

Optimal design of long-span steel portal frames using fabricated beams

Ross McKinstry, James B.P. Lim, Tiku T. Tanyimboh, Duoc T. Phan, Wei Sha*

* Corresponding author

Ross McKinstry: SPACE, David Keir Building, Queen's University, Belfast, BT9 5AG, UK

Email: rmckinstry01@qub.ac.uk

James B.P. Lim: SPACE, David Keir Building, Queen's University, Belfast, BT9 5AG, UK. Email: j.lim@qub.ac.uk

Tiku T. Tanyimboh: Department of Civil and Environmental Engineering, University of Strathclyde, Glasgow, G1 1XJ, UK. Email: tiku.tanyimboh@strath.ac.uk

Duoc T. Phan: Department of Civil Engineering, Universiti Tunku Abdul Rahman, Kuala Lumpur, 53300, Malaysia. Email: phantd@utar.edu.my

Wei Sha: SPACE, David Keir Building, Queen's University, Belfast, BT9 5AG, UK. Email: w.sha@qub.ac.uk

Ross McKinstry, PhD student

James B.P. Lim: PhD, Lecturer

Tiku T. Tanyimboh: PhD, Senior Lecturer

Duoc T. Phan: PhD, Assistant Professor

Wei Sha: PhD, Professor

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Abstract

This paper considers the optimal design of fabricated steel beams for long-span portal frames. The design optimisation takes into account ultimate as well as serviceability limit states, adopting deflection limits recommended by the Steel Construction Institute (SCI). Results for three benchmark frames demonstrate the efficiency of the optimisation methodology. A genetic algorithm (GA) was used to optimise the dimensions of the plates used for the columns, rafters and haunches. Discrete decision variables were adopted for the thickness of the steel plates and continuous variables for the breadth and depth of the plates. Strategies were developed to enhance the performance of the GA including solution space reduction and a hybrid initial population half of which is derived using Latin hypercube sampling. The results show the proposed GA-based optimisation model generates optimal and near-optimal solutions consistently. A parametric study is then conducted on frames of different spans. A significant variation in weight between fabricated and conventional hot-rolled steel portal frames is shown; for a 50 m span frame, a 14-19% saving in weight was achieved. Furthermore, since Universal Beam sections in the UK come from a discrete section library, the results could also provide overall dimensions of other beams that could be more efficient for portal frames. Eurocode 3 was used for illustrative purposes; any alternative code of practice may be used.

Keywords: hot-rolled steel; fabricated beams; portal frames; genetic algorithms; serviceability limits; buckling limits

1 Introduction

In the UK, it is estimated that steel portal frames account for 90% of all single-storey buildings [1]. The vast majority of portal frames use hot-rolled steel sections for the column and rafter members. Using such sections, frames economically achieve spans of up to 50 m [2].

For longer span frames, an alternative to the use of hot-rolled steel sections could be fabricated steel beam sections [3], [4]. Such fabricated beams, built-up through the welding of steel plates, have become increasingly popular for multi-storey buildings, where clear spans of up to 100 m are achievable [1]. In this paper, the use of such fabricated beams for portal frames will be considered using a genetic algorithm (GA) to size the dimensions of the fabricated beams.

Genetic algorithms have previously been applied to the design optimisation of hot-rolled steel portal frames [5]–[8]. In these studies, only four design variables were used; namely, the cross-section sizes of the columns and rafters, and the length and depth of the eaves haunch [8]. The design used in this present paper was elastic. Phan et al. [8] showed that elastic design was sufficient since the design was controlled by deflection limits.

On the other hand, a design optimisation of fabricated steel sections can involve up to thirteen design variables (see Section 2.2); these being, the dimensions of the plates of each of the members as well as the dimensions of the haunch. To reduce the number of function evaluations, an effective means of enhancing the reliability is required.

Three benchmark frames are considered, with the frames designed elastically under gravity load in accordance with Eurocode 3. Both ultimate and serviceability limit states are considered. A parametric study is conducted to explore the full search space for single story steel buildings. Spans of 14 m to 50 m and eave heights varying from 4 m to 12 m were considered.

2 Benchmark frames

Three frames are considered:

Frame A: Span of 40 m and height of 10 m

Frame B: Span of 50 m and height of 12 m

Frame C: Span of 60 m and height of 12 m

The pitch and frame spacing for all three frames are 6° and 6 m, respectively; such a pitch and frame spacing are typical for portal frames in the UK [2]. The column bases are assumed to be pinned. It is also assumed that the steel sections are fabricated from S275 steel [9].

2.1 Portal frames composed of universal beams

Frames are generated by selecting universal beam sections for the column and rafters from a list of 80 standard sections given in the SCI “Steel building design: Design data” Book [10]. The column, rafter and haunch sections are considered as discrete variables with haunch length (H_L) treated as a continuous variable [8].

Four universal beam cases (UBC) are considered:

- UBC1 has two decision variables: the column and rafter sections. The haunch is assumed to be the same as the rafter section and haunch length is fixed at 10% of the span
- UBC2 has three decision variables: column, rafter and haunch sections. The haunch length is fixed at 10% of the span
- UBC3 also has three decision variables: column section, rafter section and haunch length. The haunch is assumed to be the same as the rafter section.
- UBC4 has four decision variables: column section, rafter section, haunch section and haunch length.

2.2 Portal frames composed of fabricated beams

Portal frames composed of fabricated beams are generated with the dimensions described below. The plate thickness is treated as discrete variables and used for the web and flange. 34 plate thicknesses available within the UK are considered; at 1 mm spacings 6-25 mm; 5 mm spacings 30-80 mm and individually 12.5, 28 and 63.5 mm. The depths and breadths of the sections are treated as continuous variables with a range of 110 mm to 2000 mm and 50 mm to 600 mm, respectively.

Three Fabricated Beam Cases (FBC) are considered as follows with 13 decision variables in total: h_C , h_R , h_H , b_C , b_R , b_H , t_{WC} , t_{WR} , t_{WH} , t_{fC} , t_{fR} , t_{fH} , and H_L . The notations use standard

(descriptive) terminology: h for height, b for breadth and t for thickness, including web (w in subscript) and flange (f in subscript) thicknesses; in subscript, C for column, R for rafter and H for haunch.

FBC1 is defined as follows: $h_C, h_R = h_H, b_C = b_R = b_H, t_{wC} = t_{wR} = t_{wH}, t_{fC} = t_{fR} = t_{fH}$, and H_L . FBC1 also has 13 decision variables that are further constrained as shown in the equations.

FBC2 has the following properties: $h_C, h_R = h_H, b_C, b_R = b_H, t_{wC}, t_{wR} = t_{wH}, t_{fC}, t_{fR} = t_{fH}$, and H_L . Using restrictions for FBC1 and FBC2 corresponds to the operational simplicity and possibly economy of using a smaller numbers of plate sizes.

FBC3 has the following properties: $h_C, h_R, h_H, b_C, b_R, b_H, t_{wC}, t_{wR}, t_{wH}, t_{fC}, t_{fR}, t_{fH}$, and H_L .

In addition, discussions with manufacturers of fabricated beams suggest that the following geometric constraints are required in order to ensure that the plates can be welded and handled practically on the fabrication shop floor:

- $h_C > 3t_{fC}; h_R > 3t_{fR}; h_H > 3t_{fH}$
- $t_{fC} > t_{wC}; t_{fR} > t_{wR}; t_{fH} > t_{wH}$.

3 Frame actions

In this paper, the permanent actions (G) and variable actions (Q) assumed to act on the frames are as follows:

$$G: \quad 0.55 \text{ kN/m}^2 + \text{self-weight of primary steel members}$$

$$Q: \quad 0.60 \text{ kN/m}^2$$

Under vertical load, the frame should be verified at the ultimate and serviceability limit where the deflection limits and actions combination as recommended by the SCI [8,11] are adopted. Variable and permanent actions are factored in accordance with Eurocode 3: Design of steel structures [12]:

$$ULS = 1.35G + 1.5Q$$

$$SLS1 = 1.0G + 1.0Q \quad (\text{for absolute deflection})$$

$$SLS2 = 1.0Q \quad (\text{for differential deflection relative to adjacent frame})$$

where,

ULS is the ultimate limit state

SLS is the serviceability limit state

4 Ultimate limit state design

4.1 Elastic frame analysis

Modern practice has shown that plastic design produces the most efficient designs in the majority of cases [2], [13]. Elastic design is still used, particularly when serviceability limit state deflections will control frame design [14], [15]. Phan et al. [8] have demonstrated that, if the SCI deflection limits are adopted, serviceability limit states control design. Therefore, elastic design is used in this paper.

A frame analysis program, written by the authors in MATLAB, was used for the purpose of the elastic frame analysis. The internal forces, namely, axial forces, shear forces, and bending moments can be calculated at any point within the frame. It should be noted that second-order effects are not considered, since the geometry in the benchmark frames satisfy the requirements for in-plane stability of the sway check method, described in BS 5950 [16].

