Slowly moving test charge in a two-electron component non-Maxwellian plasma

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Abstract

Potential distributions around a slowly moving test charge are calculated by taking into account the dynamics of electron-acoustic waves (EAWs) in an unmagnetized plasma. The hot electrons are assumed to be suprathermal and obey the Kappa distribution function, whereas the cold inertial electrons are described by the Vlasov equation with a Maxwellian equilibrium distribution, and the positive ions form a static neutralizing background. The test charge moves slowly in comparison between both the hot and cold electron thermal speeds and is therefore shielded by the electrons. This gives rise to a short-range Debye-Hückel potential decaying exponentially with distance and to a far field potential decaying as inverse third power of the distance from the test charge. The results are relevant for both laboratory and space plasma, where suprathermal hot electrons with power-law distributions have been observed.

Lots of effort has been made in the last few decades to investigate the properties of electron components with different temperatures in laboratory [1–4] and space [5–7] plasmas. Moreover, in space environments, such as the ionosphere, the auroral zones, the mesosphere and lower thermosphere, have also observations of non-Maxwellian (nonthermal) particle distributions [12–16]. The latter have attracted significant interest, and have led to the modeling of the observed suprathermal tails in the particle distribution function (DF) by power-laws in velocity rather than by the Maxwellian DF. The simultaneous existence of hot and cold electron components in a neutralizing background of static ions supports the electron acoustic waves (EAWs) in Maxwellian plasmas [8–11]. There are also evidence involving the experimental and theoretical studies of EAWs in non-Maxwellian plasmas [4]. The phase speed associated with the EAWs lies between the hot and cold electron thermal speeds, whereas the wave frequency is much greater than the ion plasma frequency. Hence the ions can for most purposes be taken as a static neutralizing background.

In a two-electron component plasma, the suprathermal suprathermal (power-law) tails of the hot electrons can be modeled by a three-dimensional (3D) isotropic Kappa DF [14] $F_{\kappa 0}\left(\mathbf{V},\theta_\kappa,\kappa\right) = b_\kappa \left(1 + V^2/\theta_\kappa^2\ \kappa\right)^{-(\kappa+1)}$, where $b_\kappa = \left[N_{h0}\left(\theta_\kappa^2 \pi\kappa\right)^{-3/2}\right]\Gamma\left(\kappa+1\right)/\Gamma\left(\kappa_h - 1/2\right)$ is a normalization constant, N_{h0} is the hot electron equilibrium density, κ is spectral index, and Γ is the gamma function. The Kappa DF [15, 16] is a generalization of the Maxwellian DF [17], and is reduced to the Maxwellian when the spectral index Kappa tends to infinity. Vasyliunas [14] has empirically studied the effects of low spectral index in the range $\kappa \sim 2-4$ and analyzed the energy spectrum in the Earth's plasma sheet. The effective thermal speed of suprathermal hot electrons is denoted $\theta_\kappa = \left[1_h - 3/(2\kappa)\right]^{1/2} V_{th}$ where $V_{th} = \left(2T_h/m_h\right)^{1/2}$ is the hot electron thermal speed, T_h is the hot electron temperature and m_h the electron mass. The numerical value of κ must be greater than a critical value, $\kappa > \kappa_c = 3/2$, for a physical system. The Kappa DF has power law tails for speeds V much larger than the effective thermal speed (viz., $V \gg \theta_\kappa$), and the importance of non-thermal effects is most significant at small κ-values. In the opposite limit $V \lesssim \theta_\kappa$ the Kappa DF is approximately reduced to a Maxwellian DF with $\theta_\kappa \sim V_{th}$.

