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TESTING AND VALIDATION OF AN ALGORITHM FOR CONFIGURING DISTRIBUTION GRID SENSOR NETWORKS

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ABSTRACT
The control of Smart Grids depends on a reliable set of measurement information such that distributed generation and demand can be effectively managed. The cost of procuring and installing sensors at multiple nodes in the grid is prohibitive and choosing the optimum strategy with regards to sensor location, accuracy, number and type is very important. This report describes the testing of a sensor placement algorithm developed to determine measurement strategies for distribution grids. This testing was performed on a laboratory microgrid at the University of Strathclyde. The ability of the algorithm to choose the optimal subset of measurements was tested by comparing the estimated power flow with the measured power flow of a fully instrumented grid. The chosen subset is found to have the close to the lowest overall error and all estimates agree with the rejected measurements within the calculated uncertainties.

INTRODUCTION
A great deal of research has been carried out into the development of algorithms that optimise the use of measurements on distribution grids to provide the best estimate of system state (power flows and nodal voltages) with the fewest measurements (e.g. [1]-[9]). Some methods in the literature are focussed on finding the optimal measurement set to meet given accuracy constraints (e.g. [5]), whereas others focus on robustness of the state estimator to failure or degradation in sensor performance [9], fault determination [10], ability of the algorithm to cope with network changes [11], etc. However, the majority of testing and application of such algorithms is carried out in simulation and demonstrations of their application to real measurement data are limited.

SENSOR PLACEMENT ALGORITHM
An algorithm was developed to find a measurement strategy for distribution grids to determine the system state with a required accuracy with minimum instrumentation. The algorithm was developed using some functions from the MATPOWER power system simulation package of Matlab M-files [12]. Preliminary testing of part of the algorithm was reported in [13]. The procedure applies an analytical sensitivity analysis to determine which input measurements are critical to the solution and ranks the measurements according to their influence on the uncertainties in the state estimates. The number of required measurements is first minimised and the measurement locations are selected to give the lowest average uncertainty in the state estimation. The cost/benefit of adding more high accuracy measurements is then investigated. The aim is to find the optimum position and number of measurements with the best cost/accuracy trade-off.

In order to obtain a thorough understanding of the observability of the system and allow the development of reliable LV grid control schemes, knowledge of the uncertainty in the system state is required. This will be achieved through a comprehensive sensitivity analysis carried out in simulation on the grid model and state estimators, using a variety of scenarios, measurement strategies and prior information.

Uncertainties will be assigned to measured data, which is applied to the state estimator, and the sensitivity of the estimator to the accuracy of these input parameters will be assessed. An uncertainty can then be assigned to the state estimation.

Sensitivity Analysis
The sensitivity analysis can be performed as follows. For the unconstrained case, suppose the state estimation problem is given as

\[
\min_x (z - Hx)^T (z - Hx),
\]

where \( x = (x_1, \ldots, x_n)^T \) are the parameters to be estimated, \( H \) is a (linearised) \( m \times n \) observation matrix and \( z \) is a vector of length \( m \), storing measured data values. The solution \( x \) depends linearly on \( z \) and can be written as \( x = S_z z \). If the variance matrix associated with \( z \) is \( V_z \), then the variance matrix associate with the solution estimate \( x \) is given by \( V_x = S_z V_z S_z^T \). If \( V_z \) has Cholesky factors \([14]\) \( V_z = L_z L_z^T \), then writing \( z := L_z^{-1} z \) and \( H := L_z^{-1} H \), (1) can be posed in terms
of transformed data $z$ whose corresponding variance matrix is an identity matrix. In the analysis below, it is assumed that this transformation has been made.

If $H$ has QR factorisation

$$H = QR = \begin{bmatrix} Q_1 & Q_2 \\ R_1 & 0 \end{bmatrix} = Q_1 R_1,$$

and the solution $x$ is given by

$$x = R_1^{-1} Q_1^T z,$$

i.e., $S_z = R_1^{-1} Q_1^T$. The matrix $S_z$ shows precisely how the uncertainties associated with the data vector $z$ contribute to the uncertainties associated with the parameter estimates $x$. For example, $S_z^2(j,i)$ is the variance component of the variance $u^2(x_j)$ associated with the $j$th parameter estimate $x_j$ arising from the $i$th data point $z_i$. The sum of the squares of the elements in any column gives the total variation contribution to the variance of $x$ arising from the corresponding data point. The matrix $S_z$ gives valuable information on which sets of measurements contribute most to the uncertainties associated with the solution parameters and therefore indicate where resources may be most usefully concentrated in improving measurement uncertainties. The vector of residuals is given by