4.2 Ultimate limit state design requirements

Structural members are designed to satisfy the requirements for local capacity in accordance with Eurocode 3 [17]. Specifically, members are verified for capacity under shear, axial, and moment, and combined moment and axial force. For fabricated beams, the buckling curves used are taken in accordance with the UK National Annex [18]. Sections are classified based on the axial and bending force in conjunction with their geometric properties as class 1, 2 or 3. For class 1 or 2 sections, a plastic design approach is used in verification. For class 3 sections an elastic verification is substituted in the design. Sections outside this range (class 4) are excluded through use of a GA penalty.

Local buckling verifications are excluded under the proviso that a more detailed design of any necessary web and flange stiffeners will be conducted on the optimum selected sections. For example the stiffeners are generally required in the eave connections to allow for the concentrated axial forces transference from the rafter to the column.

4.2.1 Shear capacity

The shear force, V_{Ed} , should not be greater than the shear capacity, $V_{c,Rd}$.

$$V_{Ed} \leq V_{c,Rd} \quad (1)$$

The shear capacity is given by:

$$V_{c,Rd} = \frac{A_v(f_y/\sqrt{3})}{\gamma_{M0}} \quad (2)$$

where

f_y is the yield stress of steel

A_v is the shear area

γ_{M0} is partial factor for resistance

$V_{c,Rd}$ is the shear design resistance for class 1, 2 and 3 sections

4.2.2 Axial capacity

The axial capacity should be verified to ensure that the axial force N_{Ed} does not exceed the axial capacity (N_{Rd}) of the member.

$$N_{Ed} \leq N_{c,Rd} \quad (3)$$

4.2.3 Moment capacity

The bending moment should not be larger than the moment capacity of the cross section, $M_{c,Rd}$.

$$M_{Ed} \leq M_{c,Rd} \quad (4)$$

where

M_{Ed} is the moment applied to the critical section

For class 1 or 2 sections $M_{c,Rd}$ is the plastic design resistance of the section, $M_{pl,Rd}$

For class 3 sections $M_{c,Rd}$ is the elastic design resistance of the section, $M_{el,Rd}$

When members are subject to both compression and bending, the moment capacity $M_{c,Rd}$ is reduced if the axial force is significant in accordance with clause 6.2.9.1 for class 1 or 2 sections and check 6.2.9.2 for class 3 within Eurocode 3 Part 1-1 [17].

4.2.4 Buckling

Buckling is verified using equations 6.61 and 6.62 of Eurocode 3 [17]. As the frame is under single axis bending the additional second axis bending terms can be removed resulting in equation 5 and 6 below [19].

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} - 1 \leq 0 \quad (5)$$

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{b,Rd}} - 1 \leq 0 \quad (6)$$

The equivalent uniform moment factors k_{zy} and k_{yy} interaction factors are calculated based on the Annex B method of Eurocode 3 [17]. Lateral restraint is provided by the side rails and purlins; torsional restraint is provided by the addition of stays to the bottom flange. Purlin and side rail spacing (L_{cr}) is generally controlled by the spanning capability of the cladding, normally 1 to 3 m depending on the loading. The rafter buckling is considered in zone C of Figure 1 and the column is verified equally at the column top (see Figure 2). In both cases a specified L_{cr} restraint spacing is used in the calculation of stability based unity factors. In this paper feasible cladding spans of 1.5 m and 3 m are considered. Elsewhere in the frame where Lateral Torsional Buckling (LTB) is not explicitly considered it is assumed that restraints are provided at the necessary spacing to provide adequate stability.

4.2.5 Shear Buckling

The shear buckling check has been considered and was found not to control the design, based on unstiffened webs with non-rigid end posts verified, in accordance with EN-1993-1-5:2006 [20], with transverse stiffeners provided at the support and at the connections. The

maximum observed reduction in shear force design capacity was 40%, with an average reduction of 26.5% for the FC2 cases.

5 Optimisation model

The objective of the overall design optimisation is to determine the portal frame with the minimum primary member steel material weight, whilst satisfying the design requirements. The weight of the frame depends on the cross-section sizes of members. The objective function is expressed in terms of the weight of the primary members per square metre of the floor area. The weight was calculated by summation of the volume of steel material throughout the frame using element lengths and cross-sectional areas.

The design constraints unity factors are as follows:

$$g_1 = \frac{V_{Ed}}{V_{c,Rd}} \leq 1 \quad (7a)$$

$$g_2 = \frac{N_{Ed}}{N_{Rd}} \leq 1 \quad (7b)$$

$$g_3 = \frac{M_{Ed}}{M_{c,Rd}} \leq 1 \quad (7c)$$

$$g_4 = \frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1 \quad (7d)$$

$$g_5 = \frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1 \quad (7e)$$

$$g_6 = \frac{\delta_e}{\delta_e^u} \leq 1 \quad (7f)$$

$$g_7 = \frac{\delta_a}{\delta_a^u} \leq 1 \quad (7g)$$

$$g_8 = \begin{cases} 1.2 & \text{if } 3t_f > h \\ 0 & \text{Otherwise} \end{cases} \quad (7h)$$

$$g_9 = \begin{cases} 1.2 & \text{if } t_w > t_f \\ 0 & \text{Otherwise} \end{cases} \quad (7i)$$

$$g_{10} = \begin{cases} 1.2 & \text{if class 4} \\ 0 & \text{Otherwise} \end{cases} \quad (7j)$$

where δ_e and δ_a = deflections at eaves and apex, respectively. The superscript u indicates the maximum permissible deflection. The constraints for ultimate limit state design are g_1 to g_5 while the serviceability limit state design constraints are g_6 and g_7 . Constraint g_1 is for shear capacity; g_2 is for axial capacity; g_3 is for combined axial and bending capacity; g_4 and g_5 are interaction of axial force and bending moment on buckling for major axis; and g_6 and g_7 are for horizontal and vertical deflection limits. If the geometrical cross-section constraints, g_8 , g_9 and g_{10} , is exceeded, an arbitrary (lowest tier) unity factor of 1.2 is assigned. These are verified throughout the frame and are further denoted to show the maximum value within a given zone: g_{xC} within the column; g_{xR} within the rafter; g_{xH} within the haunch.

6 Optimisation methodology

The design optimisation considered in this paper contains mixed discrete and continuous design variables. This was implemented using a genetic algorithm within the optimisation toolbox in MATLAB. In order to consider discrete and continuous design variables, special crossover and mutation functions enforce variables to be integers [21]. One of the benefits of a real coded genetic algorithm is that genetic operators are directly applied to the design variables without coding and decoding as with binary string GAs. As demonstrated by Deb [22] real coded GA is appropriate for optimisation problems having continuous design variables. This allowed for the representation of the frame using realistic parameters that can be used directly in design.

It has been identified that genetic algorithms sometimes prematurely converge to a local optimum solution due to the domination of superior solutions in the current population [23], [24]. It has also been observed that using the real coded genetic algorithms, large population sizes are needed in order to obtain the optimum solution consistently [25].

6.1 Genetic algorithm configuration

Two techniques were explored in order to improve reliability and speed of convergence both applied prior to starting the GA. The first method is Search Space Reduction (SSR). Impressive reductions in GA execution times have been demonstrated recently. For example, Kadu et al. [26] used heuristics to limit the GA search to a region of the solution space in which near optimal solutions were thought to exist. Known constraint violations were used to maximise the proportion of feasible solutions even with geometric changes. This was done by initially calculating the frame bending moments using the maximum available sections then eliminating sections with significant lower moment capacities for both the column and rafter independently. This would reduce the number of sections considered from 80 to approximately 30.

The second method is Improved Initial population (IIP) where the size of the initial population was larger than subsequent generations ensuring that several viable solutions were included in the first generation. The initial population was pre-generated using a Latin hypercube (LH) sampling plan. Two sizes of initial populations 6 and 12 times the standard GA population size were considered. The larger initial population was then ranked based on fitness value. The GA's second generation was then generated with a population created using 50% from the larger initial population's best solutions and the rest randomly throughout the available search space by the GA.

The optimisation was conducted on a workstation (2.53 GHz CPU, 4GB RAM). Quoted computational times and function evaluation totals include the effort required in the phases before moving to the genetic algorithm. An outline of the GA stages is presented in Figure 3. Elite values are preserved 4 or 8 individuals, 4 for small populations and 8 for populations in excess of 150 individuals. Selection is conducted using roulette selection.

Mutation of the variables was performed using an adaptive feasible approach where mutation direction is based on the last successful or unsuccessful generation and the values are kept within the bounds of the optimisation. Selected parents are then combined using a scattered crossover operation where variables are converted to binary and the random bits from each are combined [27].

The optimisation was stopped when the maximum number of generations is reached, set at 100-250 generations. Additionally the optimisation was also terminated by convergence defined as 50 generations with no improvement; a function tolerance of 10^{-6} was specified. The maximum number of generations required generally was less than the maximum with almost all optimisations stops attributed to either the convergence criteria.

