

Aerodynamic Separability in Tip Speed Ratio and Separability in Wind Speed- a Comparison

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Abstract. From extensive application over a number of years, it has been established that the nonlinear rotor aerodynamics of typical medium and large wind turbines exhibit an effectively global separability property, in other words the aerodynamic torque of the machine can be defined by two independent additive functions. Two versions of the separability of aerodynamic torque for variable speed wind turbines are investigated here; the separated function, related to wind speed, in the first version is only dependent on that variable and not rotor speed and in the second version is only dependent on tip speed ratio. Both provide very good approximations to the aerodynamic torque over extensive neighbourhoods of T_0 , at least from 0 to $2T_0$.

1. Introduction

The aerodynamic torque, T , for a wind turbine is dependent on the wind speed, V , the blade pitch angle, β , and rotor speed, ω ; that is,

$$T = \frac{1}{2} \rho A R V^2 C_Q(\lambda, \beta) \quad ; \quad \lambda = \omega R / V \quad (1)$$

where ρ is the density of air, A is the rotor area, C_Q is the torque coefficient and λ the tip speed ratio. The separation of aerodynamics into two additive components, the first independent of wind speed and the second independent of pitch angle, is discussed in Jamieson et al [1], Leithead et al [2], [3], [4], [6] and Leith et al [7]. Surprisingly, this separability holds not just at the equilibrium operating points for variable speed wind turbines in above rated conditions, but over an extensive region of the operating space. For constant speed wind turbines, of course, neither component depends on rotor speed which does not vary.



In this paper, two different forms of separability of the aerodynamics for variable speed wind turbines are compared. The first form is purely empiric in derivation and is a direct extension of separability for a constant speed wind turbine. The second form is derived from the nature of the physical relationship (1). The paper is organised as follows. In section 2 and section 3, separability of the aerodynamics for constant speed wind turbines and for variable speed wind turbines, respectively, are discussed. In section 4, separability of the aerodynamics for variable speed wind turbines, based on empirical extension of the constant speed case, is presented. In section 5, separability of

$$T / \omega^2 \propto C_Q(\beta, \lambda) / \lambda^2 \quad (2)$$

is discussed and, in section 6, separability of the aerodynamics for variable speed wind turbines, based on the nature of this physical relationship, is presented. In section 7, the two forms of separability are discussed and, in section 8, some concluding remarks are made.

2. Constant speed wind turbines

For a constant speed wind turbine, the rotor speed is ω_0 and rated torque is T_0 . In above rated conditions, there is a pitch angle at which rated torque is attained for each wind speed; that is, there is a locus of equilibrium operating points on which the torque is T_0 . Provided $f_{\omega_0}(V)$ is monotonic, where $f_{\omega_0}(V)$ is the relationship of β to V along this locus of equilibrium points, and $h_{\omega_0}(\beta)$ and $g_{\omega_0}(V)$ are chosen such that

$$h'_{\omega_0}(\beta) = \frac{\partial T(\beta, f_{\omega_0}^{-1}(\beta))}{\partial \beta} \quad (3)$$

$$g'_{\omega_0}(V) = -\frac{\partial T(f_{\omega_0}(V), V)}{\partial V} \quad (4)$$

then, on the locus,

$$T(\beta, V) = T(\beta, V, \omega)|_{\omega_0} = \varepsilon_{\omega_0}(\beta, V) = T_0 + (h_{\omega_0}(\beta) - g_{\omega_0}(V)) \quad (5)$$

Furthermore, the partial derivatives by construction are, also, correct on the locus. The relationship of $g_{\omega_0}(V)$ to $h_{\omega_0}(\beta)$ is

$$g'_{\omega_0}(V) = h'_{\omega_0}(f_{\omega_0}(V)) \frac{df(V)}{dV} \quad (6)$$

$$\Rightarrow g_{\omega_0}(V) = h_{\omega_0}(f_{\omega_0}(V)) \quad (7)$$

Hence, on the locus, $(h_{\omega_0}(\beta) - g_{\omega_0}(V)) = 0$.