The test charge approach has relevance for various plasma physics environments, e.g. heavy-ion pumped X-ray lasers, heavy-ion stopping power in dense plasmas, ICF schemes, dust coagulation in space and astrophysical plasmas, particle acceleration, etc. A charged test particle is screened (or shielded) by a cloud of charged particles, when it is moving slowly (or is stationary) in comparison with the thermal speed of the plasma particles. Physically, a test charge polarizes the plasma medium and attains a potential distribution around it. For a stationary test charge, the potential is spherically symmetric and decreases exponentially fast with distance from the dust grain beyond the Debye radius (λ_D). This short-range potential is known as the Debye-Hückel (DH) potential [18]. On the other hand, if the test particle speed coincides with the phase speed of plasma oscillations in a resonant manner, a long-range oscillating wake-field potential can occur behind the test particle. The wakefield potential forms a pattern of positive and negative maxima with a decreasing amplitudes over a distance of many Debye lengths. An enhancement of the test charge speed then causes an increase of the amplitude of the wakefield. Finally, if the speed of the test charge is much higher than the thermal speed of all plasma species, the plasma no longer shields the test charge, and as a result a Coulomb potential forms around the test charge.

The interaction of a test charge with a plasma gives rise to several important effects, such the loss of energy due wave-particle interactions or particle-particle collisions. The dressed charge potentials and energy loss due to slow and fast charged particles were computed for a plasma with the electrons obeying the Maxwell-Boltzmann and Fermi-Dirac statistics and with a fixed background of ions [19]. Long-range potentials proportional to the inverse third power of the distance were obtained for a slowly moving particle in a Maxwellian plasma [20]. The impact of ion dynamics [21, 22], dust charge fluctuations [23], Bhatnagar-Gross-Krook (BGK) collisions [24], and free and trapped electron densities [25] on the potential profiles have also been examined. Linear dielectric theory was employed [26] for slow and fast ion test charge velocities in a Maxwellian plasma, showing numerically that an oscillatory wakes appear behind the ion test in addition to the DH potential. Attractive potentials were found in two-electron component plasma with different densities and temperatures for Maxwellian [27] and non-Maxwellian [28] plasma when the test charge moves with speeds between the thermal speeds of the hot and cold electrons.

In this Brief Communication, we investigate the potential distributions around a slowly moving test charge in an unmagnetized plasma, whose constituents are suprathermal hot electrons obeying a Kappa distribution and a colder core of electrons obeying a Maxwell distribution, with the respective temperatures T_h and T_c and number densities N_{h0} and N_{c0}, in a neutralizing background of static positive ions. The dynamics of both the hot and cold electrons is described by the Vlasov equation with the respective equilibrium distribution.

The test charge perturbs the plasma equilibrium state $N_{i0} = N_{c0} + N_{h0})$, where N_{i0} is the equilibrium ion number density and N_{c0} is the equilibrium number density of the cold electrons.

The electrostatic response due to the hot and cold electrons in a non-Maxwellian plasma is governed by the linearized 3D Vlasov-Poisson system of equations

$$\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)F_{h1} = -\frac{e}{m_e}\nabla\phi_1\cdot\nabla_{\mathbf{V}}F_{h\kappa 0}(\mathbf{V})\,, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)F_{c1} = -\frac{e}{m_e}\nabla\phi_1\cdot\nabla_{\mathbf{V}}F_{c0}(\mathbf{V})\,, \tag{2}$$

and

$$\nabla^2\phi_1 = -4\pi q_t\delta\left(\mathbf{R}-\mathbf{V}_t\ t\right)+4\pi e\left(N_{h1}+N_{c1}\right), \tag{3}$$

where $N_{c1} = \int F_{c1}(\mathbf{R},\mathbf{V},t)d\mathbf{V}$ and $N_{h1} = \int F_{h1}(\mathbf{R},\mathbf{V},t)d\mathbf{V}$ are the perturbed cold and hot electron number densities, with $F_{c1}(\mathbf{R},\mathbf{V},t)$ and $F_{h1}(\mathbf{R},\mathbf{V},t)$ being the respective perturbed DF, obeying $|F_{c1}(\mathbf{R},\mathbf{V},t)| \ll F_{c0}(\mathbf{V})$ and $|F_{h1}(\mathbf{R},\mathbf{V},t)| \ll F_{h\kappa 0}(\mathbf{V})$, where F_{c0} is the equilibrium cold electron Maxwellian DF. Here, $-e$ is the electron charge and m_e the electron mass.