6.2 Fitness and penalty functions

The objective of the optimisation is the minimum weight of the primary members of the portal frame. This includes the weight of the column, rafter and haunch elements. For the haunch sections, this weight includes any wastage in material as a result of the fabrication. Figure 4 shows details of the cutting required for the section fabrication, with the white area representing wasted material, and h_H the additional depth achieved once the section has been welded onto the rafter. The haunch sections are cut within the web outside the root radius, realistically reflecting fabrication cutting conditions.

To solve the optimisation problem, a penalty based approach was added to the calculated frame weight (F_{weight}), see Equation 8 to obtain the optimisation fitness value (F_p). Two levels of violation were considered, the standard violation penalty (P) was a weight equivalent to the heaviest available configuration used to eliminate moderate violations (maximum constraint value under 1.5). For higher violations a penalty ten times the standard penalty value was added. The maximum value of any individual unity factor (g_M) outlined in Section 5 was used to determine the extent of any violation. It was observed that high penalty values were required in order to eliminate very infeasible solutions. Otherwise the infeasible solutions would undercut feasible solutions.

$$F_p = \begin{cases} F_{\text{weight}} & \text{if } g_M \leq 1 \\ F_{\text{weight}} + P & \text{if } 1 < g_M \leq 1.5 \\ F_{\text{weight}} + 10P & \text{if } g_M > 1.5 \end{cases} \quad (8)$$

7 Results and discussion

7.1 Reliability study

For different numbers of design variables (based on optimisation case) different global population sizes were investigated using a population based on a multiplier of 5, 10, 15, 20, 25 and 30 times the number of design variables with each optimisation run 30 times.

A total of 6 optimisation control strategies variations (see Table 1 and Table 2) were used, for 7 different sets of optimisation design variables consisting of 4 Universal Beam Cases (UBC) and 3 Fabricated Beam Cases (FBC). Table 1 includes the standard deviation, an indication of the reliability of the optimisation for different GA configurations. Table 2 shows the required number of function evaluations for a given configuration to achieve the reliability shown in Table 1. This totalled 252 variations per frame considered each run 30 times. A representative result is presented in Table 1 and Table 2 for Frame A using UBC3. All of the optimisation cases (UBC1, UBC2...) produced comparable trends when switching between optimisation control strategies. It is observed that for both search space reduction and improved initial population, the standard deviation for small population multipliers is reduced significantly. As the computational effort required (in terms of mean function evaluations) increases, the standard deviation reduces to as low as 0.28 kg/m^2 .

An example GA progress against function evaluations is shown in Figure 5 comparing the performance of the standard GA with the enhancement methods SSR and IIP. Achieving the global optimum for $\times 15$ multiplier is rare without SSR or IIP. However it would appear that local optimums are being found in the very near vicinity of the global optimum. From a designer's perspective, the additional effort required to achieve the global optimum, outweighs the time required to run the simulation. Using SSR only results in a faster convergence using less function evaluations due to a lower initial starting. Combining SSR with an IIP of 12 times the standard population results in the GA starting from an even lower starting fitness position but used more function evaluations. The increased number of function evaluations was only marginal, with lower fitness more consistently found; it is therefore useful for increasing reliability without disproportionately increasing optimisation time. Similar observations can be seen in Table 2 where more results are presented for comparison.

The use of SSR allows for faster optimum convergence reducing function evaluations with the GA having a better starting position. IIP($\times 12$) has similar but less obvious benefits than simply increasing the global population multiplier. Increasing the global population multiplier significantly increases the number of function evaluations due to the stopping criteria in many cases being based on the number of stall generations. This will lead to higher numbers of individuals in each of the stall generations making it less efficient than using a combined SSR and IIP($\times 12$) approach which only has a large population in the first generation. It can also be seen IIP($\times 12$) also has a similar effect of SSR where the first generation will have a better starting position leading to a faster convergence.

7.2 Recommended genetic algorithm configuration

From reviewing the 30 run study a population multiplier of 15 seems to have an effective balance of reliability and computational effort. To further reduce the required computational effort it was necessary to decrease the number of GA iterations per optimisation run. As both reduction strategies allowed for lower optimum variation with reduced computational effort it is recommended that both search space reduction (where available) and an initial generation population multiplier of 12 is used (see bold numbers in Table 1 and Table 2). In order to check that the reliability was not unacceptably adversely affected a comparison is made between 10 and 30 runs (see Table 3).

The mean weight variation change from 30 GA runs to 10 runs is small indicating that 10 runs are sufficient for optimisation purposes. In general the optimum mean weight variation is below 2% of the optimum weight using 10 runs. The exception is FBC3, where a higher global population multiplier may be justified due to the larger number of interacting variables. The efficiency could be further improved by reducing the stall generation stopping constraint from 50 generations to 30, eliminating unnecessary function evaluations. This was not implemented in this study as it would likely permit for faster convergence but with sacrificed reliability.

It is therefore recommended that this configuration is used for fast reliable results with all beam cases, with the exclusion of FBC3 where higher variations may be observed. It will be shown in Section 7.5 that case FBC3 is not recommended. Therefore optimising the GA configuration for FBC3 will not be explored further.

7.3 Comparison of results against previously reported in literature

In order to validate the proposed modelling and optimisation method, a comparison is made against results previously published in the literature. It should be noted that all the examples in the literature are based on conventional hot-rolled steel portal frames. As each author calculated frame weight differently due to different geometric assumptions, a direct weight comparison is therefore not possible; instead the optimum chosen sections are compared.

In these comparisons it is assumed that the haunch would be fabricated i.e. the length and breadth of their haunch could be of any size. In practice the haunch is usually cut from a universal beam section (Figure 4) and this section is usually the same section as used for the rafter. The cut shown in Figure 4 is practical but the previous authors did not do this realistically. For the purposes of the comparisons described in this section, optimisation case UBC3 is considered; it should be noted that UBC1 and 2 produce similar weights.

7.3.1 Issa and Mohammad

The benchmark frame adopted by Issa & Mohammad [6], as presented by Saka [5], is optimised. The frame considered is of span of 20 m, height to eaves of 5 m and pitch of 8.53° . The load acting on the frame is 4 kN/m vertical UDL acting globally on the rafter.

Issa and Mohammad adopted deflection limits of $h_f/300$ and $L_f/360$ for the eaves and apex, respectively, which are more stringent than the SCI deflection limits. It should be noted that the deflection limits by Issa and Mohammad were verified at the same actions as the member verification, i.e. no distinction was made for the actions between ultimate limit state design and serviceability state for deflection verification. It should also be noted that although the optimisation conducted by Issa and Mohammad included member buckling effects, they reported in their paper that deflections limits governed this particular benchmark design; the frame is therefore suitable for use as a benchmark for comparison.

Table 4 shows the results of the optimisation. It can be seen that the section sizes for the column and rafter are the same as that of Phan et al. [8]. However, H_L is longer than that of Phan et al., since the haunch is assumed to be cut from the same section as the rafter.

7.3.2 SCI frame

The benchmark frame adopted by Phan et al. [8], based on an SCI worked example [2], is also compared against the developed model. The frame geometry had a span of 30 m, height to eaves of 7 m and pitch of 6°. The load acting on the frame is 11.3 kN/m vertical UDL, comprised of 0.66 and 0.6 kN/m² for permanent and variable actions, respectively, with a frame spacing of 6 m. For the comparison, the loading factors and constraints were altered to reflect the conditions reported. The SCI deflection limits were adopted.

The results of optimisation in comparison to Phan et al. are presented in Table 4. For UBC3 the same sections were selected as Phan et al. However, a longer H_L was required. The cut haunch used in UBC3 is more realistic given its prevalence in the industry but is still comparable to Phan et al. results. The same primary members were selected under similar loading conditions and the total steel volumes within the haunches are similar.

7.4 Portal frames composed of universal beams

In this section, the optimisation benchmark frames described in Section 2 are optimised using the recommended GA configuration from Section 7.2. It should be noted that Frame C span of 60 m was observed to be outside the range of UB based portal frames and therefore was only optimised using fabricated beams.

The results of the universal beam optimisation are presented in Table 5. As the number of design variables increase for both Frames A and B, the primary structural weight reduces. In all cases, Eurocode capacity based moment verification dominates over others (shear and axial based). The deflection at the apex is binding. Using universal beam sections LTB was verified at both 1.5 m and 2.5 m restraint spacings (L_{cr}). For Frames A and B, LTB was found not to be design controlling producing the same optimum results for both considered lengths.

The largest weight savings were found to be achieved by including the haunch section as a design variable within the GA. However, the sections selected tend to be significantly different than the rafter and column sections.

