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Lectures and transition: from bottles to bonfires?

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Abstract

Despite much criticism, lectures remain a common and highly valued element of university mathematics teaching, with the potential to contribute significantly to learning. This potential, however, may not be realised because students’ and lecturers’ expectations, especially at the start of a degree course, are often very different. This chapter gives an overview of the difficulties and suggests ways in which lectures can be used to ease the transition from school to university mathematics.

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1 Introduction

The title of this chapter comes from a classic essay about education:

For the mind does not require filling like a bottle, but rather, like wood, it only requires kindling to create in it an impulse to think independently and an ardent desire for the truth. Imagine, then, that a man should need to get fire from a neighbour, and, upon finding a big bright fire there, should stay there continually warming himself; just so it is if a man comes to another to share the benefit of a discourse, and does not think it necessary to kindle from it some illumination for himself and some thinking of his own, but, delighting in the discourse, sits enchanted... (Plutarch, 1927)

The recipient of Plutarch’s advice was facing the ancient equivalent of the school–university transition. He had come of age, left his tutor’s guidance, and become an independent student attending public lectures in philosophy. The essay warns him that he will encounter both bad lecturers and good, that the crowd-pleasers and charlatans among them outnumber the true philosophers, and that in order to learn one must cultivate the ability to benefit from listening to lectures. Almost two millennia later, although much of education has changed recognisably, two points remain the same. Lectures remain, especially in mathematics, a key element of higher education; and many students find the transition from tutelage to independent learning very hard.

This chapter will first summarise the debate about lectures: what, if anything, are they good for? It will then look at the challenges that lectures present when students first encounter them, and suggest how they can help equip students for university mathematics.

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2 The debate around lectures

In contrast to their enduring use in mathematics, lectures are regarded in many disciplines as outdated and ineffective, and have come under sustained criticism since the work of Bligh (1972). Although a variety of criticisms have been levelled at them (see e.g. Gibbs, 1981), the fundamental objection is that lectures are essentially transmissive: they are simply a mediaeval technology for equipping students with slightly inaccurate versions of the lecturer’s own notes. This is troubling both because better reproduction technologies now exist and because mathematics is learned actively, by tackling problems, proving or disproving claims, and testing the logic of arguments. It is argued that we cannot therefore expect our students to learn from lectures.

In fact, lectures in university mathematics form one component of a wider process, such as the traditional cycle of lectures, homework and tutorials. If this cycle is provided, and if the students engage with it, then the question to ask is not whether lectures are a good way of transmitting the material, nor whether students can be expected to “learn from lectures”, but whether lectures help students to start the learning process. In this view, an effective lecture is one that equips or inspires students to engage in further genuine mathematical activity, and it cannot be considered in isolation.

When seen this way (Pritchard, 2010a,b), lectures have several strengths and can perform three main roles. They can be an effective way to communicate ideas, to model the ways in which mathematicians think and work, and to motivate students and shape their attitudes to mathematics. Although lectures are likely to be most effective when their multiple strengths are consciously recognised, traditional practice has evolved to exploit many of them implicitly (LMS, 2010). The resulting benefits may help explain why maths students continue to attend lectures even when alternatives are provided, and why those students who rely on lectures in preference to other resources tend to be the most successful (Cretchley, 2005; Inglis et al., 2011; Trenholm et al., 2012).

The merits of lectures, however, are only potential merits. If students have no opportunity to apply the ideas they meet in lectures to fresh problems, and to receive feedback on their attempts, then the best lectures imaginable will be of little benefit. If a student turns up diligently to lectures only, in Plutarch’s phrase, to “sit enchanted”, that student is not likely to learn much — and is liable to feel less enchanted come the exam.

Ultimately, the success of the lecture–homework–tutorial cycle depends both on students’ participation and on some shared understanding between students and lecturers of what is meant to happen. A lecturer may present three examples to illustrate a principle and find that a student interprets them as three distinct problem-solving templates, concealed within a bran tub of “theory”. A lecturer can spend a lecture motivating the proof of a major theorem, explaining its importance and sharing a sense of its beauty, only to be derailed by the question “Is all this going to be in the exam?” What any form of teaching can give students is inseparable from what students are prepared to take from it.

