu, 0.7 pu; 0.5 pu, 0.4 pu, 0.2 pu) will generate the maximum power. The maximum power is calculated by \( (\beta_3 + \beta_4 + \beta_5) V_{\text{module}} = 3\beta_3 V_{\text{module}} \) for the first string and \( (\beta_6 + \beta_5 + \beta_6) V_{\text{module}} = 3\beta_6 V_{\text{module}} \). The total maximum
power is $3(\beta_3 + \beta_6)V_{\text{module}} = 2.7V_{\text{module}}$. This explains Eq. (27). Clearly, this maximum power is greater than that for the unarranged arrays ($2.6V_{\text{module}}$).

Now the general proposition can be proved by applying mathematical induction to $\alpha$. The proof for $\alpha = 1$ is easy and now consider the case to deduce the statement for $\alpha = 2$ from $\alpha = 1$, while the general proof is omitted as it is a simple repetition of this proof for $\alpha = 2$. In fact, for $\alpha = 2$, we can assume the maximum short-circuit currents of the two un-bypassed PV modules in the $l$-th string are $\gamma_1^l$ and $\gamma_2^l, l = 1, 2, \ldots, p$. Without loss of generality, we can further assume that $\gamma_1^1 > \gamma_2^1 > \gamma_1^2 > \gamma_2^2 > \cdots > \gamma_1^p > \gamma_2^p$. Then the maximum power generated, denoted by $\bar{P}^\text{max}_2$, is

$$\bar{P}^\text{max}_2 = 2(\gamma_1^2 + \gamma_2^2 + \gamma_3^2 + \cdots + \gamma_p^2)V_{\text{module}}$$ (28)

By definition of $\beta_1, \beta_2, \ldots, \beta_{ps}$ in (23), $\beta_2$ is the second largest maximum short-circuit current within these $ps$ modules. While $\gamma_2^1$ is not the largest PV module maximum short-circuit current as there is $\gamma_1^1$ which is greater than $\gamma_2^1$. Therefore, $\beta_2 \geq \gamma_2^1$. Similar reasoning deduces that $\beta_4 \geq \gamma_2^2, \ldots, \beta_{2p} \geq \gamma_2^p$. $\bar{P}^\text{max}_2 = 2(\beta_2 + \beta_4 + \beta_6 + \cdots + \beta_{2(p-1)} + \beta_{2p})V_{\text{module}} \geq 2(\gamma_2^1 + \gamma_2^2 + \gamma_2^3 + \cdots + \gamma_2^p)V_{\text{module}}$. Then, $\bar{P}^\text{max}_2$ is the maximum possible power output for $\alpha = 2$.

B. PV Module Rearrangement Algorithm

Assume the maximum short-circuit currents of all the PV modules are given by $\{i_{\text{module},j,k}: j = 1, \ldots, p; k = 1, \ldots, s\}$, and it is re-arranged from the highest to lowest as in Eq. (23) through four steps.

**Step 1:** Calculate $P_1^\text{max}, P_2^\text{max}, P_3^\text{max}, \ldots, P_s^\text{max}$ from Eq. (25).

**Step 2.** Find the maximum from $\{P_1^\text{max}, P_2^\text{max}, P_3^\text{max}, \ldots, P_s^\text{max}\}$ and define the maximum as $P_s^\text{max}$, where $s^*$ is an integer from $\{1, 2, 3, \ldots, s\}$ so that $P_s^\text{max} = \max(P_1^\text{max}, P_2^\text{max}, P_3^\text{max}, \ldots, P_s^\text{max})$.

This implies that each PV string has $s^*$ non-bypassed modules (generating electricity) when the maximum power output of the PV array is achieved.

**Step 3.** Rearrange the PV modules as follows:

**3.1** Group the modules with maximum short-circuit currents $\beta_1, \beta_2, \ldots, \beta_{s^*}$ in the first PV string.