7.5 Portal frames composed of fabricated beams

Table 6 and Table 7 show the results of the portal frame optimisation composed of fabricated beams with restraint spacings (L_{cr}) of 1.5 m and 2.5 m, respectively. For ease of

comparison, the dimensions of the fabricated beams are shown using the same designation as for standard universal beams i.e. depth × breadth × weight (in kg/m).

It can be observed that the dimensions of the fabricated beams are controlled by both deflections and buckling stability. In general the depth of the section is controlled by the deflection of the frame while the breadth controls the stability. Comparing Table 6 and Table 7, increasing the restraint spacing (L_{cr}) from 1.5 m to 2.5 m results in an average optimum frame weight increase of 2.6%. This increase in optimum frame weight is due to the additional steel needed to provide stability to the section.

The most proficient GA decision variable configuration was found to be FBC2 where the haunch section is based on the rafter. When the haunch was allowed to vary independently from the rafter (FBC3) the haunch could be very shallow or significantly out of proportion with the rafter. This would produce the lightest frame weight but would make the fabrication and joint detailing very difficult. A much better option is a configuration where the haunch is based on the rafter (FBC2) resulting in a frame that can be fabricated more easily, with a minimal frame weight increase of 1-2% over the more optimum weight of FBC3.

7.6 Observations

In all cases, portal frames composed of fabricated beams are lighter than those using universal beams. Figure 6 compares the cross-section of Frame A with UBC3 and FBC2 drawn to scale side by side. It can be seen that the fabricated beams tend to be deeper generating the required moment capacity while sacrificing the inherent stability generally found in a rolled section by increasing the depth and reducing the breadth.

The change in sectional weight is presented in Figure 7 for Frames A, B and C for all optimisation cases (UBC1,UBC2...). Frame weights are separated into the component weights of the column, rafter and haunch. It can be seen going between optimisation cases that individual section weights do not always decrease. In some cases heavier column sections result in lighter frames overall. Additionally the weight contribution of the haunch is very small in comparison to the rest of the frame, making it an effective way of reducing deflections without adding unnecessary weight to the frame.

If lateral restraint constraints are omitted for universal beams frames in the vast majority of cases the optimum configuration will not change. This is due to the inherent stability generated from their flanges this is significantly different for the fabricated beams. Without the need for stability the optimum shape for cross-sectional area is a thin rectangle due to the more efficient generation of second moment of area. This is also true for both elastic and plastic modulus resulting in very unstable unfeasibly thin rectangular sections. The inclusion of lateral restraint is therefore essential in the design of fabricated beams, controlling to an extent breadth and thickness of the flanges.

If serviceability limits are omitted, significantly lighter structures can be obtained, see Table 8, in comparison to Tables 5-7. When serviceability is ignored the controlling factor for universal beams reverts to bending moment unity factors. This results in the selection of sections based on plastic section modulus. However, the building deflection under differential loading would result in cladding damage and water ingress. For fabricated beams it becomes a combination of both bending and stability.

8 Building topography parametric study

In the previous section, it was shown that fabricated beam sections produce lighter frame weights. This section investigates this saving across the range of standard portal frame sizes. Spans considered in the study range from 14 m to 50 m; column heights range from 4 m to 12 m. The pitch and frame spacing for all frames are 10° and 6 m, respectively, resulting in a grid of 19×9 data points (highlighted in the contour plots by a dot).

The optimum designs for the frames in this section are obtained by running 10 optimisations per case using UBC3 and FBC2. UBC3 and FBC2 are chosen in the parametric study to allow the maximum number of design variables while avoiding impractical haunch configuration. For UBC3 each optimisation required on average 2906 function calls (53 generations + 540 initial population). Running each point ten times required 42 minutes of computational time. FBC2 on average required 10095 function calls (60 generations + 2025 initial population) requiring 10 runs approximately 2 hours and 20 minutes of computational time. The increases in optimisation time ($\approx \times 3.5$) moving from beam based to plate based is

noticeable but still quick enough that it could be used within the design environment making the additional weight saving achievable.

For each case, the minimum weight was obtained (in terms of kg/m^2).

Figure 8a shows the contours of minimum weight of frames with a restraint spacing of 1.5 m using universal beams. A monotonic trend is seen, of increasing frame weight with both span and column height.

This study has a number of improvements over the similar universal beam optimisation presented by Phan et al. [8]. In the previous study the frame self-weight remained constant with a lower permanent actions. It was found that self-weight significantly increased the weight of the frames particularly in long span frames accounting for in excess of half the permanent frame loading. This study therefore more accurately reflects the weight of portal frames with the range of geometric configurations presented.

Figure 8b shows equivalent weights using fabricated beams again with 1.5 m restraint spacing; the span column height trend continues, but the weights are consistently lower. Figure 9 shows the percentage reduction in weight using fabricated beams. Both fabricated and rolled optimal solutions do not include the weight of the connections and stiffeners, which for fabricated beams may be higher. For short span and column height frames the weight savings are minimal (<5%). For long span and column height frames the savings can be significant. For frame spans in excess of 30 m, weight savings of 14% in primary members can be expected (up to 19% in some instances). The savings are not consistent from case to case due to the variation in optimality of different beam cases. However, there is a strong trend in that the longer the span the higher the potential savings. It should be noted that without fabrication constraints, savings of up 30% are achievable. However this requires the web to be thicker than the flange which is inconsistent with fabrication practices and limitations.

The universal beam based frames are much less susceptible to the influence of stability. Figure 10a shows the growth in frame weight when increasing the restraint spacing from 1.5 m to 3.5 m. Frame weight changes occur in only short span buildings under 25 m with a maximum weight increase of 15% for the shortest span frames.

There is considerably more variation in the fabricated beams. Figure 10b and Figure 10c show the weight difference for a restraint spacing increased from 1.5 m to 2.5 and 3.5 m, respectively. For a 1 m increase, there is a weight increase of 0-12% and for 2 m, a weight increase of 0-16%. The largest increases are for small span frames. This can be explained by the limited available plate thicknesses.

Larger span frames have significantly lower weight increases of approximately 2% and 4% for 1 m and 2 m, respectively. This can be attributed to the ability of the GA to redistribute steel within the cross-section towards the required attributes. As steel within the flanges not only provides stability but also adds moment capacity and stiffness allowing for cross-sectional area migration from the web to the flange as needed.

9 Conclusions

Fabricated beam portal frames have a distinct weight advantage over universal beam based frames with an achievable weight saving of 15% in primary frame weight for large span frames (>40 m). Fabricated beams in portal frames are controlled by two governing constraints: the buckling stability of the sections, and serviceability limits of the frame (deflection).

For medium to long span frames, universal beam optimised frame sections are governed by serviceability, with the exception of small span frames which are more susceptible to buckling constraints that should be included in the optimisation when using universal beam based frames.

The geometric cross-sectional dimensional constraints placed on the optimisation have a significant influence on the weight obtained. Where an insufficient range of plate thicknesses is provided, the fabricated section will have weight comparable, or in excess of a rolled based design. If the flange and web thicknesses are allowed to vary independently of each other the fabricated beam weight savings are increased significantly by an additional 10-15%. The choices of optimisation design variables are important. FBC3 although consistently the lightest solution would generate less than ideal haunch configurations where the geometry would have insufficient depth for bolts or have excessively long shallow flanges. It is recommended to have a configuration similar to FBC2 where the haunch

section was based on the rafter, or have more complex haunch based geometric constraints in place. Universal beam based frames were found to have similar issues when the haunch was allowed to fully vary (UBC4) with a similar recommendation to base the haunch on the rafter (UBC3).

Using plate lines and automated fabrication will be more expensive than readily available stocked rolled sections for small frames. However, in large long span projects, the 15% weight saving may make it an economically attractive viable solution. Fabricated beams will be ill-advised for small frames where savings are minimal. However, for larger frames or frames outside the range of universal beams (50+ m) they provide a lighter solution.

A genetic algorithm with a population 15 times the number of variables with a larger initial population and reduced search space, was capable of producing reliable repeatable results in an industry acceptable time frame for both fabricated and universal beam based frames. Fabricated beams require approximately 3.5 times longer than universal beam based optimisations but are still viable for design completing in less than 3 hours. This would allow for the comfortable automated optimisation of multiple frames outside normal working hours overnight within the industry. Further work would be to examine tapered beams and the inclusion of local buckling section verifications.