Locally to the locus of equilibrium operating points, $\varepsilon_{\omega_0}(\beta, V)$ is a good representation of $T(\beta, V)$, that is, within a sufficiently small neighbourhood of the locus the difference can be made arbitrarily small. More generally, near the locus,

$$T_{\omega_0}(\beta, V) \approx \tau_{\omega_0}(\varepsilon_{\omega_0}) = \tau_{\omega_0}(T_0 + (h_{\omega_0}(\beta) - g_{\omega_0}(V))) \quad (8)$$

for any function τ_{ω_0} such that,

$$\tau_{\omega_0}(\varepsilon_{\omega_0})|_{\varepsilon_{\omega_0}=T_0} = T_0 \quad (9)^i$$

$$\tau_{\omega_0}'(\varepsilon_{\omega_0})|_{\varepsilon_{\omega_0}=T_0} = 1 \quad (10)$$

By judicious choice of $\tau_{\omega_0}(\cdot)$, the size of the neighbourhood can be maximised. In effect, in a region that includes the whole set of equilibrium operating points, the dependence of aerodynamic

ⁱ On the locus of equilibrium points $h_{\omega_0}(\beta) - g_{\omega_0}(V) = 0$ and $\varepsilon_{\omega_0} = T_0$

torque on pitch angle is through $h_{\omega_o}(\beta)$ and on wind speed through $g_{\omega_o}(V)$ whilst the dependence on displacement of the operating point from the locus is through τ_{ω_o} [5].

Somewhat surprisingly, $T_{\omega_o}(\beta, V) = \tau_{\omega_o} \left(T_0 + (h_{\omega_o}(\beta) - g_{\omega_o}(V)) \right)$ applies for a large neighbourhood for all rotors, catering satisfactorily for a very broad range of values of pitch angle and wind speed that includes all the normal operating points. Furthermore, $\tau_{\omega_o}(\cdot)$, is generally a very weak nonlinear function. Typically, the neighbourhood includes values of aerodynamic torque from 0 to $3T_0$. Indeed, it only starts to break down on the rotor entering stall.

The significance of the above, particularly in the context of wind turbine control, is that the wind turbine can be viewed as a nonlinear dynamic system dependant only on the pitch angle (the nonlinearity $h_{\omega_o}(\beta)$) with an additive external disturbance dependent only on wind speed ($g_{\omega_o}(V)$)ⁱⁱ[5].

In Figure 1, the aerodynamic torque for a 3MW HAWT is plotted against its separated representation, $\varepsilon_{\omega_o}(\beta, V)$. The range of pitch angles is from 3° to 19° and wind speed from 11.41m/s to 24.21 m/s. Rated torque is 1.586×10^6 . Clearly, $\varepsilon_{\omega_o}(\beta, V)$ is a good approximation to $T_{\omega_o}(\beta, V)$ even for operating points far from rated torque. A best fit to the data would be an appropriate choice for $\tau_{\omega_o}(\cdot)$.

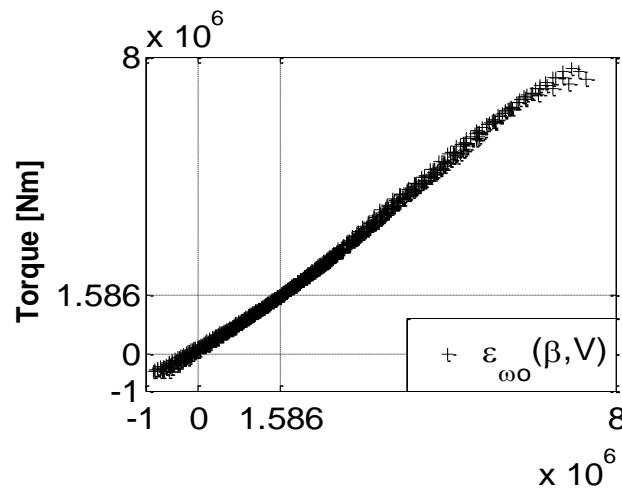


Figure 1. Separability for a constant speed pitch regulated wind turbine.