**3.2** Group the modules with maximum short-circuit currents $\beta_{s^*+1}, \beta_{s^*+2}, \ldots, \beta_{2s^*}$ in the second PV string.

**3.3** Group the modules with maximum short-circuit currents $\beta_{2s^*+1}, \beta_{2s^*+2}, \ldots, \beta_{3s^*}$ in the third PV string which is different to 3.1 and 3.2.

**3.4** Repeat the above procedure to place modules with maximum short-circuit currents $(\beta_{3s^*+1}, \beta_{3s^*+2}, \ldots, \beta_{4s^*}), (\beta_{4s^*+1}, \beta_{4s^*+2}, \ldots, \beta_{5s^*}), \ldots, (\beta_{(p-1)s^*+1}, \beta_{(p-1)s^*+2}, \ldots, \beta_{ps^*})$, and ensure that each of these $(\beta_{js^*+1}, \beta_{js^*+2}, \ldots, \beta_{(j+1)s^*})$, $3 \leq j \leq p - 1$, must occupy a different string.
3.4) Place the remaining \((ps - ps^*)\) PV modules arbitrarily in the remaining places of the \(p\) strings. Note that each string has \((s - s^*)\) unoccupied places to accommodate PV modules. Therefore, there are \((ps - ps^*)\) remaining places in these \(p\) strings.

This algorithm can be illustrated by the following flow chart.

**Fig. 6 Flow chart of the PV module reconfiguration strategy.**

V. **ANALYTICAL STUDIES**

The developed PV array reconfiguration is studied in case studies for simplified non-uniform aging and general non-uniform aging.

A. Simplified Non-Uniform Aging Cases

A \(2 \times 2\) PV array is employed in case studies where the maximum short-circuit current of each PV module is given in per unit (pu). The specifications of the PV modules are tabulated in Table I. The healthy PV module has the maximum short current, which is marked as 1 pu. The PV array aging condition can be expressed as 1 pu, 0.5 pu; 0.2 pu, 0.1pu to represent the conditions from healthy to aged. When PV array connected as (1 pu, 0.1 pu; 0.5 pu, 0.2 pu), their output curve is shown in Fig. 7.

**TABLE I Specifications of the PV Module**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-circuit voltage</td>
<td>44.8 V</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Short-circuit current</td>
<td>5.29 A</td>
</tr>
<tr>
<td>Power output</td>
<td>180 W</td>
</tr>
<tr>
<td>MPP current</td>
<td>5 A</td>
</tr>
<tr>
<td>MPP voltage</td>
<td>36 V</td>
</tr>
<tr>
<td>Current temperature coefficient</td>
<td>0.037%/K</td>
</tr>
<tr>
<td>Voltage temperature coefficient</td>
<td>–0.34%/K</td>
</tr>
<tr>
<td>Power temperature coefficient</td>
<td>–0.48%/K</td>
</tr>
<tr>
<td>Nominal operating cell temperature</td>
<td>46±2°C</td>
</tr>
</tbody>
</table>

Fig. 7 The output characteristics without the rearrangement (1 pu, 0.1 pu; 0.5 pu, 0.2 pu).

Following Step 1 of the reconfiguration algorithm, \( p = s = 2 \), these the maximum short-circuit currents can be re-ordered as: \( \beta_1 = 1 > \beta_2 = 0.5 > \beta_3 = 0.2 > \beta_4 = 0.1 \). Therefore, the maximum power output is

\[
\text{max}((1 + 0.5)V_{\text{module}}(\text{pu}), 2(0.5 + 0.1)V_{\text{module}}(\text{pu})) = 1.5 V_{\text{module}}(\text{pu})
\]

This maximum power is achieved by choosing only one module from each string for electricity generation, i.e., \( s^* = 1 \). The existing sequence of the in Fig. 7, PV modules in this PV can generate this maximum power. Therefore, there is no need for rearrangement.

