The Barnett formula under the Smith Reforms

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Abstract:

With the implementation of the Smith Commission reforms, there will be abatements to the Scottish government's block grant as calculated by the Barnett formula, to allow for the tax revenues which Westminster will forego. The income tax abatement will be increased through time in an arrangement known as Holtham indexation. The purpose of this paper is to model these new fiscal arrangements. The modelling implies that, other than in an unlikely special case, the system cannot run on indefinitely with fixed parameter values, without reaching relative values of per capita public expenditure in Scotland and England which would be politically unacceptable. Moreover, there is a likelihood of adverse dynamic effects which would further destabilise the system. The paper also puts forward an adjustment to the original form of Holtham indexation, to take account of relative population growth. While not answering every problem, this adjustment has strong equity arguments in its favour, and would also significantly stabilise the system. It is particularly important that the implications of this type of modelling should be taken on board by policy makers and parliamentarians just now, when the detailed legislative and other arrangements for implementing the Smith reforms are about to be finalised.

I Introduction

An earlier paper in the Fraser of Allander Commentary, (Cuthbert, 2001), modelled the way in which relative population change between England and Scotland interacted with the Barnett formula. It was shown there how relative population growth in England compared to Scotland could have a very marked effect on the limiting ratio of per capita public expenditure between England and Scotland, and on the trajectory towards that limit.

This paper extends the modelling in that earlier paper, to incorporate the effects of the proposed method of indexation, (Holtham indexation), of the abatement to the Barnett block grant which will come into effect as the Smith Commission reforms are implemented. It turns out that both relative population growth, and the relative rate of growth in the relevant tax base between England and Scotland, will play an important part in determining the behaviour of the Barnett formula as modified by the Smith Commission proposals. In particular, the modelling indicates the potential for the emergence of dynamic effects, in which relative population growth could interact with growth in the tax base, in a way which could adversely affect Scotland.

The paper also puts forward a suggested modification of Holtham indexation: not only is there a strong equity argument for this modification, but it would also correct some of the worst effects of unadjusted Holtham indexation.

II Background (1): The Smith Commission proposals on tax, and Holtham indexation

The Smith Commission reported on 24 November 2014, (Smith Commission, 2014), and on 21 January 2015 the then coalition government put forward its proposals for implementation in "Scotland in the United Kingdom: An Enduring Settlement", (Cm8990).This paper is concerned with the Smith proposals relating to tax. What is proposed in the light of Cm8990 is that Scotland would be given control of certain taxes - principally non-savings, non-dividend income tax, together with air passenger duty, and the aggregates levy: and that the Scottish government would receive the revenues from these taxes. In addition, while Scotland would have no control over the rates of VAT, about half of the VAT revenues attributed to Scotland would be assigned to the Scottish government. In total, Cm8990 estimates that about half of the Scottish budget would come from these tax resources, or those which Scotland already controlled.

In line with pledges made during the Scottish independence referendum campaign, however, a commitment was made to retain the Barnett formula. What would happen after the implementation of Smith is that there would be an abatement to the Scottish government's block grant as calculated by the Barnett formula, to allow for the tax revenues foregone by Westminster in relation to the various taxes to be devolved or hypothecated to Scotland. In line with the 'no-detriment' principle of Smith, the initial size of the abatements would be equal to the tax revenues raised by the various taxes at the then current UK rates of tax.

It was recognised, however, that the size of the abatements would need to be increased each year by an appropriate form of indexation. For income tax, what is proposed in Cm8990 is that indexation should be carried out using a method proposed by the Holtham Commission for Wales: (Holtham Commission, 2010). Under this "Holtham" approach, the abatement for income tax would be increased each year in line with the increase in the overall income tax base for the UK.

At time of writing, the precise details of Holtham indexation have not been specified: but it was recognised by Holtham himself that the method could penalise Scotland if the Scottish tax base did not grow as fast as that for the UK as a whole. As Professor Holtham himself said, in evidence to the Scottish Parliament Finance Committee in April 2013, the method "*might not be in Scotland's interest if [the Scottish] tax base grows more slowly than that of the UK*": (Scottish Parliament Finance Committee, 2013).

No detail is available at present on the proposed indexation methods for the abatements to the Scottish block grant for the other devolved taxes: this paper will concentrate solely on the approach currently proposed for income tax, (which is, by a good margin, the largest of the taxes to be devolved.)