3. Variable speed wind turbines

For variable speed wind turbines, separation functions, $h_{\omega_o}(\beta)$, $g_{\omega_o}(V)$ and $\tau_{\omega_o}(\cdot)$, can be determined for different values of rotor speeds, ω_o . Hence, for the locus of equilibrium operating points, i.e all operating points, (β, V, ω) , for which $T(\beta, V, \omega)$ is T_0 ,

$$T(\beta, V, \omega) = T_0 + (h(\beta, \omega) - g(V, \omega)) \quad (11)$$

where $h(\beta, \omega)$ and $g(V, \omega)$ are defined by

$$h(\beta, \omega)|_{\omega=\omega_o} = h_{\omega_o}(\beta) \quad (12)$$

$$g(V, \omega)|_{\omega=\omega_o} = g_{\omega_o}(V) \quad (13)$$

ⁱⁱ Functions $h_{\omega_o}(\beta)$ and $g_{\omega_o}(V)$ are unique for each ω_o and rotor

Note, ω_o has a range of possible values, that includes rated rotor speed, ω_o , and the locus is now a 2-dimensional surface in a 4-dimensional space. In the neighbourhood of the locus of equilibrium operating points

$$T(\beta, V, \omega) \approx \tau\left(\omega, T_0 + (h_{\omega_o}(\beta) - g_{\omega_o}(V))\right) \quad (14)$$

where $\tau(\omega, \cdot)|_{\omega=\omega_o} = \tau_{\omega_o}(\cdot)$. Consider the set of points, (β_o, ω_o, V_o) , on the locus, for which the aerodynamic torque has constant value, T_0 . These points can be continuously parameterised by α such that

$$T(\alpha) = T(\beta(\alpha), \omega(\alpha), V(\alpha)) = T_0 \quad (15)$$

Since

$$\frac{\partial T}{\partial \beta} \frac{d\beta(\alpha)}{d\alpha} + \frac{\partial T}{\partial \omega} \frac{d\omega(\alpha)}{d\alpha} + \frac{\partial T}{\partial V} \frac{dV(\alpha)}{d\alpha} = \frac{dT(\alpha)}{d\alpha} = 0 \quad (16)$$

the partial derivative with respect to ω is simply related to the other two partial derivatives. Since the partial derivatives with respect to β and V of the separated form, $(h(\beta, \omega) - g(V, \omega))$, on the locus of equilibrium points are correct by construction, the partial derivative with respect to ω is also correct.

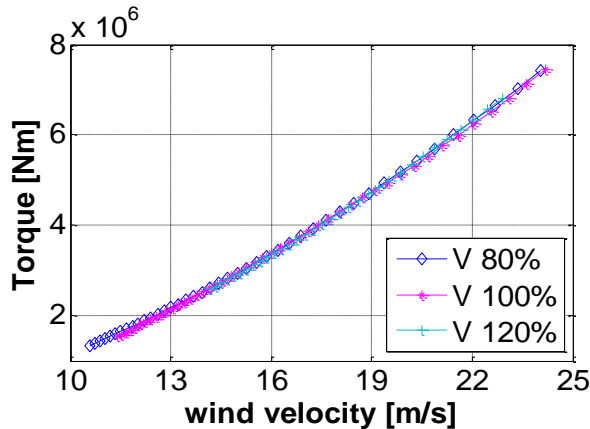


Figure 2 . Function $g(V)$ for 80%, 100% and 120% of ω .

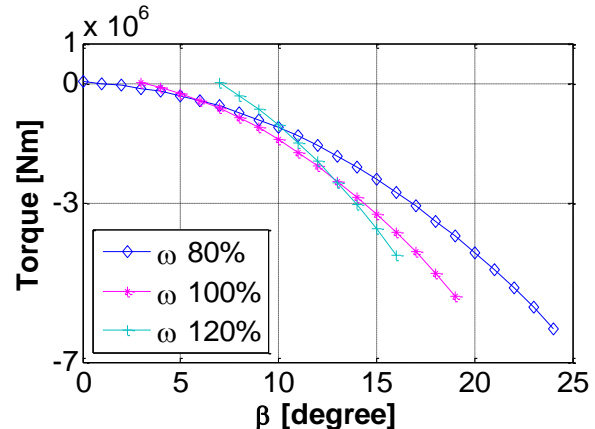


Figure 3. Function $h(\beta)$ for 80%, 100% and 120% of ω .

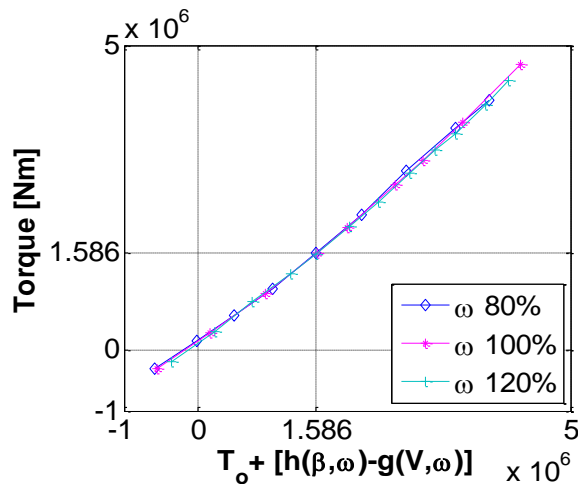


Figure 4. Function τ for 80%, 100% and 120% of ω .