Note that there are only three options for rearrangement: (1 pu, 0.1 pu; 0.5 pu, 0.2 pu), (1 pu, 0.5 pu; 0.1 pu, 0.2 pu) and (1 pu, 0.2 pu; 0.1 pu, 0.5 pu) where the notation \( (a, b; c, d) \) indicates that the two modules \( (a \text{ and } b) \) with the maximum short-circuit currents are placed in one string, and the other two modules \( (c \text{ and } d) \) are in another string. These are simulated in Fig. 10(a) and (b). It is clear that the arrangements (1 pu, 0.1 pu; 0.5 pu, 0.2 pu) and (1 pu, 0.2 pu; 0.1 pu, 0.5 pu) provide the identical maximum power (224 W) while the arrangement (1 pu, 0.5 pu; 0.1 pu, 0.2 pu) has the maximum power of 207 W. The arrangement (1 pu, 0.2 pu; 0.1 pu, 0.5 pu) has also the maximum power \( 1.5 \text{ pu} V_{\text{module}} \). Obviously, the output powers in Fig. 7 and Fig. 8(b) are both 224 W, suggesting a good agreement between the analytical and simulation results.
Fig. 8 Output characteristics with two arrangement options (case 1).

The second case is for the $2 \times 2$ PV array with the aging parameters of (1 pu, 0.3 pu; 0.5 pu, 0.4 pu). The output power is obtained by simulation and presented in Fig. 9.

From Step 1 of the reconfiguration algorithm, $p = s = 2$, these maximum short-circuit currents can be re-ordered as: $\beta_1 = 1 > \beta_2 = 0.5 > \beta_3 = 0.4 > \beta_4 = 0.3$. The maximum power is given by $P_{s^*} = \max\{(1 + 0.5)V_{\text{module}}(\text{pu}), 2(0.5 + 0.3)V_{\text{module}}(\text{pu})\} = 1.6V_{\text{module}}(\text{pu})$. That is, $s^* = 2$ and all the modules must generate electricity. From Step 3.1, the two modules with maximum short-circuit currents $\beta_1 = 1$ and $\beta_2 = 0.5$ are placed in one string while the other two modules with $\beta_3 = 0.4$ and $\beta_4 = 0.3$ are in another string. Therefore, the maximum power output can be achieved by the arrangement option (1 pu, 0.5 pu; 0.4 pu, 0.3 pu).
Similar to case 1, there are three possible options in case 2: (1 pu, 0.3 pu; 0.5 pu, 0.4 pu), (1 pu, 0.4 pu; 0.5 pu, 0.3 pu), and (1 pu, 0.5 pu; 0.4 pu, 0.3 pu). It can be seen from Figs. 9 and 10 that the maximum power output from the three rearrangements are 238 W, 244 W, and 273 W, respectively. Therefore, the rearranged PV array can gain 35 W more power than the original PV array configuration.

From the two case studies, the proposed rearrangement strategy can effectively improve the output power of non-uniformly aged PV arrays. Furthermore, in the process of the rearrangement, the MPP voltage area can be located which assists in the online maximum power point tracking (MPPT). Taking case 1 for example, the global MPP is located in the MPP area of one module. In case 2, the global MPP is located in the MPP area of two modules while the exact global MPP voltage is determined by the module temperature.

![Graphs](image)

(a) Option (1 pu, 0.4 pu; 0.5 pu, 0.3 pu). (b) Option (1 pu, 0.5 pu; 0.4 pu, 0.3 pu)

Fig. 10 Output characteristics with two arrangement options (case 2).