Acknowledgements

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Figures

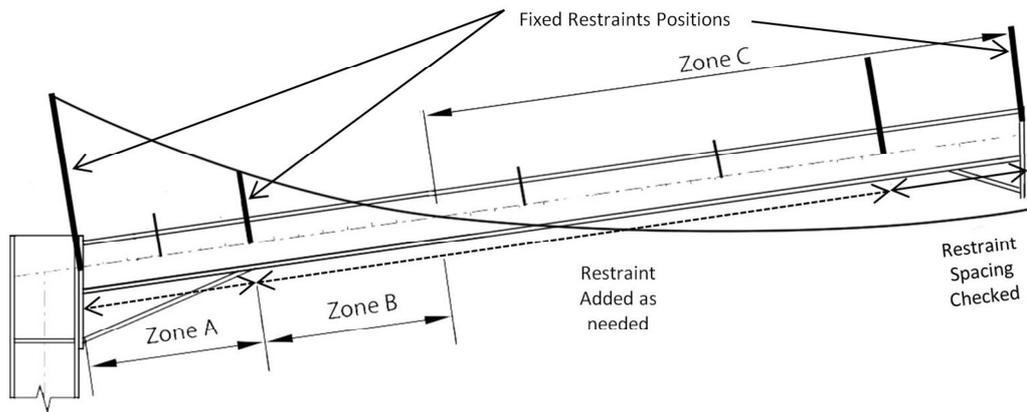


Figure 1 Buckling zones of rafter after [19]

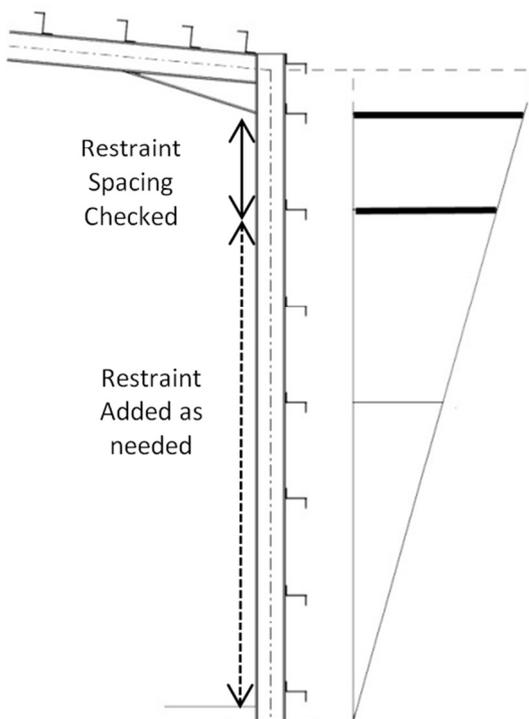


Figure 2 Critical buckling restraint positions of column after [19]

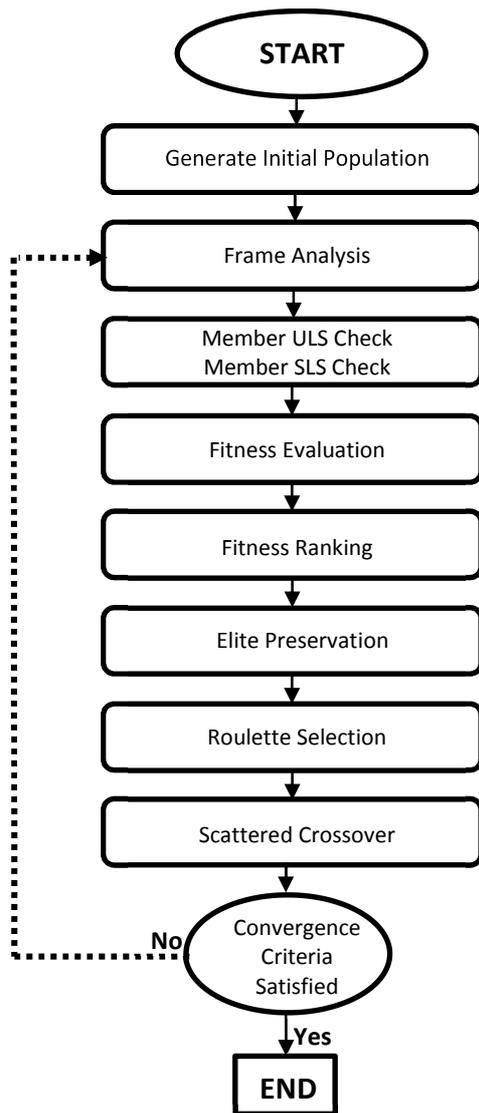


Figure 3 Flowchart of genetic algorithm

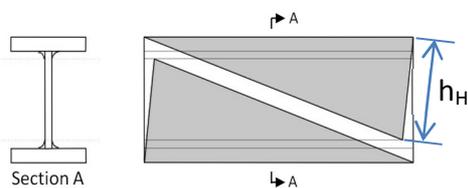


Figure 4 Details of eaves haunch cut from universal beam section. The white area in the figure on the right-hand side is wastage