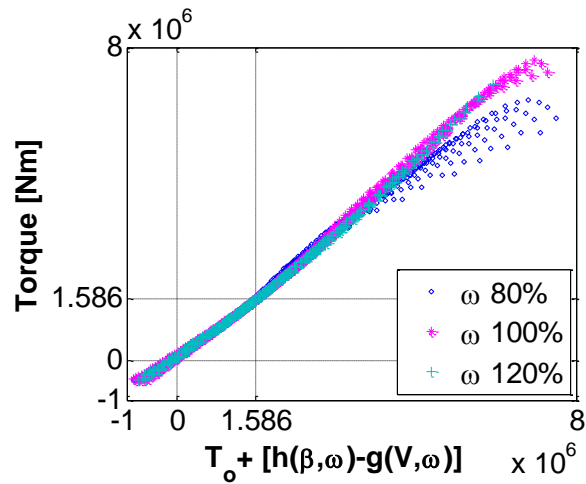


Figure 5. Separability for case studio 80%, 100% and 120% of ω .

The separated representation of the aerodynamics is determined for a variable speed HAWT with the same rotor characteristics as the constant speed HAWT in section 1. The rotor speed range is

between 80% and 120% of rated rotor speed, ω_0 . The functions, $g(V, \omega)$ and $h(\beta, \omega)$, are depicted in Figures 2 and 3, respectively, with ω values of 80%, 100% and 120% ω_0 . The similarity of the functions, $g(V, \omega)$, is striking. In Figure 4, the aerodynamic torque, $T(\beta, V, \omega)$, is plotted against its separated representation, $T_0 + (h(\beta, \omega) - g(V, \omega))$. The separated representation is again a good approximation.

4. Separability in wind speed

The observation, that the plots in Figure 2 are very similar, suggests strongly that separability could have a simplified form with $g(V, \omega)$ only dependent on V ; that is, on the locus of equilibrium operating points

$$T(\beta, V, \omega) = \varepsilon(\beta, \omega) = T_0 + (h(\beta, \omega) - g(V)) \quad (17)$$

For any choice of constant rotor speed, ω_0 , on the locus of equilibrium operating points,
 $h_{\omega_0}(\beta) - g_{\omega_0}(V) = h_{\omega_0}(\beta) + g_{\omega_0}(V) - g_{\omega_0}(V) - g_{\omega_0}(V)$

$$\begin{aligned} &= h_{\omega_0}(\beta) + g_{\omega_0}(f_{\omega_0}^{-1}(\beta)) - g_{\omega_0}(f_{\omega_0}^{-1}(\beta)) - g_{\omega_0}(V) \\ &= g_{\omega_0}(f_{\omega_0}^{-1}(\beta)) - g_{\omega_0}(V) \end{aligned} \quad (18)$$

Hence, choosing $g(V)$ to be $g_{\omega_0}(V)$, the function at ω_0 , an appropriate choice for $h(\beta, \omega)$ is

$$h(\beta, \omega)|_{\omega=\omega_0} = g_{\omega_0}(f_{\omega_0}^{-1}(\beta)) \quad (19)$$

In Figure 6, $h_{\omega_0}(\beta)$ and $g_{\omega_0}(f_{\omega_0}^{-1}(\beta))$ are compared with ω_0 values of 80%, 100% and 120% ω_0 . The pairs of functions match closely.