B. General Non-Uniform Aging Cases

For general non-uniform aging modules, it is very difficult to obtain any results similar to the obtained proposition. Consider a $p \times s$ PV array with 3 cell-units in each PV module. The total number of possible arrangements of the PV modules is $\left(\binom{ps}{s}\binom{ps-s}{s}\binom{ps-2s}{s}\cdots\binom{2s}{s}\right)/p!$, which is a huge number when $p$ or $s$ is big. For example, when $p = 5, s = 10$, this number equals $4.0279 \times 10^{29}$. Therefore, it is very difficult to calculate the maximum power for all the possible PV module arrangements for large $p$ or $s$ by enumerative search. Algorithms from combinatorial optimization (e.g. branch and bound methods) can be applied to search for the optimal maximal power when the number of possible rearrangements is huge.

Table II presents an example of a $3 \times 3$ array with the general non-uniform aging PV array with 3 cell-units in each PV module. For this PV array, there are $\binom{3}{3} \binom{3}{3} \binom{3}{3} / 3! = 280$ possible rearrangement options. Assume the PV modules are arranged as in Table II where the maximum short-circuit currents of 3 cell-units
in a PV module are put in a pair of parentheses. For instance, the option (0.9 pu, 0.8 pu, 0.7 pu) indicates the maximum short-circuit currents of the 3 cell-units in the first PV module.

Assuming the output voltage to be fixed at $aV_{cu}$, $a=1, 2, \ldots, 9$. In this case, the maximum power output can be calculated by rating the maximum power output of all possible $a$. By doing so, the maximum power is found to be 10.5pu$V_{cu}$ at $a=7$.

Table III illustrates alternative PV module rearrangements for the maximum power output out of possible 280 options. The global maximum power is $12V_{cu}(pu)$ when the voltage is $8V_{cu}$. Compared to the original maximum power ($10.5V_{cu}(pu)$), this arrangement has improved by 12.5% in power output.

### Table II The 3x3 Array Before Rearrangement

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.9 pu, 0.8 pu, 0.7 pu]</td>
<td>[0.9 pu, 0.9 pu, 0.6 pu]</td>
</tr>
<tr>
<td>[0.7 pu, 0.6 pu, 0.6 pu]</td>
<td>[0.9 pu, 0.5 pu, 0.4 pu]</td>
</tr>
<tr>
<td>[0.8 pu, 0.7 pu, 0.5 pu]</td>
<td>[0.9 pu, 0.5 pu, 0.4 pu]</td>
</tr>
</tbody>
</table>

### Table III The 3x3 Array After Rearrangement

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.9 pu, 0.8 pu, 0.7 pu]</td>
<td>[0.8 pu, 0.7 pu, 0.5 pu]</td>
</tr>
<tr>
<td>[0.9 pu, 0.9 pu, 0.6 pu]</td>
<td>[0.7 pu, 0.6 pu, 0.6 pu]</td>
</tr>
<tr>
<td>[0.8 pu, 0.5 pu, 0.4 pu]</td>
<td>[0.9 pu, 0.5 pu, 0.4 pu]</td>
</tr>
</tbody>
</table>

C. Optimal PV Rearrangement under Converter Input Voltage Limit

Due to the limitation of inverter operations while PV arrays are connected to the grid, the minimum bus voltage of a single phase inverter should be higher than 311V (220V/50Hz); and the minimum bus voltage of three phase inverter should be higher than 538V (380V/50Hz). The corresponding PV array operation points must be higher than the minimum bus voltage in a single stage converter. Therefore, the working voltage limit is introduced to the PV module reconfiguration strategy. The Proposition and algorithm in Section IV.B can be revised as follows to cater for this voltage limit. In fact, assume that a converter input voltage limit requires the input voltage to be $\theta V_{module}$ at least. Then it is straightforward that the maximum possible power output is: $\max \{P_i^{\text{max}}: s \geq i \geq \theta\}$.

And the searching algorithm in Section IV.B only needs to search for those $P_i^{\text{max}}$ with $i \geq \theta$. 
VI. IMPLEMENTATION AND EXPERIMENTAL VALIDATION

In order to validate the proposed strategy, a 9 kW 5×10 array under a non-uniform aging condition is used for simulation and experimental tests.