The symbol $\delta\left(\mathbf{R}-\mathbf{V}_t\ t\right)$ in (3) denotes the 3D Dirac's delta function, where the argument describes the position of the test charge. The electrostatic potential due to a positive test particle of charge q_t is denoted by ϕ_1 and V_t is the velocity of the test charge along the z-axis. Fourier transforming Eqs. (1)–(3) in space and time and solving for the Fourier transformed potential $\tilde{\phi}_1$ leads to the expression $K^2D\left(K,\omega\right)\tilde{\phi}_1\left(K,\omega\right) = 8\pi^2q_t\delta\left(\omega-\mathbf{K}\cdot\mathbf{V}_t\right)$ where ω represents the angular frequency and $\mathbf{K}$ the wave vector. In the absence of a test charge (viz., $q_t = 0$), the linear dispersion relation of the EAWs is obtained, accounting for the suprathermal hot electrons. On the other hand, applying the inverse Fourier transformation with respect to ω, the electrostatic potential [29] at $\omega = \mathbf{K}\cdot\mathbf{V}_t$ is obtained as

$$\phi_1\left(\mathbf{R},t\right) = \frac{q_t}{2\pi^2}\int\frac{d\mathbf{K}}{K^2}\frac{\exp\left[i\mathbf{K}\cdot\left(\mathbf{R}-\mathbf{V}_t t\right)\right]}{D\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)}. \tag{4}$$

The modification of the potential due to the hot and cold electrons appears in the dielectric constant D of EAWs as

$$D\left(K,\mathbf{K}\cdot\mathbf{V}_t\right) = 1+\frac{2\omega_{ph}^2}{K^2\theta_\kappa^2}\left[1-\frac{1}{2\kappa}+C_hZ_\kappa(C_h)\right]+\frac{2\omega_{pc}^2}{K^2V_{Tc}^2}\left[1+C_cZ(C_c)\right], \tag{5}$$

with $C_h = \omega/(K\theta_\kappa)$, $C_c = \omega/(KV_{Tc})$, $\omega = \mathbf{K}\cdot\mathbf{V}_t$, $\omega_{pc} = (4\pi N_{c0}/m_e)^{1/2}$, and $V_{Tc} = \sqrt{2T_c/m_e}$, where the modified plasma dispersion function for a κ-distributed plasma is [16]

$$Z_\kappa(\xi) = \frac{\kappa^{\kappa-1/2}\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{3/2}\Gamma(\kappa-1/2)} \int_{-\infty}^{\infty} \frac{ds}{(s-\xi)(1+s^2/\kappa)^{\kappa+1}}, \qquad \Im(\xi) > 0, \tag{6}$$

and the standard plasma dispersion relation for a Maxwellian plasma [31]

$$Z_\kappa(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{ds\, e^{-s^2}}{(s-\xi)}, \qquad \Im(\xi) > 0, \tag{7}$$

which are analytically extended to arguments with zero or negative imaginary parts. For a test charge with intermediate velocities, resonant interactions of the test charge with the EAWs leads to an attractive wakefield potential in a non-Maxwellian dusty [28] and Maxwellian [27] plasmas. However, in the present model, the wakefield due to a test charge is excited because the test charge propagates with much lower speed than the thermal speed of the cold electrons, and therefore it does not resonate with the phase speed of the EAWs. For small velocities $|\mathbf{V}_t| \ll \theta_\kappa$, V_{Tc}, we can use the small argument expansions (for $|\xi| \ll 1$) of the dispersion functions,

$$Z_\kappa(\xi) \approx -2\left(1 - \frac{1}{2\kappa}\right)\xi + i\frac{\sqrt{\pi}\Gamma(\kappa)(1+\xi^2/\kappa)^{-\kappa}}{\sqrt{\kappa}\Gamma(1-1/2)} \approx i\frac{\sqrt{\pi}\Gamma(\kappa)}{\sqrt{\kappa}\Gamma(1-1/2)} \tag{8}$$

and

$$Z_\kappa(\xi) \approx -2\xi + i\sqrt{\pi}e^{-\xi^2} \approx i\sqrt{\pi}. \tag{9}$$