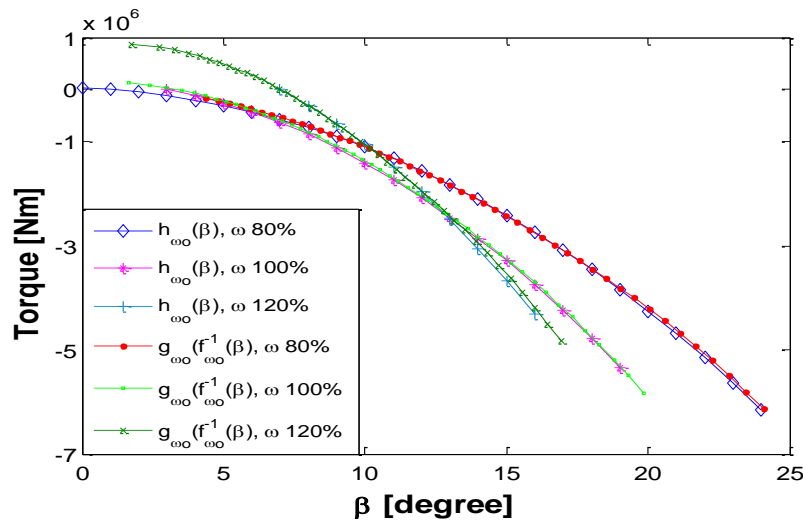


Figure 6. Validation of simplified wind speed separability

5. Separability in tip speed ratio

The aerodynamic torque is defined as

$$T(\omega, \beta, V) = \frac{1}{2} \rho A R V^2 C_q(\beta, \lambda) = K \omega^2 \frac{C_q(\beta, \lambda)}{\lambda^2} \quad (20)$$

where K is a constant. Separability is applied to $C(\beta, \lambda) = \frac{C_q(\beta, \lambda)}{\lambda^2}$ along the locus of operating points on which $C(\beta, \lambda) = C(\beta_0, \lambda_0) = C_0$ with $\lambda_0 = R\omega_0/V_0$ such that

$$C(\beta, \lambda) = C_0 + (H(\beta) - G(\lambda)) \quad (21)$$

on the locus. It can be seen from Figure 7 that the separated form, $C_0 + (H(\beta) - G(\lambda))$, is a good representation for $C(\beta, \lambda)$ for a very large neighbourhood enclosing the locus of equilibrium points. It includes values of $C(\beta, \lambda)$ that vary from 0 to $2C_0$.

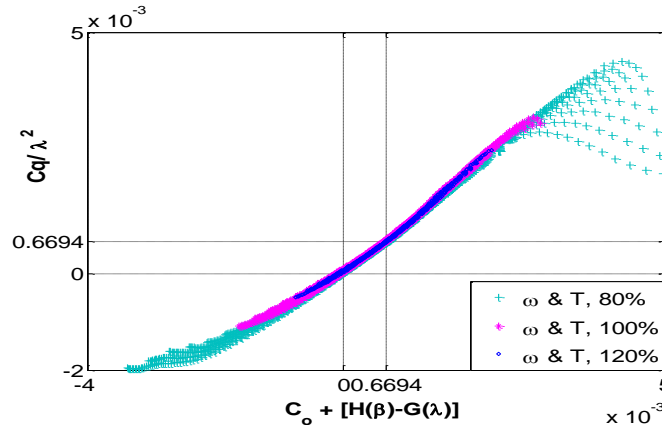


Figure 7. Separability for constant C_0 for variations of 80%, 100% and 120% of ω and T

This suggests strongly that separability could have the form

$$T(\beta, \lambda, \omega) = \omega^2(T_0/\omega_0^2 + (h(\beta) - g(\lambda))) \quad (22)$$

on the locus of equilibrium points with $T(\beta, \lambda, \omega) = T_0$. Note, the locus, on which $T(\beta, \lambda, \omega) = T_0$, is different from that, on which $C(\beta, \lambda) = C_0$. More generally,

$$T(\beta, \lambda, \omega) \approx \omega^2 \mu \left(T_0/\omega_0^2 + (h(\beta) - g(\lambda)) \right) \quad (23)$$

for some $\mu(\cdot)$ such that $\mu(T_0/\omega_0^2) = T_0/\omega_0^2$ and $\mu'(T_0/\omega_0^2) = 1$. The functions, $h(\beta)$ and $g(\lambda)$, are determined for the separated form of $T(\beta, \lambda, \omega)/\omega^2$ using data sets for 80%, 90%, 100%, 110%, 120% and 140% ω_0 for which $T(\beta, \lambda, \omega)$ is close to T_0 . Of course, these no longer remain local to T_0/ω_0^2 as ω varies. It can be seen from Figure 8 that the separated representation, $(T_0/\omega_0^2 + (h(\beta) - g(\lambda)))$, is a good representation for $T(\beta, \lambda, \omega)/\omega^2$. The function, $\mu(\cdot)$ is also determined independently using data sets for 80%, 100% and 120% ω_0 . In Figure 9, the $\mu(\cdot)$ scaled by ω^2 are shown.