A. Simulation

Case 1:

A PV array model is built in Matlab. The per-unit maximum short-circuit current for each PV module in the 5 × 10 PV array (p = 5, s = 10) is tabulated in Table IV. The corresponding output characteristics are calculated and presented in Fig. 11. Without a rearrangement, the maximum output power is 4587 W.

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
</tr>
<tr>
<td>0.3 pu</td>
<td>0.6 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.9 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.6 pu</td>
</tr>
<tr>
<td>0.9 pu</td>
<td>0.9 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.9 pu</td>
</tr>
<tr>
<td>0.6 pu</td>
<td>0.9 pu</td>
</tr>
<tr>
<td>0.7 pu</td>
<td>0.9 pu</td>
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<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
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<tr>
<td>0.7 pu</td>
<td>0.8 pu</td>
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<td>0.8 pu</td>
<td>0.7 pu</td>
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<td>0.8 pu</td>
<td>0.8 pu</td>
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<td>0.7 pu</td>
<td>0.8 pu</td>
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<td>0.7 pu</td>
<td>0.8 pu</td>
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<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
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<tr>
<td>0.8 pu</td>
<td>0.7 pu</td>
</tr>
<tr>
<td>0.9 pu</td>
<td>0.6 pu</td>
</tr>
<tr>
<td>0.5 pu</td>
<td>0.7 pu</td>
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<tr>
<td>0.7 pu</td>
<td>0.7 pu</td>
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<tr>
<td>0.7 pu</td>
<td>0.7 pu</td>
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<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.7 pu</td>
</tr>
<tr>
<td>0.9 pu</td>
<td>0.6 pu</td>
</tr>
<tr>
<td>0.5 pu</td>
<td>0.4 pu</td>
</tr>
<tr>
<td>0.4 pu</td>
<td>0.4 pu</td>
</tr>
<tr>
<td>0.7 pu</td>
<td>0.7 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
</tr>
<tr>
<td>0.8 pu</td>
<td>0.8 pu</td>
</tr>
<tr>
<td>0.7 pu</td>
<td>0.9 pu</td>
</tr>
<tr>
<td>0.5 pu</td>
<td>0.5 pu</td>
</tr>
<tr>
<td>0.4 pu</td>
<td>0.4 pu</td>
</tr>
<tr>
<td>0.9 pu</td>
<td>0.9 pu</td>
</tr>
</tbody>
</table>

Fig. 11 Output characteristics of the 5×10 array without the rearrangement (case 1).

In this PV array, there are \( \alpha \) modules generating electricity in each string, while the rest \((10 - \alpha)\) modules are bypassed by diodes. \( \alpha \) is between 1 and 10. It suffices to calculate the maximum power for each \( \alpha \) and then find the greatest power from the 10 calculations. Firstly, let us sort the maximum short-circuit currents for PV strings from largest to smallest, as in Table V.
TABLE V REARRANGED STRINGS IN CASE 1

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9 pu 0.9 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.6 pu 0.6 pu 0.3 pu</td>
</tr>
<tr>
<td>0.9 pu 0.9 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.7 pu 0.7 pu 0.4 pu</td>
<td></td>
</tr>
<tr>
<td>0.8 pu 0.8 pu 0.7 pu 0.7 pu 0.7 pu 0.6 pu 0.5 pu 0.5 pu</td>
<td></td>
</tr>
<tr>
<td>0.9 pu 0.8 pu 0.8 pu 0.8 pu 0.7 pu 0.7 pu 0.4 pu 0.4 pu</td>
<td></td>
</tr>
<tr>
<td>0.9 pu 0.9 pu 0.8 pu 0.8 pu 0.7 pu 0.7 pu 0.5 pu 0.4 pu</td>
<td></td>
</tr>
</tbody>
</table>