This gives

$$D\left(K, \mathbf{K}\cdot\mathbf{V}_t\right) = 1 + \frac{2\omega_{ph}^2}{K^2\theta_\kappa^2}\left[1 - \frac{1}{2\kappa} + C_h i \frac{\sqrt{\pi}\Gamma(\kappa)}{\sqrt{\kappa}\Gamma(1-1/2)}\right] + \frac{2\omega_{pc}^2}{K^2V_{Tc}^2}(1 + C_c i\sqrt{\pi}), \tag{10}$$

where $\omega_{ph} = (4\pi N_{h0}e^2/m_e)^{1/2}$. In many situations, the plasma has a dense core of cold electrons and a dilute halo of hot electrons. In this case, $\omega_{ph}^2/\theta_\kappa^3 \ll \omega_{pc}^2/v_{Tc}^3$, and the Landau damping term of the hot electrons (proportional to C_h) in (10) can be neglected compared to the Landau damping term due to the cold electrons (proportional to C_c). The hot electrons then contribute primarily to the shielding of the test charge, but not to resonant wave-particle interactions. For this case, D can be expressed as

$$D\left(K, \mathbf{K}\cdot\mathbf{V}_t\right) \simeq 1 + \frac{1}{K^2\left(\lambda_h^\kappa\right)^2} + \frac{\omega_{pc}^2}{K^2V_{tc}^2}(1 + i\sqrt{\pi}C_c). \tag{11}$$

Here $\lambda_h^\kappa = \lambda_h/\sqrt{c_\kappa}$ represents the shielding length [30] of the hot, supra-thermal electrons, $\lambda_h = \left(T_h/4\pi N_{h0}e^2\right)^{1/2}$, and $c_{\kappa_k} = (2\kappa-1)/(2\kappa-3)$. For large values of κ, the parameter

$c_\kappa \to 1$ and consequently, the modified shielding length approaches the Maxwellian case, i.e., $\lambda_h^\kappa \to \lambda_h$ in a two-electron component plasma. The Landau damping term in (11) vanishes for a static test charge particle ($V_t = 0$). For this case, Eq. (4) simply yields the short range DH potential of the form $\phi_{DH}(\mathbf{R}) = (q_t/|R|)\exp(-|R|/\lambda_{DH}^\kappa)$ having the effective shielding length λ_{DH}^κ and the distance $|R| = (\rho^2+\xi^2)^{1/2}$ from the test charge in terms of the radial (ρ) and axial ($\xi = z$) distances. However, the investigations of Montgomery *et al.* [20] showed that if the test charge moves slowly in comparison with the cold electron thermal speed, one cannot neglect the Landau damping term in Eq. (11). In the limit of a low test charge speed ($V_t \ll V_{tc}$), the reciprocal of Eq. (11) can be expressed as

$$D^{-1} \simeq \frac{K^2(\lambda_{DH}^\kappa)^2}{K^2(\lambda_{DH}^\kappa)^2+1} - i\frac{\sqrt{\pi}}{2}\frac{\mu V_t}{V_{tc}}\frac{K^2\omega_{pc}^2(\lambda_{DH}^\kappa)^4}{V_{tc}^2\left[K^2(\lambda_{DH}^\kappa)^2+1\right]^2}. \tag{12}$$

Here $\lambda_{DH}^\kappa = \left((\lambda_h^\kappa)^{-2}+\omega_{pc}^2/V_{tc}^2\right)^{-1/2}$ and $\mu\,(=\cos\theta_k)$ represents the angle between the wave vector $\mathbf{K}$ and velocity vector $\mathbf{V}_t$. For further simplifications, we express the electrostatic potential in a spherical polar coordinate system by writing $\mathbf{V}_t = (0,0,V_t)$, $\mathbf{R} = (R\sin\theta_R\cos\phi_R, R\sin\theta_R\sin\phi_R, R\cos\theta_R)$ and $\mathbf{K} = (K\sin\theta_K\cos\phi_K,\ K\sin\theta_K\sin\phi_K, K\cos\theta_K)$. Finally, Eqs. (7) and (4) can be solved as

$$\phi_1(\mathbf{R},t) = \phi_{DH}(\mathbf{R},t) + \phi_{FF}(\mathbf{R},t). \tag{13}$$