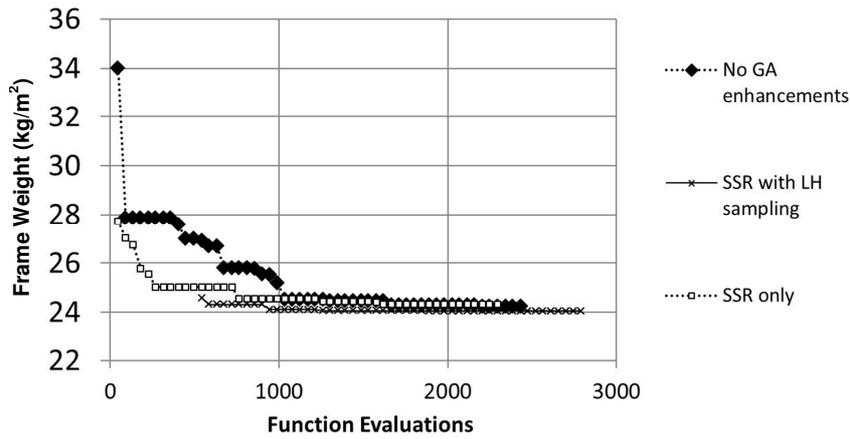


Figure 5 Convergence history for different genetic algorithm strategies

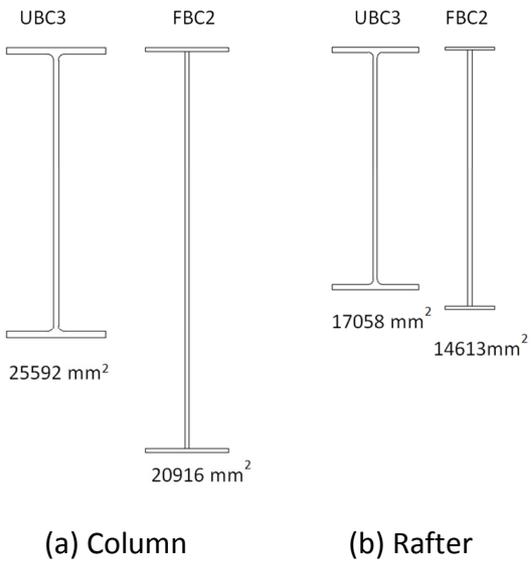


Figure 6 Comparison of optimum cross-sections for Frame A using UBC3 and FBC2

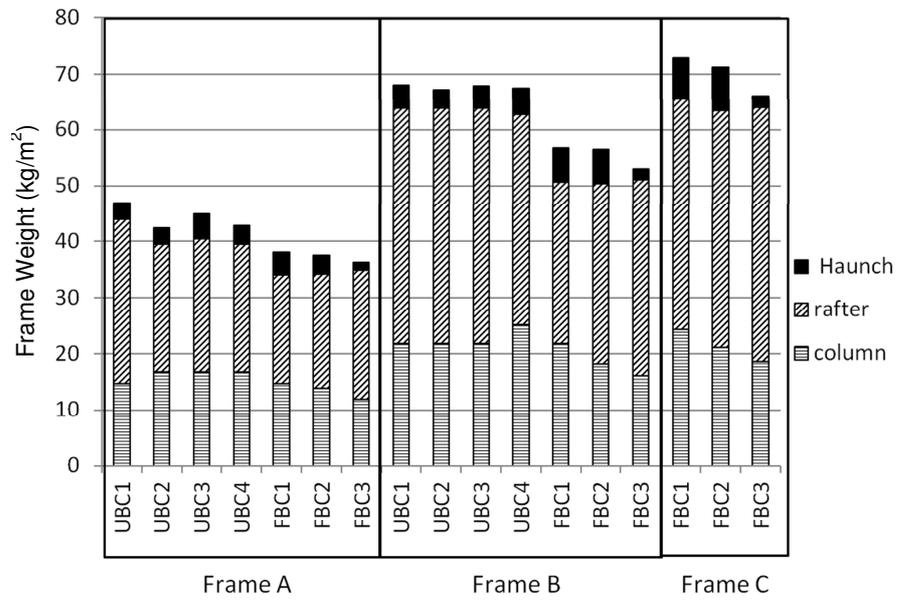
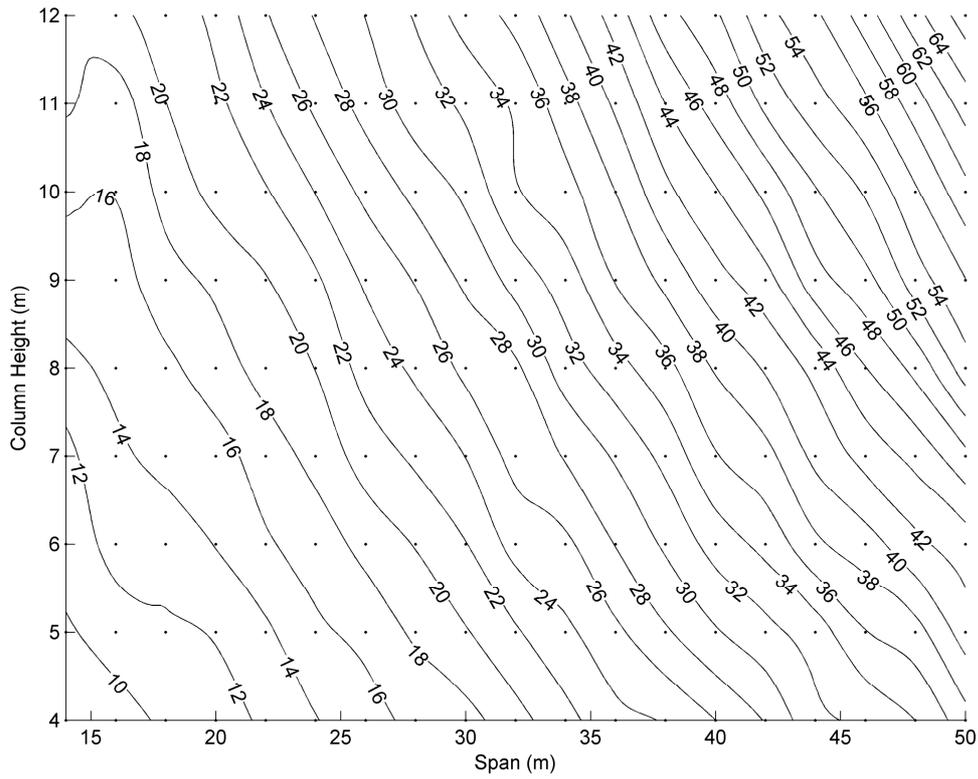
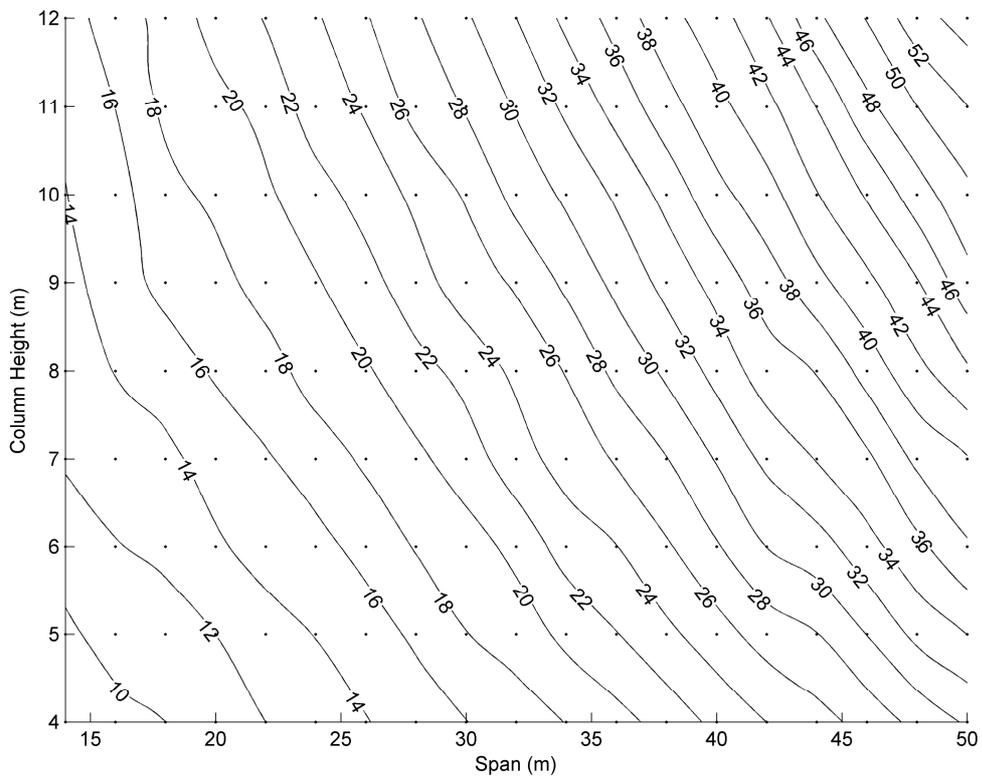


Figure 7 Optimum frame weight for $L_{cr} = 1.5$ m



(a) UBC3



(b) FBC2

Figure 8 Contour of frame weight (kg/m²) for (a) UBC3 and (b) FBC2 with $L_{cr} = 1.5$ m