For $\alpha = 1$, the maximum power is given by

$$V_{module} \times (0.9 + 0.9 + 0.8 + 0.9 + 0.9) \text{ pu} = 4.4V_{module}(\text{pu})$$

(29)

For $\alpha = 2$, the maximum power is

$$V_{module} \times 2 \times (0.9 + 0.9 + 0.8 + 0.8 + 0.9) \text{ pu} = 8.6V_{module}(\text{pu})$$

(30)

Similarly, for $\alpha = 3, 4, ..., 10$, the maximum powers are calculated as: $12.3V_{module}(\text{pu})$, $15.6V_{module}(\text{pu})$, $19.5V_{module}(\text{pu})$, $22.8V_{module}(\text{pu})$, $25.9V_{module}(\text{pu})$, $26.4V_{module}(\text{pu})$, $24.3V_{module}(\text{pu})$, $20V_{module}(\text{pu})$. Therefore, the maximum power output is $26.4V_{module}$ when there are 7 PV modules in each PV string generating electricity.

Now consider the optimal PV module rearrangement. The maximum short-circuit currents are re-organized from the highest to lowest as follows:

- 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.8 pu 0.8 pu;
- 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu;
- 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.7 pu 0.7 pu;
- 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.6 pu;
- 0.6 pu 0.6 pu 0.5 pu 0.5 pu 0.5 pu 0.4 pu 0.4 pu 0.4 pu 0.3 pu.

According to Eq. (23), $\beta_1 = 0.9, \beta_2 = 0.9, ..., \beta_{50} = 0.3$. Following Steps 1 and 2 in the reconfiguration algorithm, the maximum power is now calculated by:

$$V_{module} \times \max\{4.5 \text{pu}, 8.8 \text{pu}, 12.6 \text{pu}, 16.8 \text{pu}, 20.5 \text{pu}, 24 \text{pu}, 28 \text{pu}, 30.4, 32.4, 32\} = 32.4V_{module}(\text{pu})$$

This corresponds to the case that the output voltage is $9V_{module}, s^* = 9$. There are 9 PV modules in each string which generate electricity. Given that the original maximum power output is only $26.4V_{module}(\text{pu})$, this re-arranged PV array can generate $32.4V_{module}(\text{pu})$. This is because those six modules are brought back to the generation side by the reorganization (see Table VI). The corresponding output characteristics are illustrated in Fig. 12. As can be seen that the maximum output power is 5242 W with the rearrangement, which is 655 W more than that without the rearrangement (4587 W). Obviously, its energy efficiency is improved by increasing 14.28% power generation.
Table VI Rearrangement of the 5×10 Array in Case 1

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.8 pu 0.4 pu</td>
</tr>
<tr>
<td></td>
<td>0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.4 pu</td>
</tr>
<tr>
<td></td>
<td>0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.4 pu</td>
</tr>
<tr>
<td></td>
<td>0.8 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.7 pu 0.4 pu</td>
</tr>
<tr>
<td></td>
<td>0.7 pu 0.7 pu 0.7 pu 0.6 pu 0.6 pu 0.6 pu 0.5 pu 0.5 pu 0.3 pu</td>
</tr>
</tbody>
</table>

Fig. 12 Output characteristics of the 5×10 array with the rearrangement (case 1).

Table VI is constructed by the rearrangement algorithm as follows. From Step 3.1, 9 modules with the maximum short-circuit currents (0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.9 pu 0.8 pu) are grouped in the first string. From Step 3.2, further 9 modules with the current of (0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu 0.8 pu) are placed in the second string. When j = 2, 3, 4, 27, their respective modules are put in the third, fourth and fifth strings following Step 3.4. Now, 45 PV modules are reorganized in the PV array, leaving 5 modules un-sorted. These 5 modules have the maximum short-circuit currents of 0.4 pu, 0.4 pu, 0.4 pu, 0.4 pu and 0.3 pu. Since each string has only one unoccupied place, the 5 modules can be arbitrarily placed to fill the gap, as instructed in Step 3.5. The remaining modules in each of the 5 strings is bypassed and become idle; they are not in operation. The bucket effect determines that all first 9 PV modules in each string with higher maximum short circuit currents are operational to generate power.

