

Laminar shocks in high power laser plasma interactions.

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We propose a theory to describe laminar ion sound structures in a collisionless plasma. Reflection of a small fraction of the upstream ions converts the well known ion acoustic soliton into a structure with a steep potential gradient upstream and with downstream oscillations. The theory provides a simple interpretation of results dating back more than forty years but, more importantly, is shown to provide an explanation for recent observations on laser produced plasmas relevant to inertial fusion and to ion acceleration.

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I. INTRODUCTION

Some recent experiments on the interaction of high power lasers with plasmas have shown evidence of shock-like structures with very high electric fields existing over very short distances [1, 2]. Amendt et al [3] say that data from proton radiography in inertial confinement fusion capsules suggest the existence of fields of more than 10^{10} Vm⁻¹ over distances of the order of 10-100 nm. For the non-cryogenic targets used in the experiment Amendt et al (2009) demonstrated that the experiment behaves as a quasi-collisionless classical plasma, this justifies the use of a collisionless theory especially when considering the extremely small length scales over which the electric field exist. In a more recent paper Amendt et al [4] suggest that barodiffusion (ie pressure-driven diffusion) may be a possible explanation, but this does not seem to produce very short length scales. Another relevant recent paper is that of Haberberger et al [5] who describe experiments in which collisionless shocks generate high energy proton beams with small energy spread.

Our objective here is to show that there is a simple analytic treatment of collisionless shock structure in unmagnetized plasmas which can reproduce the essential features of these experiments and which may be useful in predicting the properties of shocks in collisionless plasmas. The basic method goes back to early studies of collisionless shocks, in particular the work of Sagdeev [6], in which it is shown that solitary wave structures can be described by an equation analogous to that of a particle moving in a potential (now usually referred to as the Sagdeev potential). The Sagdeev potential is a function of the electrostatic potential ϕ and a solitary wave occurs when the Sagdeev potential has a maximum at the origin then goes through zero again at some finite value of ϕ . In terms of the particle motion analogy there is a homoclinic orbit in its phase space leaving the origin, going to the other zero of the Sagdeev potential, then returning to the origin over an infinite time period. Sagdeev suggested that a shock like structure could be produced by introducing some damping into the system, so that the orbit, instead of returning to the origin ended up at the bottom of the potential well. A comprehensive review of analytical work describing electrostatic shocks is treated by Tidman and Krall [7]. Here we show that a shock structure can be produced by having a finite ion temperature so that some ions are reflected by the potential maximum at the shock. This produces the asymmetry between the upstream and downstream sides which destroys the familiar symmetrical ion sound solitary wave. The idea of reflection from the shock front has been familiar for many years, especially in studies of perpendicular shocks in magnetized plasmas where the reflected ions are turned around by the magnetic field and produce a foot structure (see for example the analysis of Woods [8]). In a collisionless unmagnetized plasma the reflected ions simply travel upstream unimpeded. Early observations of electrostatic shocks were made by Taylor et al [9] showing the kind of structure we describe, a potential ramp followed by downstream oscillations, at low Mach numbers. Computer simulations by Forslund and Freidberg [10] later showed shocks, with more complicated dissipative structures at higher Mach number. More recent PIC simulations by Fiuza et al [11] also report shocks at higher Mach numbers than the ones used in this paper. Some work on this latter problem has been carried out by Smirnovskii [12, 13] using basic ideas similar to ours presented here. The problem has been generalized to the relativistic case by Stockem et al [14] Our objective is to give a more transparent account of the theory and to relate it to the recent experimental results mentioned above.