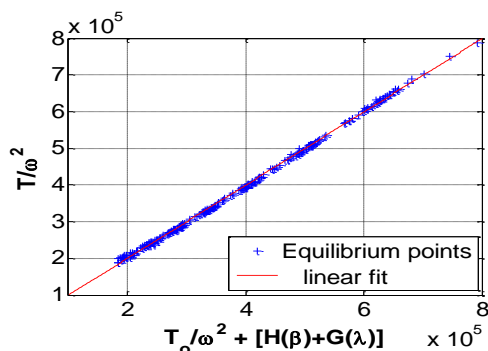


Figure 8. Equilibrium points after separation

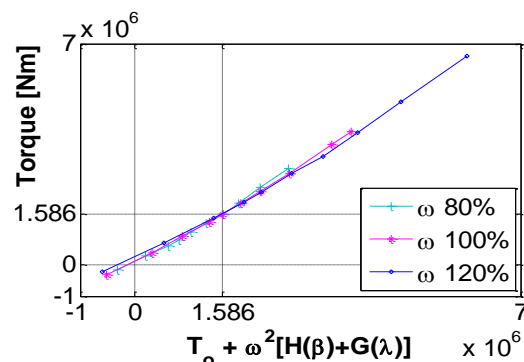


Figure 9. Function $\mu(\cdot)$ scaled by ω^2

6. Comparison of the two versions of separability

In Sections 4 and 5, two different versions of separability are discussed, specifically, a separation using wind speed, (17), and a separation using tip speed ratio, (23); that is,

$$T \approx \tau(T_0 + (h(\omega, \beta) - g(V))) \approx \omega^2 \mu(T_0/\omega_0^2 + (h(\beta) - g(\lambda))) \quad (24)$$

For wind speed separability, $h(\omega, \beta)$ and $g(V)$ together with optimised polynomial fits, $h(\omega, \beta) = a_\omega \beta^2 + b_\omega \beta + c_\omega$ and $g(V) = aV^2 + bV + c$, are depicted in Figure 10. For tip speed ratio separability, $h(\beta)$ and $g(\lambda)$, together with optimised fits $H(\beta) = a\beta^4 + b\beta^3 + c\beta^2 + d\beta + e$ and $G(\lambda) = a\lambda^{-2} + b\lambda^{-1} + c \ln(\lambda) + d\lambda + e$, are depicted in Figure 11.

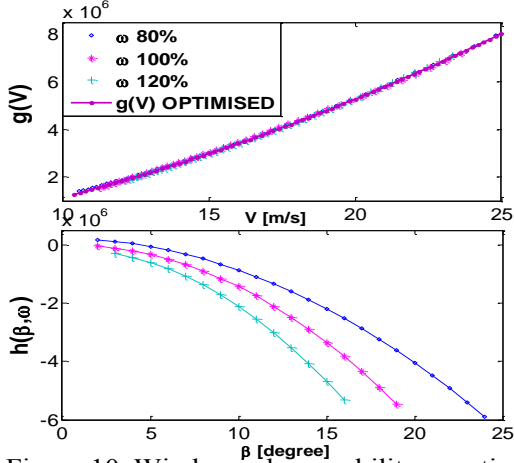


Figure 10. Wind speed separability equations

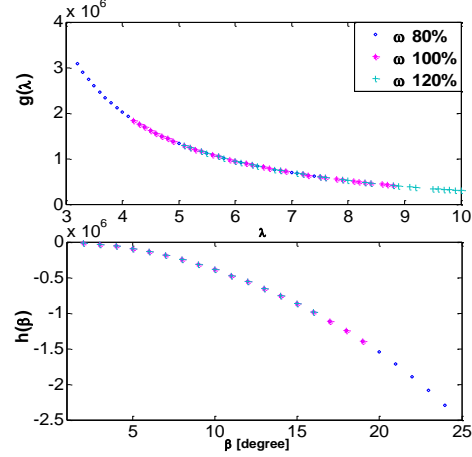


Figure 11. Tip speed ratio separability equations

In Figures 12, Figure 13 and Figure 14 the two versions of separability are directly compared for rotor speed of 80%, 100% and 120% ω_0 , respectively. Both are good approximations to the aerodynamic torque for large neighbourhoods enclosing the locus of equilibrium points. As would be expected, the 100% ω_0 case exhibits the closest match. The least accurate is the 80% ω_0 case but that is not unexpected since its operating points are the closest to stall.