III Background (2): The effect of relative population change on the Barnett formula

As already noted, a previous paper in this commentary, (Cuthbert, 2001), analysed what the effect would be on the Barnett formula if population was growing in England relative to Scotland - as has been the case for many years. This section recapitulates, (without going into any proofs), on the notation and formulae established in that paper.

Under the Barnett formula, the change in the Scottish government's Departmental Expenditure Limit (DEL) each year is determined as a per capita share of the change in expenditure on corresponding services in England. In Cuthbert (2001) a simplified model of Barnett was developed, under which the Scottish DEL for any given year is adjusted only once by Barnett, when the new baseline for that year is first established. Another simplification is the assumption that public expenditure in England is growing by a constant percentage each year.

Specifically, the following notation and assumptions were used:-

Let E_t denote expenditure in England in year t, and $E^S{}_t$ expenditure in Scotland: (strictly, "expenditure" here is that covered in the relevant DEL).

Let p_t denote population in England in year t, and $p^S{}_t$ population in Scotland.

Let R_t denote the ratio of per capita expenditures between Scotland and England at time t.

Let k denote lag, (in years).

It was assumed that

a) $E_{t+1} = \theta E_t$: (i.e. expenditure in England grows at a constant rate.)

b) $\frac{p_{t+1}}{p_t} = \lambda \frac{p^S{}_{t+1}}{p^S{}_t}$ for all t, where $\lambda \geq 1$: (i.e., there is a constant relative rate of growth of population in England relative to Scotland).

c) In the annual public expenditure planning round, the new final year baseline is determined as being equal to the previous end year figure: and Barnett applies only to that end year, with population shares determined at a lag k.

The above model was solved, to show how the per capita spending relativity between Scotland and England, denoted by R_t, evolves through time from its initial starting value in year 0.

The relevant formula for R_t is as follows:

$$R_t = (\frac{\lambda}{\theta})^t [R_0 - \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}] + \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)} \quad \text{, for } \lambda \neq \theta . \qquad (1)$$

The derivation of formula (1) is given in the Annex to Cuthbert (2001)

What formula (1) means is that, in the circumstance where $\frac{\lambda}{\theta} < 1$, then the initial per capita spending relativity, R_0, will decay geometrically to the limiting value $\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$, which is a function of the expenditure growth rate in England, the rate of relative population growth, and the lag.

What this implies is that, when the nominal rate of growth in public expenditure is greater than the relative rate of population growth, then the Barnett formula will deliver convergence of the ratio of per capita spending levels in Scotland to England towards a limiting value. The significance of formula (1), however, is that it indicates that the limiting value will be 1 only if λ =1. If $\lambda > 1$, then the limiting value will be greater than 1: and in the circumstance where θ is not much greater than λ, the limiting value could be very much greater than 1. For example, a value of $\lambda = 1.002$, (a common historic value), and $\theta = 1.025$, (together with k = 4), would imply a limiting situation where public expenditure per head in Scotland was almost 10% above that in England.

These questions are explored in more detail in Cuthbert, (2001): but formula (1) explains a lot about why the Barnett formula has not actually brought about the convergence towards equality in per capita spending levels which many commentators were expecting.

Another important implication of formula (1) is that, if $\frac{\lambda}{\theta} > 1$, that is, if the rate of growth in nominal public expenditure in England falls below the relative rate of population growth in England compared to Scotland, then per capita expenditure will increase in Scotland relative to England, and will go on increasing. This implication was not studied in detail in the earlier paper, because at that time this situation was not expected to occur. But this has been the situation since 2010, given the cutbacks in public expenditure following the effects of the 2008 crash. And again, formula (1) explains how the Barnett formula, in the presence of relative population change (as between Scotland and England), has to some extent protected Scotland from the worst effects of UK public expenditure cuts.

IV Extending the model to include Holtham indexation of a Barnett abatement

In this section, the model outlined in the previous section is extended to cover the situation where there is an abatement to the Barnett formula block grant for tax revenues which the Scottish government will receive direct: and where this abatement is indexed using the Holtham approach. In developing this model, a number of simplifying assumptions are made (in addition to the simplifications in the original approach.) In particular, it is assumed that the Scottish government adopts a neutral tax policy, under which it does not change its tax rates away from those current in the rest of the UK, (rUK): and it is assumed that the ratio of tax receipts to the tax base stays constant through time, in both Scotland and the UK as a whole.