II. THEORY

Consider collisionless ions flowing into a region where the potential increases from zero to some positive value ϕ_{\max} . Taking the incoming ions to have a Maxwellian distribution with average velocity V the density where the potential is ϕ , normalised to the initial density of the incoming flow is

$$n_i(\phi, \phi_{\max}) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp \left[-\frac{(\sqrt{v^2 + 2\phi} - V)^2}{2} \right] dv + \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{2(\phi_{\max} - \phi)}} \exp \left[-\frac{(\sqrt{v^2 + 2\phi} - V)^2}{2} \right] dv \quad (1)$$

with ion velocities normalised to the thermal velocity $V_i = \sqrt{\kappa T_i}$ and the potential to $\frac{m_i V_i^2}{Ze}$ with Z the ion charge state, and $\kappa = k/m_i$ where k is Boltzmann's constant and m_i is the ion mass. We assume that V is sufficiently large that the backward part of the Maxwellian in the shock frame is negligible. The second term here takes account of particles reflected from the potential maximum. This, of course, cannot be chosen as an independent parameter but has to be consistent with the plasma dynamics, which is why it is included as an argument in n_i . Its evaluation will be discussed later.

For the electrons we assume thermal equilibrium in the potential, with the electrons flowing to produce charge equilibrium far upstream where the potential tends to zero, so that

$$n_e(\phi, \phi_{\max}) = Z n_i(0, \phi_{\max}) \exp \left(\frac{\phi}{T} \right) \quad (2)$$

where $T = \frac{ZT_e}{T_i}$.

Poisson's equation then gives

$$\frac{d^2\phi}{dx^2} = n_e(\phi, \phi_{\max}) - Z n_i(\phi, \phi_{\max}) \quad (3)$$

with distances scaled to $\frac{V_i}{\omega_{pi}}$. In order to find ϕ_{\max} self-consistently we introduce the Sagdeev potential $\Phi(\phi, \phi_{\max})$ through

$$\Phi(\phi, \phi_{\max}) = \int_0^\phi [Z n_i(\phi', \phi_{\max}) - n_e(\phi', \phi_{\max})] d\phi' \quad (4)$$

so that (3) becomes

$$\frac{d^2\phi}{dx^2} = -\frac{\partial\Phi}{\partial\phi}, \quad (5)$$

analogous to the equation of motion of a particle in a potential.

The quantity ϕ_{\max} is still an unknown. To determine it we note that if the motion of a notional particle according to (5) starts at $\phi = 0$ then it will increase monotonically to ϕ_{\max} if the Sagdeev potential is zero at ϕ_{\max} and negative when ϕ lies between zero and ϕ_{\max} . So the condition that the value of ϕ_{\max} be consistent with the system dynamics is that

$$\Phi(\phi_{\max}, \phi_{\max}) = 0, \quad (6)$$

an equation which determines ϕ_{\max} . The dimensionless parameters governing the system are V and T and it can soon be found that not all combinations of these yield a system in which the Sagdeev potential has a zero for positive ϕ and is negative in the interval $(0, \phi_{\max})$. The complicated nature of the Sagdeev potential and its dependence on the unknown quantity ϕ_{\max} mean that we are unable to make any analytic progress in determining the parameter range for which a suitable potential exists. The observations we make on this are determined by numerical experimentation, taking $Z = 1$. It is found that an acceptable solution only exists if the electron temperature is sufficiently high. We have found that a value of T of around 15 or more is needed and with this value and $V = 4.5$ we obtain the Sagdeev potential of Figure 1.

In our normalized units the ion sound Mach number is $\frac{V}{\sqrt{T}}$, in this case 1.162. For this value of T it appears that an acceptable solution only exists in a narrow range of Mach numbers between about 1.13 and 1.19

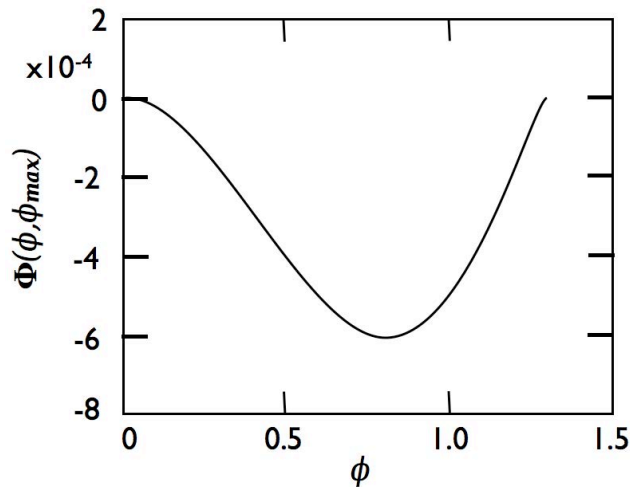


FIG. 1: The Sagdeev potential for $T=15$, $V=4.5$.