For the tip speed ratio separability, the neighbourhood, for which the separated representation is a close approximation, extends in every case at least from 0 to $2T_0$, expanding as the set of operating points for the different rotor speeds move away from the stall region. It is particularly good approximation when $T \gg T_0$. For wind speed separability, the neighbourhood, for which the separated representation is a close approximation, also extends in every case at least from 0 to $2T_0$. It is particularly good when $T \ll T_0$. Although there is no first principle justification for the wind speed version of separability, i.e. for a $g(V)$ being independent of ω , the latter even locally to T_0 , it is at least as good as tip speed ratio separability.

Each figure is further accompanied by a blow up of the data close to T_0 , i.e. the data local to the equilibrium operating points. It can be seen that there is a small inaccuracy, particularly clear in Figure 12, in the tip speed ratio version of separability. The underlying reason for this inaccuracy is the implicit assumption in Section 5 that the data sets for 80%, 90%, 100%, 110%, 120% and 140% ω_0 should all lie on a straight line; that is $\mu'(T_0/\omega_0^2) = 1$ on the operating region defined by these data sets. That this assumption is not appropriate can be seen from Figure 7 or from Figure 9.

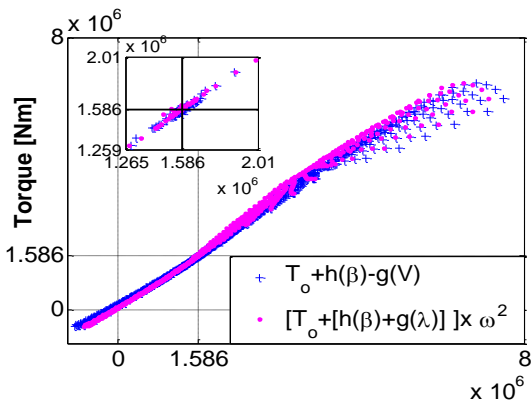


Figure 12. Separability comparison ω 80%

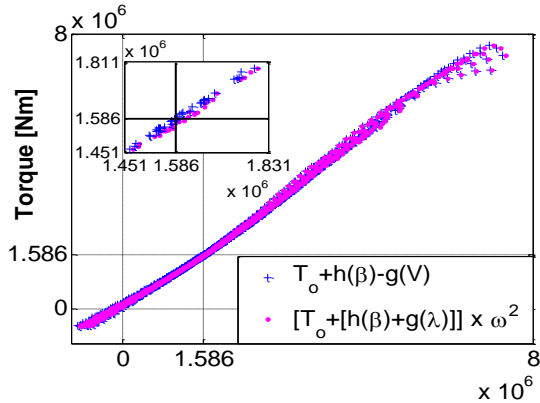


Figure 13. Separability comparison ω 100%

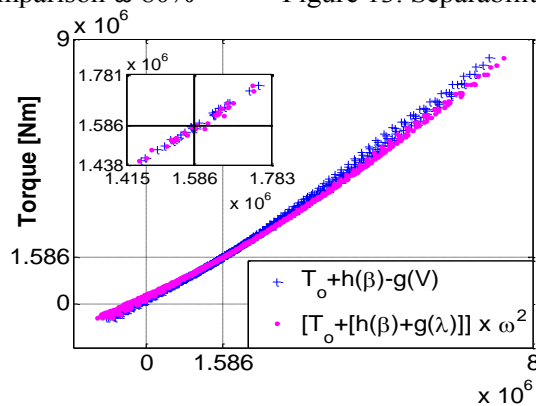


Figure 14. Separability comparison ω 120%

7. Conclusion

Two versions of separability of aerodynamic torque for variable speed wind turbines are investigated; the separated function, related to wind speed, in the first version is only dependent on that variable and not on rotor speed and in the second version is only dependent on tip speed ratio. Both provide very good approximations to the aerodynamic torque over extensive neighbourhoods of T_0 , at least from 0 to $2T_0$. If anything, the wind speed separability is the more accurate but has the least analytic support being purely empirically motivated. The two versions of separability are illustrated by application to a 3MW HAWT.

Acknowledgments

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