Additional notation, and further assumptions, are as follows.

Let $T_t^E, T_t^S, \text{and } T_t$ represent, respectively, tax revenues in England, Scotland and the whole UK in year t.

Let ϕ be the relative rate of growth in the tax base in England as compared to Scotland. It is assumed that ϕ is constant from year to year. In line with the above assumption that tax take is proportional to tax base, it follows that

$$\frac{T_t^E}{T_{t-1}^E} = \phi \frac{T_t^S}{T_{t-1}^S} \text{, for all t.}$$

Let a_t represent the abatement to the Barnett formula block grant in year t: then $a_0 = T_0^S$, (given the no-detriment assumption in setting the initial abatement), and

$a_t = \frac{T_t}{T_0} a_0$, under Holtham indexation, given the assumption that tax take is proportional to the tax base.

Let $\dot{E}_t^S$ represent abated expenditure in Scotland in year t:

therefore $\dot{E}_t^S = E_t^S - a_t + T_t^S$.

Finally, let $\dot{R}_t$ represent relative per capita spending levels in Scotland and England, when Scotland receives the abated block grant, plus its own revenues on devolved taxes.

Then it is shown in Annex 1 that

$$\dot{R}_t = R_t - (\frac{a_0}{T_0})(\frac{p_0}{p_0^S})(\frac{T_t^E}{E_t})\lambda^t(1-\phi^{-t}) \qquad (2)$$

where R_t is as given in formula (1).

Note that, if it is assumed that the term $(\frac{T_t^E}{E_t})$ remains roughly constant, (that is, if the share of "devolved" expenditure in England funded by "devolved" taxes remains roughly constant), then the second term on the right in formula (2) will be of the form $-K\lambda^t(1-\phi^{-t})$, where the constant K in this expression is a fraction, with a value approximately equal to 0.5. This follows since the first term in brackets in equation (2) is Scotland's initial share of UK taxes, and the second term is the ratio of English to Scottish population, so the product of these two terms will be approximately 1. The third term is the share of expenditure in England, (on the same services as are devolved in Scotland), which is funded by taxes which are devolved in Scotland: the corresponding figure is approximately 0.5 in Scotland, and the English figure is likely to be broadly similar.

So, roughly speaking,

$$\dot{R}_t = R_t - K\lambda^t(1-\phi^{-t})$$, where K is approximately equal to 0.5. (3)

In deriving the approximate expression in formula (3), a number of further assumptions are clearly being made: for example, it involves sweeping up all forms of taxation into a composite aggregate: it involves assuming that the abatements for the non-income tax element of the aggregate are indexed by something like Holtham indexation: and it involves assuming that the relative growth rates for the different tax bases, (the ϕ values for each element), are the same. Nevertheless, while bearing these assumptions in mind, the approximation in formula (3) is a useful guide as to how the dynamics of the post-Smith system are likely to evolve.

V Implications

So what are the implications of the above analysis?

i) **When tax bases grow at the same rate, the Barnett formula works as at present:** An immediate implication of formula (3) is that, if ϕ = 1, (that is, when the tax base in Scotland is growing at the same rate as that for the UK as a whole), then the last term is equal to zero, and hence the evolution of relative per capita spend would be exactly as under the original Barnett formula. This is as expected; Holtham indexation is neutral when the tax bases grow at the same rate.

ii) **But things are very different if the tax bases do not grow at the same rate.** If $\lambda \geq 1$, (that is, if population is growing relatively faster in England as compared with Scotland, as has been the case for many years), and if $\phi \neq 1$, (that is, if the tax bases are not growing at the same rate), then the formulae imply that relative per capita spend will eventually move to values which are, under any criterion, untenable. In the most stable case, when $\lambda = 1$ and $\phi > 1$, formula (3) implies that, for large t, $\dot{R}_t$ will behave asymptotically as $R_t - K$: which would imply, if the unadjusted Barnett formula was leading to long term convergence of per capita spending levels to something like parity, that public expenditure per head in Scotland would converge to something like 50% of the value in England.
In the less stable case, where $\lambda > 1$, then the final term in formula (3) diverges – upwards if $\phi < 1$, and downwards if $\phi > 1$.

iii) **And increases in relative population change magnify the effects.** Finally, it is worth noting that, because of the λ^t component in the final term of equation (3), any increase in λ, (that is, the rate of relative population change in England compared to Scotland), magnifies the effect of Holtham indexation.