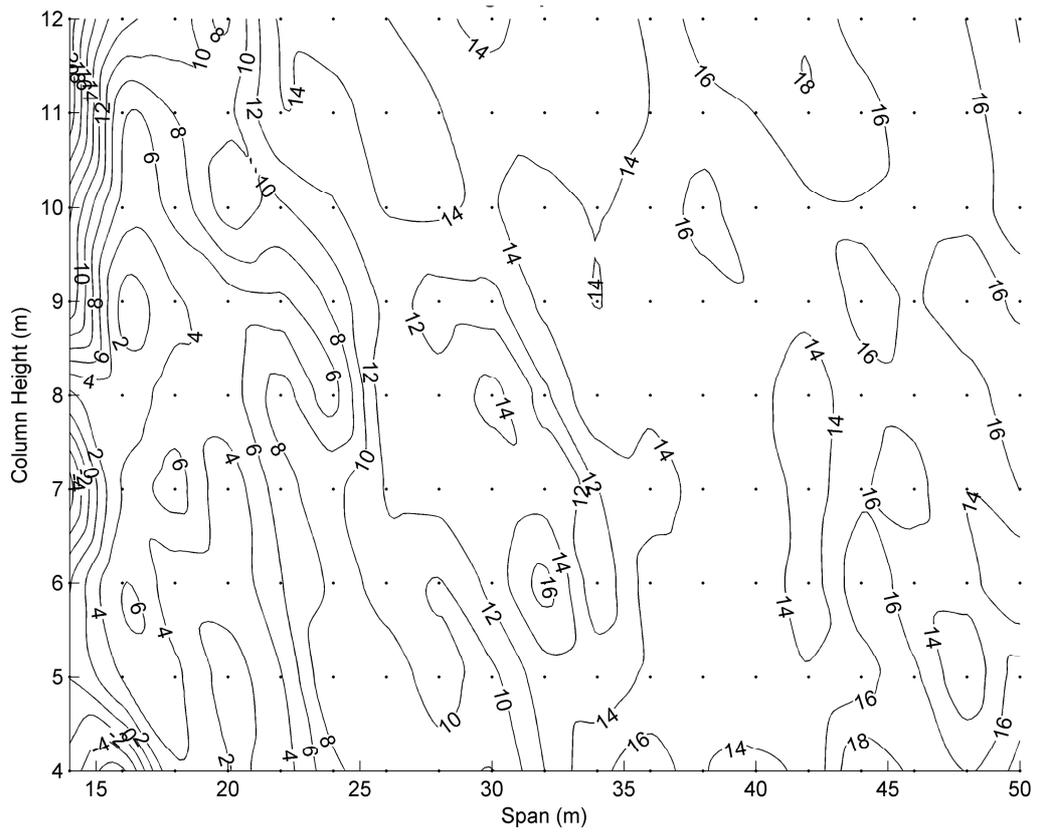
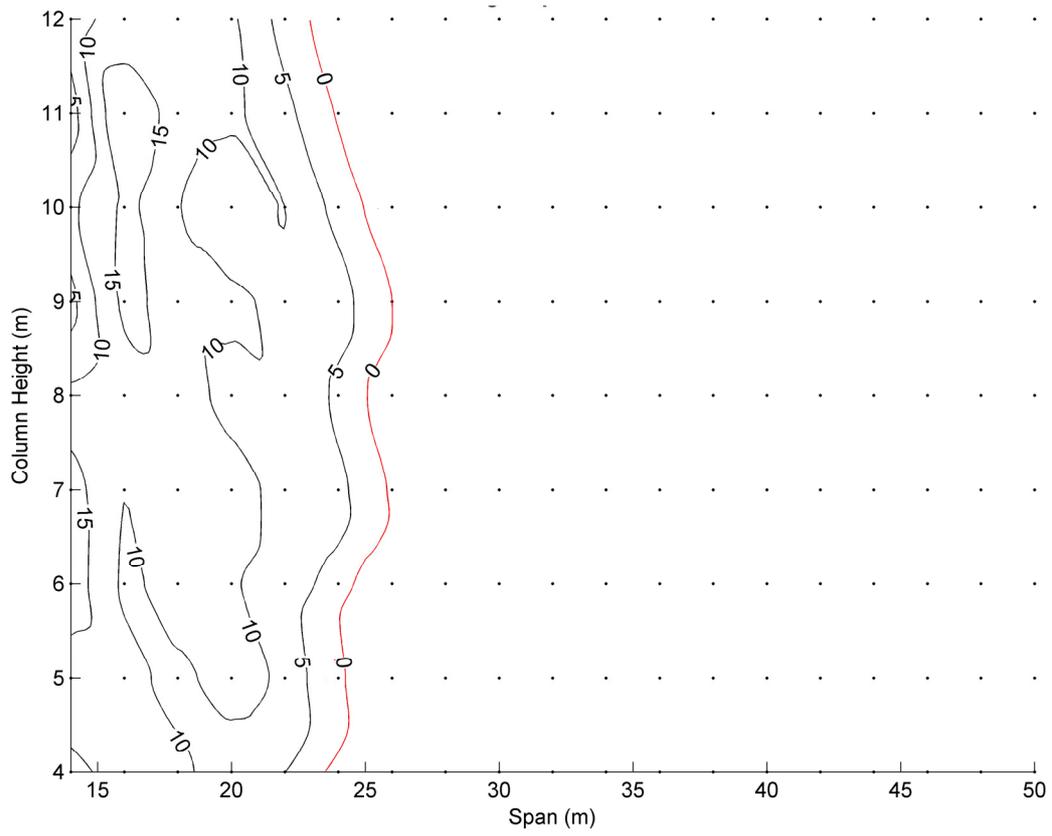
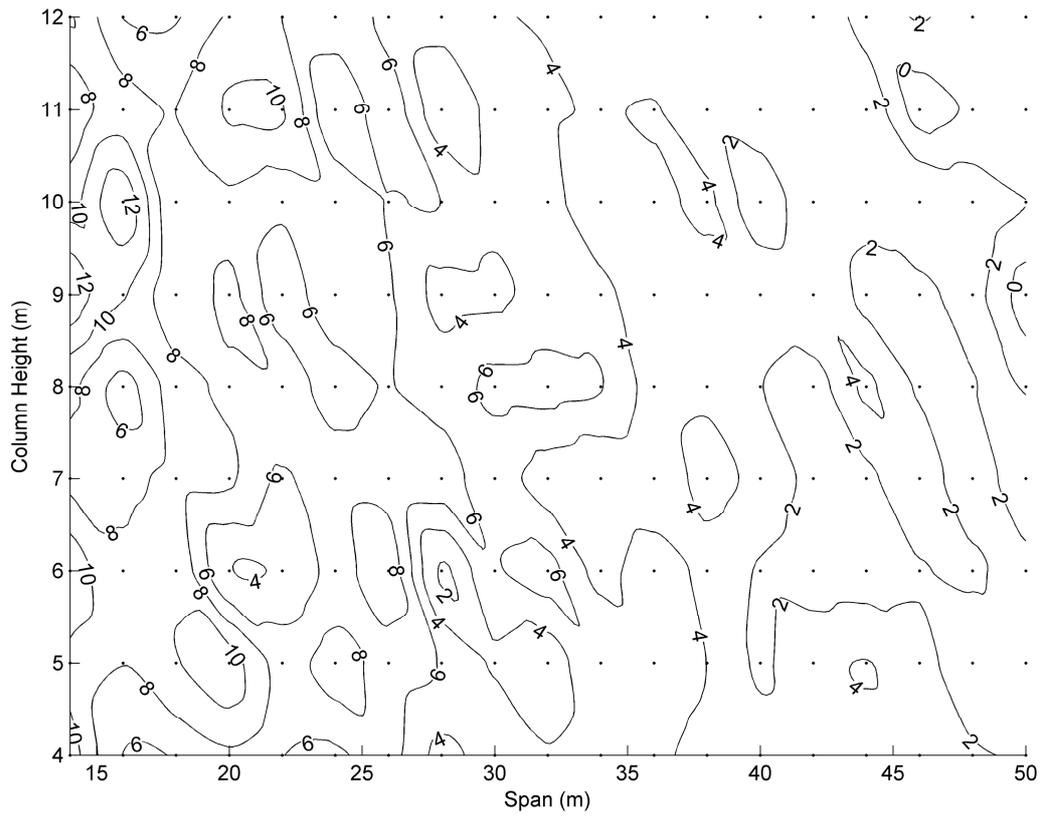


Figure 9 Contour of percentage saving in frame weight of FBC2/UBC3 with $L_{cr} = 1.5$ m



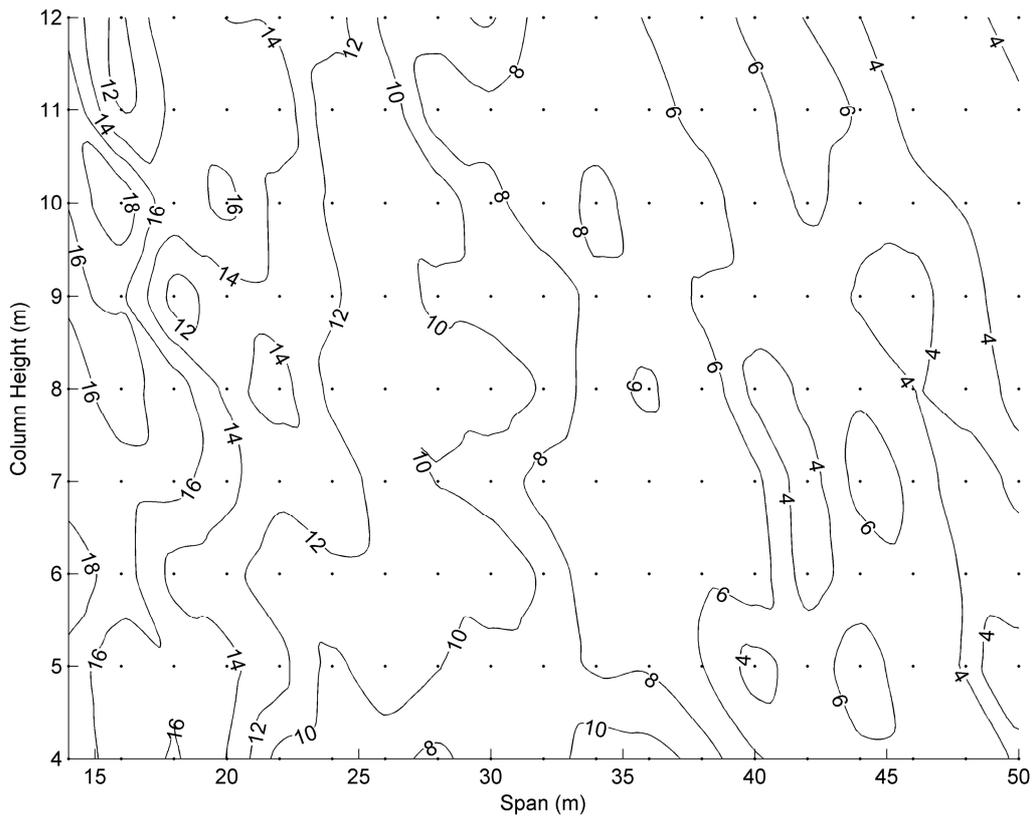
(a)

Figure 10 Contour of percentage increase in frame weight for (a) UBC3 when increasing L_{cr} from 1.5 m to 3.5 m, and FBC2 when increasing L_{cr} from 1.5 m to (b) 2.5m and (c) 3.5 m



(b)

Figure 10 Contour of percentage increase in frame weight for (a) UBC3 when increasing L_{cr} from 1.5 m to 3.5 m, and FBC2 when increasing L_{cr} from 1.5 m to (b) 2.5m and (c) 3.5 m



(c)

Figure 10 Contour of percentage increase in frame weight for (a) UBC3 when increasing L_{cr} from 1.5 m to 3.5 m, and FBC2 when increasing L_{cr} from 1.5 m to (b) 2.5m and (c) 3.5 m

Tables

Table 1 Reliability of genetic algorithm for Frame A using UBC3

Optimisation strategy		GA population multiplier					
		5	10	15	20	25	30
Initial Population (IP)	Search Space Reduction (SSR)	Mean weight (kg per m ²) (Standard deviation)					
Normal	None	47.82 (6.44)	44.44 (1.84)	43.87 (1.10)	43.31 (0.71)	43.28 (0.49)	43.27 (0.68)
×6 Larger	None	47.76 (6.27)	43.73 (1.20)	43.34 (0.49)	43.21 (0.65)	43.07 (0.63)	43.23 (0.65)
×12 Larger	None	46.05 (1.76)	44.58 (1.09)	44.53 (0.83)	44.27 (0.44)	44.39 (0.63)	44.43 (0.51)
Normal	Yes	45.21 (1.91)	43.53 (0.88)	42.97 (0.50)	43.00 (0.52)	42.89 (0.50)	42.89 (0.48)
×6 Larger	Yes	44.39 (1.45)	43.28 (0.65)	43.12 (0.49)	42.89 (0.44)	42.86 (0.43)	42.85 (0.45)
×12 Larger	Yes	45.00 (1.00)	44.28 (0.82)	44.07 (0.47)	44.06 (0.43)	43.97 (0.27)	43.99 (0.28)