The modified DH and far-field potentials are, respectively, given by

$$\phi_{DH}(\mathbf{R},t) = \frac{q_t}{R}\exp\left(-\frac{R}{\lambda_{DH}^\kappa}\right), \tag{14}$$

and

$$\phi_{FF}(\mathbf{R},t) = \frac{2q_t\xi}{\sqrt{\pi}R}\frac{\omega_{pc}^2 V_t\lambda_{DH}^\kappa}{V_{tc}^3}\frac{(\lambda_{DH}^\kappa)^3}{R^3}. \tag{15}$$

Here, $R = (\rho^2+\xi^2)^{1/2}$ is the distance from the test charge in terms of the radial and axial distances ρ and $\xi = z - V_t t$. Equation (13) represents the sum of the short range DH potential [18] and the long-range, far-field potential due to a slowly moving test charge. We note that the Kappa distribution of the hot electrons modifies significantly the expressions (14) and (15) for the short and long range potentials. In particular, Eq. (15) is derived in the long range limit $R/\lambda_{DH}^\kappa \gg 1$, and shows a decrease of the shielded potential as the inverse third power of the both axial and radial distances.

For numerical examples, we normalize Eqs. (14) and (15) as $\bar{\phi}_{DH} = \phi_{DH}/(q_t/\lambda_h)$, $\bar{\phi}_{FF} = \phi_{FF}/(q_t/\lambda_h)$, $\bar{\rho} = \rho/\lambda_h$, $\bar{\xi} = \xi/\lambda_h$, and $\bar{V}_t = V_t/V_{tc}$. Numerical values are chosen from a laboratory experiment [4], namely, $N_{h0} \sim 2 \times 10^7 \mathrm{cm}^{-3}$, $N_{c0} \sim 6 \times 10^7 \mathrm{cm}^{-3}$, $T_c \sim 0.7$eV, and $T_h \sim 12$eV. The cold electron plasma frequency is derived as $\omega_{pc} \sim 4.36 \times 10^8$Hz, the cold electron thermal speed is $V_{tc} \sim 3.50 \times 10^7$cm/s, and the hot electron Debye length $\lambda_h \sim 0.575$cm. Assuming the spectral index to be small, i.e., $\kappa = 1.6$, we find the modified effective shielding length as $\lambda_{DH}^{\kappa} \sim 0.126$cm for the parameter $c_\kappa = 11$. The impact of the suprathermal tails of the hot electrons is reduced for the large value $\kappa = 50$ and consequently, we have $c_\kappa \sim 1.02$ and $\lambda_{DH}^{\kappa} \to \lambda_{DH} \sim 0.138$cm for a two-electron component Maxwellian plasma. Hence, the effective shielding length is slightly shorter with the power law Kappa distribution as compared to Maxwellian case.