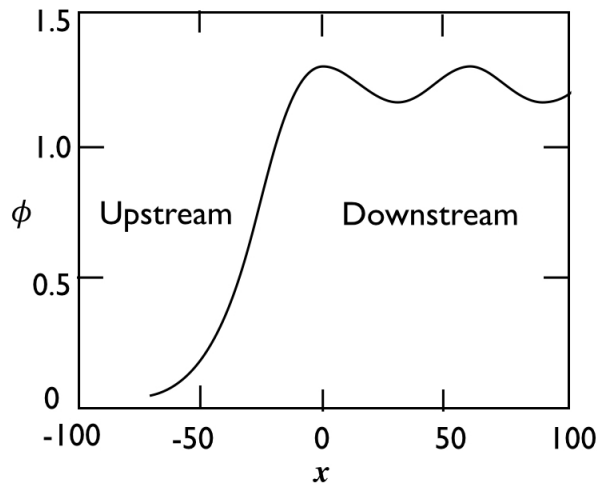


FIG. 2: The electrostatic potential for the parameters of Figure 1

If the Sagdeev potential was the same downstream of the point where the potential reaches its maximum then we would just get a standard solitary wave solution, symmetric about this maximum. However, in the downstream region there is no reflected component and the second term in (1) is absent. This changes the Sagdeev potential and in the downstream region the notional particle motion which determines the solution is oscillation in a potential well. A composite solution can be obtained by starting at the maximum, with zero potential gradient and integrating upstream and downstream with the appropriate charge densities in (3). The result is shown in Figure 2, with the distance x in units of the ion thermal velocity divided by the ion plasma frequency.

If some dissipation were introduced then the notional particle would end up at the minimum of the potential well, corresponding to decaying oscillations on the downstream side and a shock like structure as described by Sagdeev [6]. It is worth noting that the solution is very sensitive to small changes in the charge density. In this case the density as $\phi \rightarrow 0$ on the upstream side goes to 1.0019, so that very few ions are reflected, but there is nevertheless a radical change in the nature of the solution between the upstream and downstream sides. For higher electron to ion temperature ratios the Mach number can be larger leading to a higher percentage of reflected ions as will be shown in the next section.

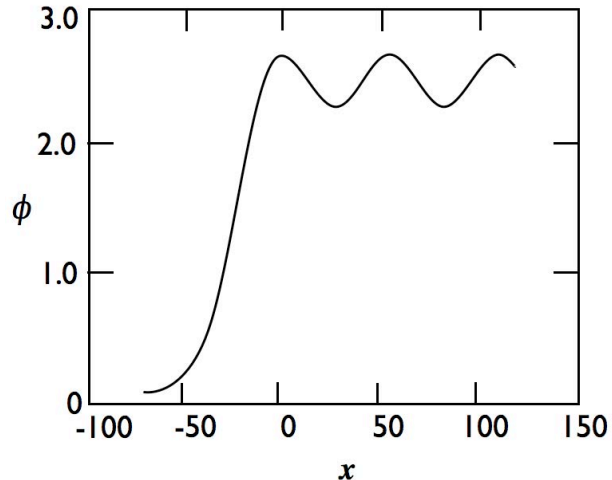


FIG. 3: The potential for the D-T plasma with $T_e = 20$ and $V = 4.75$.