Case 2:

For a middle aged PV array, some modules are broken in the array; usually, the faulty modules are replaced by new modules. In this scenario, the typical 5 × 10 PV array is presented in Table VII, in which there are new modules with high performance scattered in the array. Due to the non-uniform of aging, the
corresponding output characteristics are calculated and presented in Fig. 13; the maximum output power is 1661W.

**Table VII The 5x10 PV Array without Rearrangement for Case 2**

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 pu 0.7 pu</td>
</tr>
<tr>
<td></td>
<td>1 pu 0.2 pu</td>
</tr>
<tr>
<td></td>
<td>1 pu 0.3 pu</td>
</tr>
<tr>
<td></td>
<td>1 pu 0.4 pu</td>
</tr>
<tr>
<td></td>
<td>1 pu 0.2 pu</td>
</tr>
<tr>
<td></td>
<td>1 pu 0.2 pu</td>
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<td>1 pu 0.2 pu</td>
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<td>1 pu 0.2 pu</td>
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<td>1 pu 0.2 pu</td>
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<tr>
<td></td>
<td>1 pu 0.2 pu</td>
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<tr>
<td></td>
<td>1 pu 0.2 pu</td>
</tr>
</tbody>
</table>

Following a similar procedure as the previous example, the maximum power output equals $10V_{\text{module}}$, which is achieved when all the 50 modules are activated to generate electricity. Now consider the optimal rearrangement. From the algorithm in Section IV.B, it is easy to find that the maximum power is $11.4 V_{\text{module}}$ (pu), which can be achieved by allowing 6 modules generating electricity in each PV string, and the corresponding I-V curves are presented in Fig.14.

**Table VIII The 5x10 PV Array with Rearrangement in Case 2**

<table>
<thead>
<tr>
<th>Row (string)</th>
<th>Column (module)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1 pu 1 pu</td>
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<tr>
<td></td>
<td>1 pu 1 pu</td>
</tr>
<tr>
<td></td>
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Fig. 13 Output characteristics of the 5x10 array without the rearrangement (case 2).
Fig. 14 Output characteristics of the 5×10 array with the rearrangement (case 2).

Case 3:
In a 7×10 PV array, there are 7 PV strings and 10 PV modules in each strings. Each PV module consists of 3 cell-units. The aging conditions for cells are not uniform, as shown in the color scale in Fig. 15(a). The maximum short-circuit current in a healthy cell-unit is set as 1 pu under the standard testing condition (STC) which is the 1,000 W/m² irradiance, AM 1.5 solar spectrum and 25°C module temperature. Without a rearrangement, the PV array has a typical output characteristic, as illustrated in Fig. 15(b). The maximum output power is 4217 W and PV array output voltage for the global MPP is 285 V. After the proposed array reconfiguration without setting a voltage limit (Fig. 15(c)), the maximum power is achieved at 4793 W and the PV array output voltage for the global MPP becomes 233 V, as shown in Fig. 15(d). The total output power is increased by 13.6% and the lower voltage limit is 275 V. When the minimal output voltage limit is set at 275 V for the reconfigured PV array, the maximum power is 4616 W and the voltage for global MPP is 279 V, as shown in Fig. 15(f). Compared with the original PV array, the rearranged PV power (with a voltage limit) increases by 9.5%, which is about 4% lower than the proposed array reconfiguration without a voltage limit. This is because that the voltage for the global MPP (233V) is outside of this voltage limit (>275V). The consequent output power with a voltage limit is lower than the case without a voltage limit.
(a) The aging conditions of the array before the rearrangement

(b) The array output before the rearrangement

(c) The array after the rearrangement (without a voltage limit)
Fig. 15 The application of the proposed reconfiguration to the 7×10 array.
B. Experimental Tests