$$r = z - H x = (I - HR_1^T Q_1^T) z,$$

showing the dependence of $r$ on the data vector. The matrix $I - Q_1 Q_1^T$ is the variance matrix $V_r$ associated with the residuals and the variance $u^2(r_i)$ associated with the $i$th residual is simply $1$ minus the sum of squares of the elements of the $i$th row of $Q_1$. (If the orthogonal factorisation is computed using Householder transformations [5], then $Q_1$ is calculated efficiently as $Q_1 = Q_1^T Q_1$.) Since $Q_1$ is a submatrix of an orthogonal matrix, $0 \leq u(r_i) \leq 1 = u(z_i)$. If $u(r_i) = 0$, it means that the $i$th model prediction must match the $i$th observation exactly, in other words, the $i$th model prediction is determined by the $i$th observation and the model fit must pass through $z_i$. This situation indicates that $z_i$ is pivotal and that removing that date point would lead to rank deficiency. It also indicates that if the measured value $z_i$ was an outlier due to sensor malfunction, for example, there would be no way of detecting that $z_i$ was defective. Conversely, if $u(r_i) = 1$, it means that the $i$th observation plays no part in determining the $i$th model prediction; it is determined using other information and that the measurement $z_i$ is redundant.

### Choice of Measurements

As explained in above, the relative importance of each measurement to the state estimate can be obtained from the variance matrix associated with the residuals (differences between estimated and measured parameters), $u^2(r_i)$. Using this matrix, redundant measurements can be identified and their removal will not have a significant effect on the accuracy of the state estimation.

The state estimation involves finding the voltage magnitudes and angles (the state variables) at each bus by solving a number of simultaneous equations. As such there must be at least as many equations as unknowns, which implies a minimum set of input measurements [15]. There are many procedures for testing for observability and identifying observable islands (e.g. [16]). For this work, the values of the residual variances are used to assess the importance of each measurement and remove those that are redundant. The rank of the observation matrix is used to assess whether the network remains observable. Measurements are removed until the minimum set of measurements is reached.

There may be several configurations which satisfy the requirements for observability but do not lead to the minimum uncertainty for the state estimation, but the use of the residual variances to iteratively remove measurements leads to a minimum set of measurements that is close to optimal.

In the case of larger real networks, where pseudo measurements based on load forecasting would also be used in place of real measurements, the most important measurements can also be selected using the matrix $S_z$ by summing the contributions of all the variances of the solution, $x$, corresponding to each measurement point, $z_i$. The measurement points can then be ranked based on these sensitivities and those with the highest sensitivity values chosen for real measurements. A cost measure can also be included based on the difficulty of placing a measurement at a given location, so that the measurements with the highest sensitivity are only selected if their cost can be justified.

### THE STRATHCLYDE MICROGRID MEASUREMENT SYSTEM

A full description of the microgrid at Strathclyde University can be found in [17] and this section describes the setup that was applied for validation of the algorithms. A schematic diagram of the microgrid is shown in Figure 1. In conjunction with the development of algorithms for state estimation (SE) and sensitivity analysis, the characteristics of the sensors to be used on the Strathclyde microgrid have been identified. The sensors installed on the grid are specified to a precision of 1% and isolation amplifiers are designed to be within
1 %; the total error introduced by the data acquisition system is estimated to be below 2.14 % for the magnitude of current and voltage, and below 1.65° for their phase. Consequently a precision of 4.5 % is achieved measuring the power flow. These figures can be improved by the application of a SE algorithm and by an accurate recalibration of the system. The application of the SE algorithm can reduce the uncertainty of the measured data both taking advantage of the possible redundancy in the data and by exploiting the dependency among the data due to the topology of the network.

**Measurements Specifications**

In literature the uncertainty of the real measurement data available for SE is generally assumed to be around a few per cent; for example in [18] three cases are presented in which the uncertainty of real measurements is assumed to be between 1 % and 3 %. For phasor measurement units (PMUs), the accuracy requirements are more demanding and in [19] the necessary accuracy for voltage magnitude is indicated to be between 0.5 % (with phase angle error equal to zero) and 0.2 % (with phase angle error equal to 0.09°) to achieve 1 % accuracy in power flow. The sensors installed on the microgrid and the data acquisition system guarantee a precision of 2.14 % in the measurement of voltage and current magnitude and consequently a precision of 4.5 % in measuring the power flow. This level of precision is considered enough to simulate the real operating conditions of an energy management system at the low voltage level.

**Sensors installed on the Strathclyde Microgrid**

Details about the instrumentation and the acquisition data flow implemented at Strathclyde Microgrid are given in the following paragraphs. On the switchboards in correspondence with the five bus bars (see Figure 1) current transformers (250/1A P1 1 % accuracy, 5 VA) and 400/110 star-star voltage transformers instrumentation are installed. These instruments measure the voltages/currents on the outgoing lines and the voltages on the bus bar. The current transformers are burdened with 10 Ω 5 W thick film / ceramic resistors, giving good performance with temperature. These give 0.040 V RMS for every 1 A RMS flowing in the lines. At the maximum expected current of 125 A, the voltage across the 10 Ω burdens will be 5 V RMS. Instrument transformers are commonly used in power systems to scale voltage or current when they are too large to be measured directly by an instrument.

To measure voltage and current (normally three-phase) class 1 (1 % accuracy) VTs and CTs are used. The VTs provide isolation and the VT secondary neutral can be floated to any arbitrary voltage level. The secondary phase-neutral voltages are stepped down with a passive resistive divider, isolated/clamped, filtered/clamped, and passed to the computer hardware.