VI Potential Dynamic Effects

What the previous section means is that, (other than in the unlikely case where $\phi = 1$), the system is such that it cannot proceed indefinitely with constant values of θ, λ and ϕ: eventually, relative values of per capita spend in Scotland as compared to England would move to values which would be politically unacceptable, and something would have to change.

Such a change might come about as a result of policy action: either by the Westminster government, (e.g. in the form of a fiscal transfer): or through specific policy action by the Scottish government. There is more discussion of this possibility later. But left to itself, it appears likely that the way the parameters in the system will evolve will be destabilising, rather than stabilising.

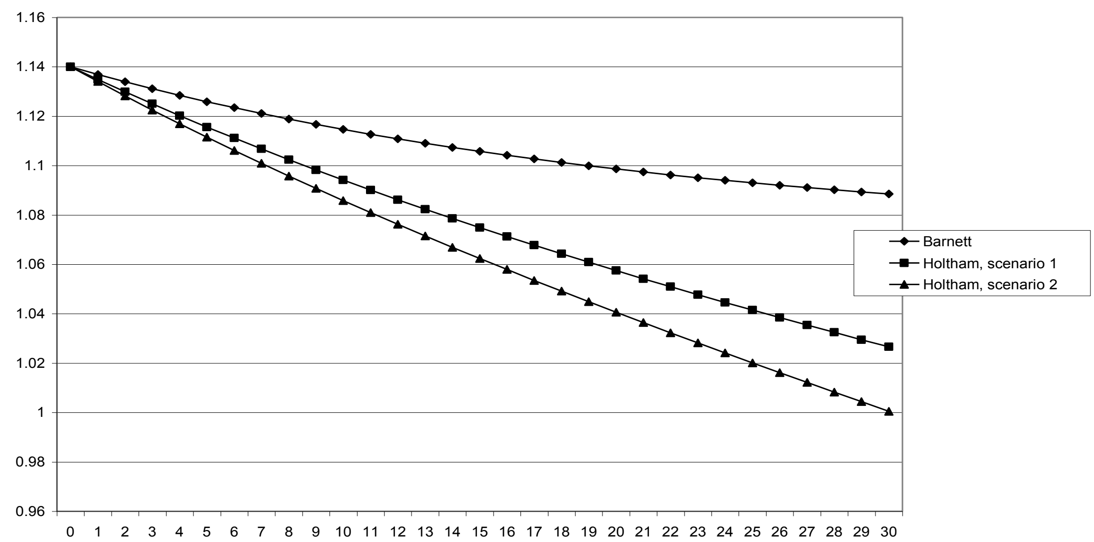
To illustrate this kind of possibility, consider the following hypothetical scenario. Suppose that, due to a boom in financial services in the City, there was an increase in English tax receipts, and that the Westminster government responded to this by increasing the rate of planned public expenditure growth. The implication, in terms of the model, is that the parameter θ, (the rate of growth of public expenditure in England), and ϕ, (the relative rate of growth in the tax base), would both increase at some specific time. Looking at formula (3), the effect of the increase in θ will be to reduce the limiting value to which the unadjusted Barnett formula is converging, (i.e. the term $\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$): to increase the rate of convergence to that value, (as the term $\frac{\lambda}{\theta}$ becomes smaller), and to increase the amount subtracted off in the final term of formula (3), (since $K\lambda^t(1-\phi^{-t})$ is an increasing function of ϕ). In other words, the increases in θ and ϕ will both have the effect of reducing $\dot{R}_t$, (that is, relative per capita spend in Scotland as compared to England).

But the effects are unlikely to simply end there: the decline in $\dot{R}_t$ is likely to have a depressing effect on the Scottish economy: and that, together with the initial stimulus to the economy in London and the South East, is likely to contribute in turn to an increase in the relative population growth parameter, λ. The effect of an increase in λ works in different directions in the different components of formula (3): in the Barnett element, it will increase the limiting value, and slow the rate of convergence: but the λ^t term will magnify the Holtham reduction element. For reasonable values of the parameters, the latter effect will dominate, however. So the overall effect of increasing λ will be to further reduce $\dot{R}_t$.