In the experiment, a 1620 W 3×3 array is employed to verify the proposed technique based on the availability. The PV module parameters are identical to those in Table I. The aging condition is realized by covering the two modules (PV21 and PV31) with plastic membrane. The test results are given in Fig. 16. The array output characteristics are obtained and presented in Table VII and Fig. 16 before and after the rearrangement. For the PV array without the reconfiguration presented in Fig. 16(a), its maximum output power is 520 W, shown in Fig. 16(b). After applying the proposed strategy to the PV array, the PV array is rearranged (by swapping PV21 and PV32 positions), as shown in Fig.16(c). Experimental results show that the maximum output power in the rearranged array is 590 W, illustrated in Fig.16(d), which increases 13.5%. Furthermore, because of this rearrangement, the global MPP shifts from a two-module MPP area to a three-module MPP area, which can be directly used for the online global MPPT.

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<th>Component</th>
<th>Before rearrangement</th>
<th>After rearrangement</th>
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<tr>
<td>MPP voltage</td>
<td>67.6 V</td>
<td>100 V</td>
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<tr>
<td>MPP current</td>
<td>7.7 A</td>
<td>5.9 A</td>
</tr>
<tr>
<td>Power level ($P_{out}$)</td>
<td>520 W</td>
<td>590 W</td>
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</table>

(a) PV array without the rearrangement         (b) Output characteristics without the arrangement
Table VIII presents a comparison of the proposed reconfiguration strategy and existing online reconfiguration strategies in the literature [27][32][33]. For condition monitoring, the online reconfiguration methods require continuous monitoring that increases the system cost and computational burden while the proposed method only needs periodic monitoring (e.g. during maintenance). For PV cell reconfiguration, existing online reconfigurations strategies need a large number of relays (e.g. high costs and high-end controllers). For example, for a 10×10 array, in order to have a complete flexible reconfiguration, a relay...
between any two modules from the total of 100 modules is needed, that is, a total of $\binom{100}{2} = 100 \times 99/2 = 4950$ relays are needed; and one of relays fault may cause two modules disconnection that also makes the PV array output power decrease dramatically. More importantly, the number of relays used by existing online reconfiguration methods increases exponentially with the size of PV systems which limits their practical applications to small-scale PV arrays only. In the new offline reconfiguration algorithm proposed in this paper, it is simpler to compute and implement, and can be applied to any sizes of the systems without significant investment in hardware.

VII. CONCLUSION

Non-uniform aging of PV modules is a common phenomenon in the PV power plants since they often operate a long time in harsh environmental conditions. The non-uniform aging decreases the PV array maximum output power and can damage the PV modules if left untreated. Without rearranging non-uniformly aged PV arrays, typical online global-MPPT schemes can only track a compromised maximum rather than its potential maximum power.

This paper has presented a new PV array reconfiguration strategy to maximize the power generation of non-uniformly-aged PV arrays without replacing aged PV modules. It is found that the bucket effect is the key factor affecting the operating mechanisms of PV arrays under non-uniform aging conditions. The cell-unit structure of PV module is investigated to study the aging characteristics of PV modules. The corresponding mathematical model for non-uniform aging PV array is built. By investing the model, the optimized reconfiguration algorithm is developed to explore potential power generation of aging PV array. The proposed strategy has been tested by simulated models which consider three cases, including cases with and without working voltage limits, and non-uniform aging inside each PV modules. A 3×3 1620-W PV array experimental rig is built to validate the proposed method.

While the existing online reconfiguration methods require large amount of relays, auxiliary power supply, and high-end controllers, the proposed method only needs inexpensive equipment to perform periodic inspections of PV cells (during maintenance). Therefore, the developed technique can improve energy efficiency and cost efficiency of PV systems and can be applied to any size of PV power plants.

REFERENCES