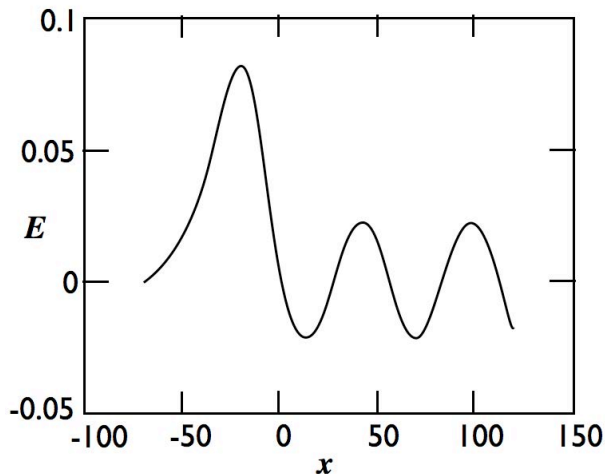


FIG. 4: The normalized electric field corresponding to the potential of Figure 3

III. RESULTS

To explore the possible relevance to a laser fusion pellet compression we can do a similar calculation with a 50/50 mixture of deuterium and tritium upstream. With the potential and flow speed normalised in terms of the deuterium thermal velocity the ion density is half the expression in (1) plus a corresponding tritium contribution in which ϕ is replaced with $\frac{2}{3}\phi$ to take account of the higher mass. The calculation then goes through as before but we find that an acceptable solution only appears to exist for somewhat higher values of T . For $T = 20$ and $V = 4.75$, corresponding to a Mach number of 1.06, we get the solution shown in Figure 3. The corresponding electric field, normalized to $\frac{m_i V_i^2}{Z e} \frac{\omega_{pi}}{V_i}$, is shown in Figure 4.

Now let us relate these normalized values to physical parameters. If we assume that $Z = 1$, then we have for the electric field and length scale

$$\begin{aligned} E(V/m) &= 4.27 \times 10^{-3} E_{norm} T_i (keV)^{1/2} n_i (m^{-3})^{1/2} \\ L(m) &= 2.34 * 10^5 L_{norm} T_i (keV)^{1/2} n_i (m^{-3})^{-1/2}. \end{aligned} \quad (7)$$

If we look at the D-T result given above and assume an ion temperature of 500 eV and density 10^{28} m^{-3} then we get a peak electric field of $2.4 \times 10^{10} \text{ V/m}$ and, taking the normalized length of the main potential ramp to be 50,

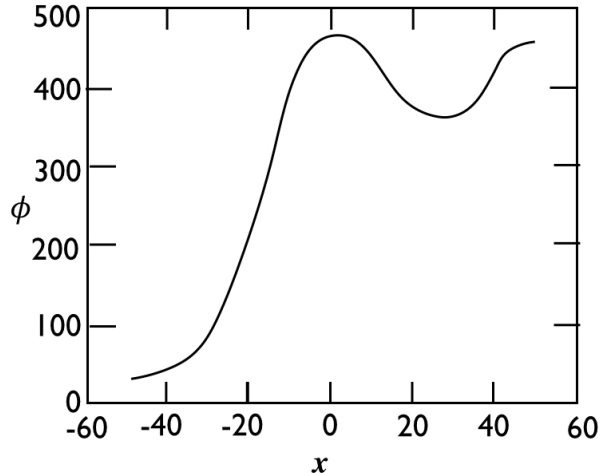


FIG. 5: The potential for $T_e = 500$ and Mach number 1.38.

corresponding to a length of 83 nm. These parameters are in striking agreement with those quoted by Amendt et al [3]. Note that because of the restricted range of Mach numbers within which these structures exist there is little scope for changing these values by adjusting the Mach number. Also there is a fairly weak dependence on density and ion temperature so that the orders of magnitude will remain in general agreement over a wide range of parameters. The main requirement is a high electron temperature compared to the ion temperature, a condition likely to be satisfied in high power laser plasma interactions. Going back to the early experiments of Taylor et al [9] we can take this example and, instead of scaling to a high temperature, high density fusion target scale it to their ion temperature of around 0.2 eV and density of around 10^{15} m^{-3} . The resulting distance between the first two potential peaks is around 5 mm, again in excellent agreement with the observations.