The danger is that the effect of reducing relative per capita spend in Scotland, $\dot{R}_t$, will be to further depress the Scottish economy, with further knock on effects on the relative growth rates of the tax base and of population. In other words, the adverse effects of the long term behaviour of $\dot{R}_t$ with fixed parameter values, which as has already been noted are untenable, (unless $\phi = 1$), are likely to be accentuated by further dynamic effects on the parameters.

Of course, for a mechanism of this type to take effect, it would have to be the case that changes in the model parameters, of the order of magnitude that might reasonably be expected in the real world, would have material effects on $\dot{R}_t$. Figure 1 below illustrates that modest changes in parameter values can indeed have material effects. The figure considers two scenarios. Under Scenario 1, the values of the parameters are θ = 1.05, λ = 1.0028, and ϕ = 1.005: and under Scenario 2, θ = 1.06, λ = 1.004, and ϕ = 1.008 . What figure 1 shows is how the ratio of relative per capita spend in Scotland to England would evolve from an initial value of 1.14, first of all under the pure Barnett formula, (with values of θ and λ as in Scenario 1): then under Holtham indexation, with Scenario 1 parameters: and finally under Holtham indexation and Scenario 2 parameters. The figure illustrates how the fairly modest changes in parameter values between the two scenarios do indeed have a quite marked effect on the rate of decline of $\dot{R}_t$. (The figure also illustrates how, for the scenario 1 parameter values, Holtham indexation is indeed much less favourable for Scotland than the original pure Barnett formula.)

Figure 1: S/E per capita expenditure ratio, under Barnett, and two scenarios with Holtham



The overall implication is that, without active policy intervention, the system currently being set up is likely to be unstable. Which then raises the question: are the available policy levers such that the system can be adequately controlled?

At this point the argument becomes more speculative. But as regards the policy levers wielded by the Scottish government, it appears unlikely that they would be sufficient to counter the kind of adverse dynamics outlined above. The economic powers which the Scottish government will possess after the implementation of Smith will themselves be fairly limited. The Scottish government is likely to have little scope to counter a decline in $\dot{R}_t$ by raising tax rates – because if tax rates get badly out of line with rUK, that in itself will depress the Scottish economy, giving a further adverse push to ϕ and λ. And in a situation where $\dot{R}_t$ is already declining, the scope for boosting the economy by radically cutting tax rates will be limited.

Overall, the conclusion is that the system is unlikely to be stable unless Westminster is prepared to actively deploy the other potential type of policy measure – namely, adjustments to overall fiscal transfers.

The above discussion illustrates the possibility of Scotland becoming locked into a cycle where relative expenditure per head, compared to England, is aggressively reduced, much faster than would happen under the Barnett formula: and where, unlike the Barnett formula, these reductions would not stop once parity was reached. However, in the long run, the chances of a converse cycle, where $\dot{R}_t$ progressively increases, seem unlikely. This is because, assuming that there is long run economic normalisation of the UK economy, and that nominal public expenditure maintains a roughly constant share of nominal GDP, then relatively high trend values of UK nominal public expenditure growth could be expected, which would imply a high value of θ. If the Scottish economy did start to boom, then this is likely to depress the relative population growth parameter, λ. The combination of a high θ and low λ would imply that the unadjusted Barnett formula would have an R_t value which would converge downwards fairly rapidly towards 1. Against the background of a declining Barnett term, R_t in formula (3), the chances of an unstable upswing in relative per capita spend, $\dot{R}_t$, look remote.

VII A suggested adjustment to Holtham indexation.

Under Holtham indexation as currently proposed, neutrality will occur if the tax base in Scotland grows at the same rate as the tax base in the UK as a whole. If the UK population is growing relative to that in Scotland, then for this condition to be satisfied, it must hold that the tax base per head in Scotland must grow at the same rate as the tax base per head in the UK, multiplied by the relative rate of population growth in the UK as a whole compared to Scotland. This implies that, for Holtham indexation to be neutral, Scotland must grow its tax base per head at a faster rate than the per capita tax base in the UK as a whole.

An alternative criterion for neutrality would be that the system should be neutral if the *per capita* tax bases in Scotland and the UK were growing at the same rate. To achieve an indexation system which satisfied this neutrality condition, what would be required would be to use an indexation factor which was equal to the growth in the UK tax base over the relevant period, divided by the relative growth in the UK to Scottish populations over the period.