A useful way to express this is in terms of the “didactic contracts” that develop between students and teachers (Mason, 2002: 166–168). These contracts are generally never made explicit, and indeed, they may contradict the explicit expectations that students and teachers profess to have of each other. Rather, they are negotiated implicitly as each party responds to the behaviour of the other. We will see examples of these contracts, and how they may be negotiated, in the following sections.
3 Challenges around transition

As lecturers we tend to be painfully familiar with our students’ mathematical weaknesses when they arrive at university. These weaknesses fall into three categories (LMS, 1995): lack of technical facility (see also Gibson et al., 2005); inability to cope with extended problems; and failure to appreciate the logical nature of mathematics. Whether or not the picture is improving, concern about these weaknesses has continued (Royal Society, 2006), and there is still frequently a gap between what we would like to be able to expect from our incoming students and what they can immediately do.

This gap is important, but it is not the whole story. If we see our students’ difficulties solely in terms of the mathematics that they have failed to learn before they reach university, this may suggest what we ought to teach them, but it will not suggest how we ought to teach them. To approach this problem, it is necessary to ask more elusive questions: what expectations about maths and about learning do our students carry into the school–university transition, and what effect does the transition have on them?

3.1 Threats to the existing contract

Many students who choose to study maths at university do so principally because they were “good at maths” at school (Wiliam, 2005: 3; Robinson et al., 2010: §2.1). In practice this means that such students were good at carrying out complex but tightly specified procedures, in an environment where such procedural tasks were set regularly and frequently, and that their enjoyment of mathematics comes largely from performing these tasks efficiently (Wiliam, 2005; Quinlan, 2009). Effectively, these students and their schools have negotiated a contract in which the students undertake to follow accurately the procedures supplied by their teachers and textbooks; in return, the teachers undertake to provide regular affirmation of the students’ performance, and also to provide them with enough procedures to pass the assessments. A secondary element in many schools is that collaborative learning and communication are heavily emphasised, while classes are “kept interesting” by frequent changes of focus. Although this emphasis develops useful skills, the corollary is that individual responsibility for learning, and sustained attention to problems, receive less attention.

When students arrive at university, the familiar contract is threatened in several ways. First, most students discover that they are no longer among the top performers, and may feel for the first time that they are struggling with mathematics (Goulding et al., 2003: 376–377; Wiliam, 2005: 3–4). Students who are motivated by performance can be very vulnerable to failure, especially if they also believe their mathematical ability is innate and unalterable (Dweck and Molden, 2005). They are prone to stress, may be keen to blame their anxiety on external factors (Rodd, 2009), and may even fall into self-handicapping behaviour in which they follow ineffective study strategies that provide ready excuses for failure (Rhodewalt and Vohs, 2005).

A second threat is that the nature of the subject has changed. Procedures are less important; their place has been taken by argument and explanation, which students may perceive as discursive or directionless and thus less “logical” than following procedures (Daskalogianni and Simpson, 2001). A related threat is the higher level of abstraction, which students may see as irrelevant to real life (Robinson et al., 2010: §3.1) or — to quote one of my own disgruntled students — “making maths seem academic”.

Underlying these threats is a more fundamental one, which is that university education assumes a new balance of responsibility between students and teachers. The authority of a teacher or a textbook is no longer absolute, and students are expected to make their own judgements of validity and relevance (Mason, 2002: 177). The classroom is no longer where
all learning happens but where it starts: new ideas may be supplied very rapidly and without expecting everything to be immediately understood (Mason, 2002: 9). Homework is typically not compulsory and so requires more self-discipline, in addition to the ability to concentrate on single problems for extended periods. Even tutorials, though superficially similar to school classes, require more willingness to take the initiative (Alcock, 2012: §10.2) and to deal with genuinely open questions to which the questioner may not know the answer (Mason, 2002: 76).