**Estimation of the uncertainty of the measurements**

Uncertainty introduced by the different stages at 50 Hz: VTs and CTs: 1 % (total vector error).

Burden resistors: 1 % in magnitude, negligible for the phase.

Passive resistive attenuation: 1 % in magnitude, negligible for the phase.

Differential amplification: active component negligible, passive components 1 % in magnitude.

Capacitive barrier: 0.5 % worst case (0.05 % typical case).

Butterworth filter, due to the tolerance of the components, introduces an uncertainty of 0.77 % for the magnitude and 1.31° for the phase.

RC low pass filter: negligible for magnitude and phase.

ADC: 1 %.

Taking the root of the sum of the squares of the above uncertainties gives a total uncertainty of 2.14 % for the measurement magnitude and 1.65° for the measurement phase.

**TESTING OF SENSOR PLACEMENT THROUGH SENSITIVITY ANALYSIS**

![Figure 1 – Setup of microgrid for experimental testing of the sensitivity analysis based sensor placement algorithm](image)

To test the algorithm the microgrid was adapted as shown in Figure 1. Busbar 0 is connected to the main power supply and acts as the slack bus. Variable load banks are connected to busbars 1 (Load B) and 3 (Load C). An induction machine acting as a generator is connected at busbar 1 (Generator B). Busbars 2 and 4 are not connected to loads or generators. The grid is operated at a base voltage of 400 V (230 V per phase), however in the experiment the voltage at the main grid was set a little higher at around 412 V (238 V per phase).

The grid was configured with a number of different load
scenarios. Loads B and C were set to draw various amounts of active and reactive power as described below.

**Scenario 1 – Loads B & C 2 kW, 0.85 power factor.**

In scenario 1, loads B and C were set to 2 kW with a power factor of 0.85. Several measurements were taken at all available points on the grid. The results of these measurements are shown in Table 1 below and these were used as input measurements for the sensitivity analysis. The sigmas derived from the calculated uncertainties in the measurements are given in Table 2.

<table>
<thead>
<tr>
<th>Table 1 – Full set of input measurements for Scenario 1</th>
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<td>PF (pu)</td>
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<table>
<thead>
<tr>
<th>Table 2 – Input sigmas for Scenario 1</th>
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<td>PF (pu)</td>
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The algorithm described above was run with these inputs and the measurements shown in Figure 2 were obtained. For the purposes of testing the algorithm, measurements of active and reactive power and voltage magnitude and phase are treated as separate measurements to give more measurement locations for the testing of the algorithm. Of course, in reality if a meter were placed in a given branch at a bus both active and reactive power and voltage would be available. Here they are treated separately so that more redundant measurement locations are provided and more calculated values can be compared with measured values.

**Figure 2 – Measurement subset chosen by sensitivity analysis algorithm for Scenario 1**

The state estimation was then performed with this subset of measurements, with the remaining measurements rejected. The derived estimates at all nodal points were compared to the input measurements from Table 1. The resulting differences are given in Table 3. The differences are well within the uncertainties of the measurements for all nodes. The RSS difference of all points was then calculated. The RSS of the uncertainties in the state variables calculated by the sensitivity analysis was also noted.

<table>
<thead>
<tr>
<th>Table 3 – Difference between input measurements and estimates (%) for Scenario 1 (chosen measurement subset)</th>
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<tr>
<td>Node</td>
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**Figure 3 – Average state estimation errors with different measurement subsets for Scenario 1**

The chosen optimum measurement set gives the lowest RSS difference between the estimates and the measurements and the lowest calculated state variable uncertainty, which are comparable to that of a fully instrumented grid, as can be seen from Figure 3.

Further tests were performed with different load scenarios. These included Loads B and C set to nominally 6 kW with a nominal power factor of 0.95, Loads B and C set to nominally 1 and 4 kW, respectively, with a nominal power factor of 0.7 and Loads B and C set to nominally 2 and 5 kW, respectively, with power factors of 0.7 and 0.9, respectively. In all cases the chosen measurement set gave the lowest RSS difference between the estimates and the measurements and the lowest calculated state variable uncertainty.

**CONCLUSIONS AND FUTURE WORK**

A sensitivity analysis based algorithm for the placement
of measurement equipment in distribution grids has been briefly presented. Validation of the method on the laboratory microgrid at the University of Strathclyde shows that the method provides the optimum minimum observable set of measurements or close to this in all tested cases. The results show that both the uncertainty calculation and placement methods are working as expected. The performance with larger networks has been verified in simulation but still needs to be validated on full-scale real working networks. Also an algorithm that can decide on where pseudo measurements might be used instead of real measurements for under-instrumented networks has been tested in simulation, but has yet to be applied on real grids. The application of the algorithm to real working grids to inform network operators of their measurement requirements is the next logical step for this work. It is possible that the algorithms can further be adapted to reveal lacking information about grid topology and structure, for example it may be possible to derive line impedances from voltage and power measurements and also to identify which phase certain connections are made, where such information is poorly documented. The expansion of the method to deal with 3-phase models would be required for this.

REFERENCES