There is a good argument in terms of equity for making this adjustment to Holtham indexation. In addition, making the change has the effect of somewhat dampening the instability that is a danger with an unadjusted Holtham indexation. Without going into the detail of the algebra, it can be shown that the effect of the adjustment is to replace formula (2) above by an expression for $\dot{R}_t$ which can be approximated by

$$\dot{R}_t = R_t - (\frac{a_0}{T_0^E})(\frac{p_0}{p_0^S})(\frac{T_t^E}{E_t})(1 - (\frac{\lambda}{\phi})^t) \qquad (4)$$

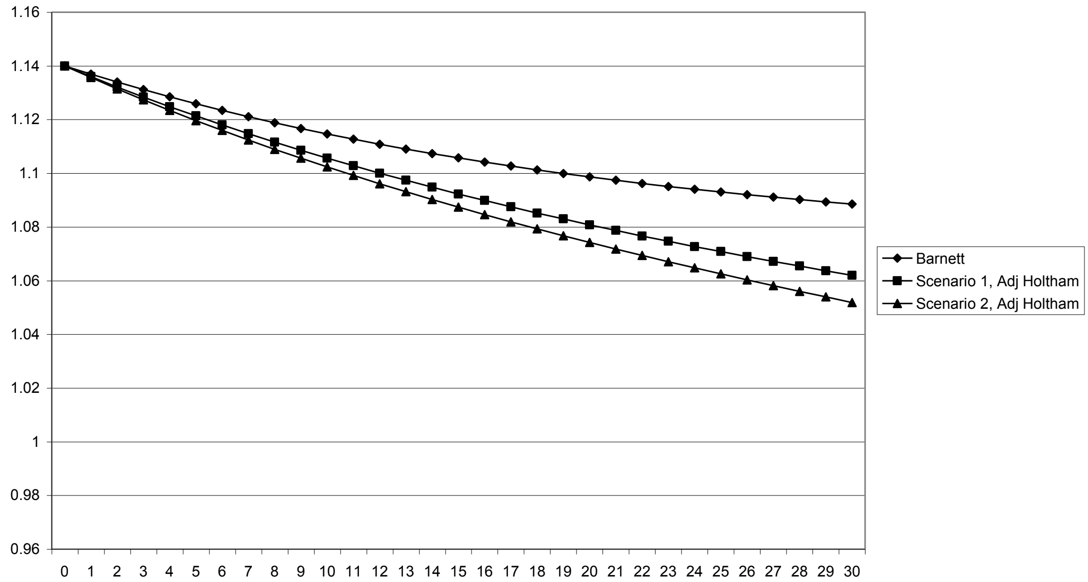
The expression in formula (4) will converge if $\phi > \lambda > 1$, (assuming $\theta > \lambda)$, whereas unadjusted Holtham indexation would diverge in these circumstances.

The adjusted system is still not satisfactory: the limit of equation (4) would still, in the long run, represent an untenable ratio of per capita expenditure. But the behaviour of the system will be much more damped: so the potential for adverse dynamic effects is reduced.

As an illustration of the effect of the adjustment to Holtham indexation, Figure 2 shows exactly the same scenarios as Figure 1, but now with adjusted Holtham indexation. It can be seen that the adjustment has had the effect of significantly reducing the departure from the original Barnett formula: and of reducing the effect of moving from Scenario 1 to Scenario 2.

There are therefore technical, as well as equity, arguments for making the suggested adjustment to Holtham indexation: although, as has been seen, the long run position even with the adjustment is still likely to be untenable.

Figure 2: S/E per capita expenditureratio: under Barnett, and two scenarios with adjusted Holtham



VIII Conclusion

This paper has modelled the way in which the revenues received by the Scottish government will behave, under the type of arrangement currently proposed for the implementation of the Smith Commission reforms as set out in Cm8990: specifically, what is considered is the kind of arrangement being proposed for income tax, under which there will be an abatement to the Barnett formula block grant in relation to the tax revenues foregone by Westminster, and this abatement will then be indexed by what is known as the Holtham approach.

A number of simplifying assumptions have been made: e.g. that the Scottish government adopts a neutral tax policy: and that tax revenues maintain a constant proportion of the tax base, in both Scotland and the UK. Nevertheless, despite the magnitude of these simplifying assumptions, the results of the modelling are of considerable interest, since they illustrate the underlying pressures which are likely to shape the dynamics of the system.