The demands that the lecture–homework–tutorial cycle makes on students are, at its best, what make it effective, but they are also dangerous. One danger is that students may break under these demands; another, paradoxically, is that they may not realise the demands are there. Students who are used to being strongly directed often find it hard to appreciate that homework and independent study are essential to learning even when they are not compulsory (Macrae et al., 2003: 58–59; Wiliam, 2005: 7). The contract that they deduce from the lack of compulsion is one in which university education is largely transmissive and transmission mainly occurs in lectures; accordingly they turn up, in Plutarch’s phrase, like empty bottles waiting to be filled. When this approach inevitably fails, such students feel this as a breach of contract: they become confused and either angry or apathetic (Macrae et al., 2003; Rodd, 2009), and it is very hard for them to have a fulfilling mathematical education.

### 3.2 Developing new contracts

Many students, of course, do not give up following a difficult transition to university mathematics. During the transition period, they develop new habits of study and settle into new implicit contracts with their teachers, and once these are established, their confidence and enjoyment generally recover (Robinson et al., 2010: §3.1). The first few weeks or months of university are therefore a crucial time — the more so because some of the contracts that form are no more appropriate to university than those that were inherited from school. To understand them we need to examine both what students say about lectures and how they behave.

When asked why they attend or value maths lectures, students give fairly consistent responses (Cretchley, 2005; Hubbard, 2007; Slomson, 2010). Lectures are described as a good way to meet new material and acquire an overview of an area; to identify what lecturers think is important; to gain motivation and feel a sense of community; and, even since the advent of online resources, to acquire good notes. These are generally responses with which lecturers would concur — although it is possible that “what lecturers think is important” is merely code for “what will be in the exam”. Such findings suggest that students’ expectations are not hugely different from lecturers’.

Students’ behaviour, however, is not always consistent with their responses in surveys. In particular, although they appreciate lectures they prefer to behave very passively during them, avoiding interaction with each other and with the lecturer. Yoon et al. (2011) suggested that this was due both to fear of public embarrassment and to a belief that the purpose of lectures was for the lecturer to “cover the material”. This suggests a view of lectures as distinctly transmissive or even ritual: a contract where students’ main duty is to turn up because “you are supposed to” (a view expressed by 40% of respondents in Hubbard, 2007).

Many of the students interviewed by Yoon et al. (2011) claimed they expected to have to learn the material outside class. Self-reported study patterns, though, suggest that this expectation is optimistic. Figures from England (Bekhradnia, 2012) suggest that maths students typically work a little under 30 hours a week, of which half is spent in class: this ratio of self-study to class time suggests that many students rely on lectures for a large portion of their learning, whether or not they actually expect to learn much from them. Like many educationalists, these
students have fallen into the trap of trying to make sense of lectures in isolation from other activities.

The implicit contracts between students and teachers are not, of course, created solely by the students, and their confused expectations of lectures may reflect our own. As noted in §2, it is common to represent lectures as essentially transmissive, and this attitude must communicate itself to the students. It is also common to use “lectures” as shorthand for all university teaching, just as we use “lecturer” as shorthand for “university teacher”, and it is common when asking for student feedback and when trying to improve our teaching to focus on what can be done to improve or replace lectures. This focus is probably misplaced: as noted in §2, students’ performance is not improved if they rely on alternatives to lectures; and even innovations such as classroom response systems which appear to make lectures more effective have little measurable effect on achievement (King and Robinson, 2009). Although lectures lie more within our direct control than other aspects of the teaching cycle, the real problems may lie elsewhere.

All this is not to say that lectures are unimportant in the school-university transition. The fact that some students develop unrealistic expectations of lectures, and that we unconsciously collude with them in this, suggests that it is worth examining how our use of lectures shapes our contract with our students. At the same time, lectures can be used directly to address some of the threats arising from transition. This is the topic of the next section.

4 Addressing transition-related challenges through lectures

The preceding section argued that new university mathematics students face difficulties that arise both directly through weaknesses in their mathematical education and indirectly through the educational “contracts” that they inherit from school (cf. Daskalogianni and Simpson, 2001). Some of these difficulties may be addressed most effectively outwith lectures, but must be allowed for when we lecture. Others can be tackled directly using lectures alongside other elements of our teaching. This section will first look at what lecturers can do to help students deal with threats and weaknesses, and then at ways in which institutions can support or hinder lecturers in doing so.