Now let us look at the results of Haberberger et al [5] mentioned above, where they attribute ion beams well collimated in energy to a shock wave in an expanding plasma. The electron temperature they find is about an MeV and we will assume that the already heated and expanding ions are at 2 keV, so that $T = 500$. The potential in this case, with a Mach number of 1.38 is shown in Figure 5.

The normalized length scale is again about 50 which translates into a physical length of about $2 \mu\text{m}$ if we take $n = 10^{26} \text{ m}^{-3}$, while the peak electric field is around $3.6 \times 10^{11} \text{ V/m}$. To compare with the experimental results, we look at the energy spectrum of the reflected ions. Adding the measured expansion velocity of $0.1c$ to the reflected ion velocity we get the spectrum shown in Figure 6. This bears a striking resemblance to the experimental results, not only in the width of the spectrum and its energy but even in the detailed shape with a sharp edge on the high side. The density of reflected ions is about 36% of the background ion density, though this can go down if the Mach number is reduced. Again we should point out that because of the limited range of possible Mach numbers and the weak dependence of the physical values on the plasma parameters, we do not have a great deal of freedom to adjust parameters so that our results lie in the correct range. The result given here appears to match the experiment much better than the computer simulation shown in the Haberberger et al [5] paper. One possible explanation is that the shock in the simulation has been launched with larger Mach number of about 2. This is well above the limit beyond which our laminar solutions do not exist (around 1.4), so it may be that what is being seen is some kind of turbulent shock, producing a much broader spectrum of fast ions.

Going back to the early simulations of Forslund and Freidberg [10] we see just this behaviour with the sort of structure we describe at low Mach number but a change to a more complex dissipative structure with many more reflected ions at higher Mach numbers. Our results suggest that what is seen in the experiment is the result of a low Mach number laminar structure rather than a higher Mach number dissipative shock.

IV. CONCLUSION

In conclusion, we have given a simple analytic description of laminar shock structures in unmagnetized plasmas and shown that the theory, despite its simplicity, can provide an explanation of results from important recent experiments

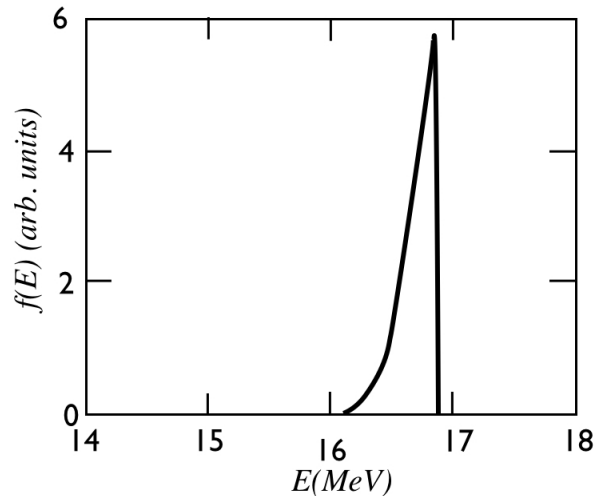


FIG. 6: The energy spectrum of reflected ions for the parameters given in the text.

on high power laser plasma interactions. It explains the existence of very high electric fields in inertial fusion targets and should be useful in guiding developments in the use of lasers to produce high quality energetic ion beams. On a more fundamental level there is a large body of literature on solitary waves where a heavy component, whether ions or dust, is assumed cold. Including thermal effects, in the way done here could lead to a reappraisal of these structures.

In future work we intend to investigate the conditions under which different types of collisionless shock can be generated in laser plasmas. A better understanding of these should help efforts to eliminate them where they produce unwanted effects or to better control them when they give useful effects like ion acceleration.

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VI. REFERENCES

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