Table 2 Average number of function evolutions for Frame A using UBC3

Optimisation strategy		GA population multiplier					
		5	10	15	20	25	30
Initial Population (IP)	Search Space Reduction (SSR)	Average number of function evaluations					
Normal	None	815	1645	2490	3261	3996	4876
×6 Larger	None	885	1778	2728	3485	4471	5539
×12 Larger	None	984	1975	2985	3961	4939	5869
Normal	Yes	783	1592	2379	3129	3954	4747
×6 Larger	Yes	884	1755	2647	3513	4376	5266
×12 Larger	Yes	1000	1968	2959	3871	4851	5923

Table 3 Effect on reliability of different number of genetic algorithm runs

Frame	UBC1	UBC2	UBC3	UBC4	FBC1	FBC2	FBC3
	$(\text{meanweight}_{10\text{runs}} - \text{meanweight}_{30\text{runs}}) / \text{meanweight}_{30\text{runs}}$						
A (40 m)	0.02%	0.39%	0.13%	-0.48%	2.35%	1.25%	1.63%
B (50 m)	0.00%	0.20%	0.00%	0.22%	-0.38%	1.69%	2.50%
C (60 m)					0.93%	0.39%	4.03%

Table 4 Comparison with previous results in the literature

Benchmark	Researchers	Column sections	Rafter sections	Depth of haunch (m)	Length of haunch (m)
Issa and Mohammad (2010)	Issa and Mohammad (2010)	457×152×52	406×140×46	0.11	2.45
	Phan et al. (2013)	457×152×52	356×127×33	0.49	3.60
	UBC3	457×152×52	356×127×33	n/a	5.13
Phan et al. (2013)	Phan et al. (2013)	610×229×113	533×210×82	0.515	4.20
	UBC3	610×229×113	533×210×82	n/a	4.99

Table 5 Optimisation of Frames A & B composed of universal beams

Case	Frame	Column section	Rafter section	Haunch section	H _L (span %)	ULS			LTB		SLS	Frame weight (kg/m ²)
						g _{3C}	g _{3R}	g _{3H}	Max of g _{4C} /g _{5C}	Max of g _{4R} /g _{5R}	Max of g _{6R} /g _{7R}	
UBC1	A	838×292×176 UB	762×267×173 UB	762×267×173 UB	10.0%	0.80	0.47	0.48	0.85	0.52	0.94	46.75
	B	1016×305×272 UB	1016×305×249UB	1016×305×249 UB	10.0%	0.74	0.47	0.45	0.78	0.49	0.97	68.07
UBC2	A	914×305×201 UB	762×267×134 UB	610×305×179 UB	10.0%	0.68	0.74	0.61	0.72	0.58	1.00	42.54
	B	1016×305×272 UB	1016×305×249 UB	762×267×197 UB	10.0%	0.74	0.57	0.46	0.78	0.49	0.99	67.10
UBC3	A	914×305×201 UB	686×254×140 UB	686×254×140 UB	14.3%	0.70	0.65	0.51	0.74	0.57	1.00	43.81
	B	1016×305×272 UB	1016×305×249 UB	1016×305×249 UB	7.6%	0.72	0.51	0.51	0.77	0.50	1.00	67.12
UBC4	A	914×305×201 UB	762×267×134 UB	610×305×179 UB	14.1%	0.68	0.74	0.61	0.72	0.58	1.00	42.52
	B	1016×305×272 UB	1016×305×222 UB	838×292×226 UB	11.5%	0.76	0.58	0.46	0.81	0.51	1.00	65.51

Table 6 Optimisation of Frames A, B & C composed of fabricated beams using L_{cr} = 1.5 m

Case	Frame	Column section dimensions (mm)	Rafter section dimensions (mm)	Haunch section dimensions (mm)	H _L (span %)	ULS			LTB		SLS	Frame weight (kg/m ²)
						g _{3C}	g _{3R}	g _{3H}	Max of g _{4C} /g _{5C}	Max of g _{4R} /g _{5R}	Max of g _{6R} /g _{7R}	
FBC1	A	1336×186×177 FB	692×186×114 FB	692×186×114 FB	16.1%	0.82	0.78	0.54	1.00	0.69	1.00	37.59
	B	1708×171×273 FB	891×171×170 FB	891×171×170 FB	17.2%	0.77	0.73	0.49	1.00	0.66	1.00	56.09
	C	1893×227×365 FB	1110×227×243 FB	1110×227×243 FB	15.3%	0.79	0.65	0.46	0.94	0.57	1.00	72.51
FBC2	A	1262×204×167 FB	784×122×120 FB	784×122×120 FB	12.6%	0.82	0.75	0.63	0.96	0.88	0.98	37.04
	B	1352×272×228 FB	1000×89×190 FB	1000×89×190 FB	15.4%	0.89	0.61	0.49	0.94	0.91	0.99	55.47
	C	1653×398×316 FB	1241×125×250 FB	1241×125×250 FB	15.8%	0.83	0.61	0.44	0.83	0.82	1.00	70.49
FBC3	A	965×277×143 FB	940×182×136 FB	302×148×103 FB	5.7%	0.94	0.94	0.83	0.98	0.84	0.99	36.06
	B	1086×385×202 FB	1425×159×206 FB	231×182×127 FB	7.6%	0.99	0.81	0.69	0.98	0.99	0.98	53.13
	C	1530×281×280 FB	1544×239×268 FB	114×509×233 FB	4.3%	0.93	0.94	0.84	0.98	0.68	0.97	66.21

Table 7 Optimisation of Frames A, B & C composed of fabricated beams using $L_{cr} = 2.5$ m

Case	Frame	Column section dimensions (mm)	Rafter section dimensions (mm)	Haunch section dimensions (mm)	H_L (span %)	ULS			LTB		SLS	Frame weight (kg/m ²)
						g_{3C}	g_{3R}	g_{3H}	Max of g_{4C}/g_{5C}	Max of g_{4R}/g_{5R}	Max of g_{6R}/g_{7R}	
FBC1	A	1344×232×189 FB	708×232×124 FB	708×232×124 FB	10.7%	0.71	0.74	0.73	0.98	0.75	1.00	39.26
	B	1627×251×274 FB	888×251×182 FB	888×251×182 FB	14.1%	0.75	0.68	0.53	1.00	0.68	1.00	57.59
	C	1775×324×356 FB	1050×324×248 FB	1050×324×248 FB	16.7%	0.80	0.65	0.43	0.96	0.56	1.00	73.78
FBC2	A	1139×252×174 FB	722×167×117 FB	722×167×117 FB	16.0%	0.80	0.74	0.53	0.97	0.94	0.99	37.91
	B	1493×321×256 FB	978×167×194 FB	978×167×194 FB	11.5%	0.75	0.66	0.60	0.87	0.89	0.98	57.49
	C	1710×295×324 FB	1126×129×254 FB	1126×129×254 FB	14.7%	0.80	0.64	0.46	0.96	0.98	1.00	71.20
FBC3	A	986×321×156 FB	917×161×133 FB	395×182×128 FB	5.2%	0.84	0.89	0.75	0.94	0.99	0.97	36.93
	B	1299×333×213 FB	1253×197×207 FB	290×219×111 FB	4.1%	0.87	0.87	0.81	1.00	0.93	0.95	52.92
	C	1395×395×301 FB	1549×213×266 FB	168×355×176 FB	4.8%	0.86	0.92	0.82	0.92	0.92	0.98	67.10

Table 8 Optimal designs with serviceability limits removed

Case	Frame	Column section dimensions (mm)	Rafter section dimensions (mm)	Haunch section dimensions (mm)	H_L (span %)	ULS			LTB		Frame weight (kg/m ²)
						g_{3C}	g_{3R}	g_{3H}	Max of g_{4C}/g_{5C}	Max of g_{4R}/g_{5R}	
UBC1	A	610×229×113 UB	610×229×113 UB	610×229×113 UB	10%	0.89	0.91	0.86	0.93	0.77	35.47
	B	686×254×152 UB	686×254×152 UB	686×254×152 UB	10%	0.95	0.99	0.96	0.99	0.80	46.03
FBC1	A	533×337×110 FB	533×337×110 FB	533×337×110 FB	12.1%	0.89	0.99	0.99	1.00	0.85	34.57
	B	671×310×148 FB	671×310×148 FB	671×310×148 FB	10.9%	0.83	0.99	0.99	1.00	0.85	47.01
	C	760×374×197 FB	760×374×197 FB	760×374×197 FB	12.3%	0.91	1.00	0.88	1.00	0.76	57.50