Figure 1 displays the profiles of the normalized effective shielding length as a function of spectral index (κ) for different hot-to-cold electron temperature ratios and cold-to-hot electron number density ratios $\sigma(\sim 9, 12, 17)$ and $\alpha\,(= 0.1, 1, 5)$ in the left [Figs. 1(a)–(c)] and right [Figs. 1(d)–(f)] panels, respectively. Since the speed of the test charge is much lower than the cold electron thermal speed, the test charge is shielded by both the hot and cold electrons and an effective shielding length is obtained as $\lambda_{DH}^{\kappa} = \lambda_h \left(c_\kappa + \sigma\alpha\right)^{-1/2}$, where $\sigma = T_h/T_c$ and $\alpha = n_{c0}/n_{h0}$. For $n_{c0} \approx 0$, the numerical results shown in Fig. 1 coincide exactly with the earlier analysis [32] for electron-ion non-Maxwellian plasmas. Additionally, Bryant [32] has highlighted the role of characteristic scale length both in the Kappa distributed and Maxwellian plasmas with the same particle temperatures and number densities. He has numerically found that at low values of the spectral index, the Debye length is shorter, while it approaches asymptotically the Debye length of a Maxwellian plasma with the enhancement of the spectral index. However, the inclusion of a cold electron density leads to a reduction of the strength of the normalized effective Debye length [see the plots in Fig. 1]. We have found that the normalized effective shielding length decreases with the increasing values of hot-to-cold electron temperature ratio and the cold-to-hot electron density ratio. Figure 2 examines the impact of the hot electron supra-thermal effects on the short range DH potential at fixed radial distance $\rho = 0.2\lambda_h$. The magnitude of the DH potential is modified at low Kappa values and approaches the Maxwellian case when $\kappa \to 20$. The impact of the hot electron supra-thermal effects, the cold-to-hot electron density ratio and the hot-to-cold electron temperature ratio is examined on the profiles of far field (long range) potential in Fig. 3 for a fixed $\rho = 0.2\lambda_h$ and $V_t = 0.0028V_{tc}$. We find that that the far field potential distributions appear at an axial distance $\xi = 1.5\lambda_h$ and are significantly influenced by the variation of plasma parameters.

To conclude, we have studied the propagation of a slowly moving test charge of velocity $\mathbf{V}_t$ along the z-axis in an unmagnetized plasma, which is comprised of one hot Kappa distributed electron species and one cold inertial electron species with a Maxwellian equilibrium distribution, as well as a static background of positive ions. The hot electrons are assumed to be inertialess, whereas the cold electrons are assumed inertial and its dynamics described by the Vlasov equation. Fourier analysis is used to obtain the electrostatic response potential, showing the impact of the supra-thermal hot electrons. When a test charge speed is taken much smaller than the cold electron thermal speed, the potential is divided into a short-range DH potential and a far-field shielded potentials. The long-range potential decays as the inverse third power of the distance and is significantly affected by the variation of plasma parameters. The present results are relevant for laboratory plasmas, where the potential distributions around the slow test charge are studied in the presence of supra-thermal hot electrons.

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Figure captions

Fig.1 (color online) The variation of the normalized effective Debye length ($\lambda_{DH}^{\kappa}/\lambda_h$) as a function of the spectral index (κ) for different values of the hot-to-cold electron temperature ratio and cold-to-hot electron number density ratio $\sigma(\sim 9, 12, 17)$ and $\alpha\,(= 0.1, 1, 5)$ in the left [Fig1(a)-(c)] and right [Fig1(d)-(f)] panels with fixed values of $\alpha = 3$ and $\sigma = 17$, respectively.

Fig.2 (color online) The impact of hot electron spectral index on the normalized DH potential $\phi_{DH}/(q_t/\lambda_h)$ versus the normalized axial distance ξ/λ_h for $\kappa = 1.6$ (black dotted curve), $\kappa = 1.8$ (red dashed curve), and $\kappa_h = 20$ (blue solid curve) with $\alpha = 3$, $\sigma = 17$ and $\rho = 0.2\lambda_h$.

Fig.3 (color online) The profile of the far field potential $\phi_{FF}/(q_t/\lambda_h)$ as a function of axial distance ξ/λ_h for (a) $\kappa = 1.6$ (black dotted curve), $\kappa = 1.8$ (blue dashed curve), and $\kappa = 20$ (red solid curve) with fixed $\sigma = 17$ and $\alpha = 3$, (b) $\alpha = 1$ (black dotted curve), $\alpha = 3$ (blue dashed curve), and $\alpha = 5$ (red solid curve) at fixed $\kappa = 1.6$ and $\sigma = 17$, (c) $\sigma = 9$ (black dotted curve), $\sigma = 13$ (blue dashed curve), and σ =17 (red solid curve) at $\kappa = 1.6$. Other numerical values are taken as $\rho = 0.2\lambda_h$ and $V_t = 0.0028V_{tc}$.