There are three key parameters in the system: the rate of growth of public expenditure in England, (θ): the relative rate of growth of population in England compared to Scotland, (λ): and the relative rate of growth of the tax base in England compared to Scotland, (ϕ).

What the model shows is that, other than the unlikely case when ϕ = 1, then for fixed values of $\theta, \lambda \text{ and } \phi$, the system will evolve towards a position where relative per capita spending levels in Scotland and England are *so different* that the situation is politically untenable. What this implies is that the system cannot run on indefinitely for fixed values of $\theta, \lambda \text{ and } \phi$: something would have to change.

In fact, the situation is worse than this, because, as the paper explains, there are likely to be dynamic feedback effects on θ, λ and ϕ which will make the system more unstable.

The implication is that, to maintain the operation of the fiscal system in some reasonable form of stability, active policy intervention will be required, by the Scottish government, by Westminster, or by both. The policy levers available to the Scottish government are so limited that it is unlikely to be able to maintain stability on its own. (It is worth remembering that the Scottish government will have control of only a single major tax, income tax: that it will have restricted borrowing powers: and that it lacks control of competition policy, international trade development, licensing of North Sea oil, utility regulation, and a number of labour market responsibilities.) This implies that an active monitoring of the system by Westminster, and adjustment of fiscal transfers, are likely to be required.

The paper also proposes an adjustment to crude Holtham indexation which, while by no means providing a complete answer to the likely problems, has strong equity arguments in its favour, and would also have a stabilising effect on the system.

References

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Annex 1: Proof of formula (2)

The notation is as in the main part of the paper. The proof proceeds in a number of steps.

1) Express T_t^S in terms of T_t^E.

Since, by definition, $\frac{T_t^E}{T_{t-1}^E} = \phi \frac{T_t^S}{T_{t-1}^S}$, for all t, it follows that

$$\frac{T_t^S}{T_t^E} = \phi^{-1} \frac{T_{t-1}^S}{T_{t-1}^E} = ... = \phi^{-t} \frac{T_0^S}{T_0^E} :$$

therefore $T_t^S = \phi^{-t} \frac{a_0}{T_0^E} T_t^E$, since $T_0^S = a_0$.

2) Calculate the Holtham indexation factor.

$$T_t = T_t^E + T_t^S = T_t^E (1 + \phi^{-t} \frac{a_0}{T_0^E}) :$$

therefore, indexation factor = $\frac{T_t}{T_0} = \frac{T_t^E}{T_0} (1 + \phi^{-t} \frac{a_0}{T_0^E})$.

3) Calculate adjustment to Barnett block grant.

Adjustment to Barnett block grant

= - (indexed abatement) + (Scottish tax revenues)

$$= - \frac{T_t^E}{T_0} (1 + \phi^{-t} \frac{a_0}{T_0^E}) a_0 + \phi^{-t} \frac{a_0}{T_0^E} T_t^E$$

$$= - T_t^E (\frac{a_0}{T_0}) [1 + \phi^{-t} \frac{a_0}{T_0^E} - \phi^{-t} \frac{T_0}{T_0^E}]$$

$$= - T_t^E (\frac{a_0}{T_0}) [1 + \phi^{-t} \frac{a_0}{T_0^E} - \phi^{-t} \frac{(a_0 + T_0^E)}{T_0^E}]$$

$$= - T_t^E (\frac{a_0}{T_0}) [1 - \phi^{-t}] .$$

Hence $\dot{E}_t^S = E_t^S - T_t^E (\frac{a_0}{T_0}) [1 - \phi^{-t}]$.

4) Calculate $\dot{R}_t$.

$$\dot{R}_t = \frac{\dot{E}_t^{\,S}}{p_t^{\,S}}\frac{p_t}{E_t}$$

$$= R_t - (\frac{a_0}{T_0})(\frac{T_t^{\,E}}{E_t})[1-\phi^{-t}](\frac{p_t}{p_t^{\,S}})$$

$$= R_t - (\frac{a_0}{T_0})(\frac{p_0}{p_0^{\,S}})(\frac{T_t^{\,E}}{E_t})\lambda^t[1-\phi^{-t}] \text{ , since } \frac{p_t}{p_t^{\,S}} = \lambda^t(\frac{p_0}{p_0^{\,S}}):$$

thus establishing formula (2).

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