4.1 The role of lecturers

4.1.1 Accommodating lack of technical facility

Of students’ mathematical weaknesses, lack of technical facility is the one that lectures can do least about. What is needed, as many students realise (Gibson et al., 2005), is practice. Homework and tutorials, supplemented with diagnostic tests and extra support where necessary, may be the best way to address this weakness. Another promising approach, which lecturers can sometimes employ, is to revisit school mathematics from different angles, reinforcing basic material without presenting it as remedial (Robinson et al., 2010: §7).

This weakness does carry implications for effective lecturing. Because some students see basic algebraic processes as major operations in themselves, they are easily disconcerted when these processes are passed over swiftly as part of a larger argument. This is particularly the case if students are used to looking for local rather than global understanding of a mathematical argument (Lithner, 2003). Surrendering to the demand to fill in all the details is time-consuming and reinforces the perception that these details are difficult and important. An alternative is to leave gaps for students to fill in later: it is essential in this case to advertise that this is what is intended, and to anticipate that some students will not do so.
4.1.2 Addressing inability to solve extended problems

Although the inability to tackle extended problems also results largely from lack of practice, it has an extra dimension because one of the purposes of lectures (Pritchard, 2010b) is to provide students with a model of how mathematicians think. Faced with an extended problem, we naturally try to break it into smaller problems and tackle them systematically: by demonstrating and narrating this process, we may help students to break through the mental block that descends on them when they are first confronted with unfamiliar problems. Such modelling will not be effective if students do not realise it is happening, so it is important to be explicit (cf. Mason, 2002: 56–57) and to advertise the occasions when we put the notes to one side and work from reason rather than from memory. Students also readily confuse what we write as part of a solution with the comments we make about the solution process, so it can be helpful to distinguish between these — for example, using a black pen for the solution, and a blue pen for comments and explanations — and encouraging students to do likewise in their own notes.

Using lectures to demonstrate problem-solving is popular with students (Robinson et al., 2010: §5.1). The trap that it presents is the “tyranny of examples” (Barton, 2011), a pernicious contract resembling that inherited from school, in which the lecturer undertakes to provide templates for every question that could appear in the exam, and the students undertake to memorise and reproduce them. It is necessary to resist the pressure — expressed persistently and plaintively through student feedback — to succumb to this tyranny, which arises because students misunderstand what mathematics is about.

4.1.3 Clarifying the nature of mathematics

The lecturer’s modelling role is even more important in addressing students’ perception of mathematics as a set of ritual procedures rather than a process of creative logical argument. In many cases, they misconceive what real mathematics looks like because they have never or rarely seen it. We can help to address this weakness, and to make the change of perspective less threatening, by doing maths, live, in front of them: “thinking and struggling” through problems (Atkinson et al., 2000); even, at best, providing a performance that engages their imaginations like those of a theatre audience (Rodd, 2003; Körner, 2013). Of course it is unrealistic to expect every lecture to be an inspiring piece of theatre, but most courses provide a few opportunities to share something of the intellectual fire inside the subject.

The dangers of presenting lectures as performances are that we focus too much on putting on a show, while students come to treat the lecture as a form of entertainment. Like those caricatured by Plutarch, they will enjoy the warmth without kindling fires for themselves. A related danger is that students may treat a charismatic lecturer as the source not just of social but of intellectual authority, and so never come to rely on their own reasoning. Part of the solution may be for lecturers to advertise our own fallibility, which brings us to the next point.

4.1.4 Mitigating anxiety and vulnerability

The anxiety and vulnerability experienced at the school–university transition probably cannot be eliminated, but they may be reduced by demonstrating that all mathematicians are fallible and by giving mathematics a human face. Wood et al. (2007) argue that the threat that abstraction presents to new students can be reduced when “a lecturer personifies the content of his/her lecture”, making an emotional connection to abstract concepts and connecting multiple representations of ideas. Lecturers can also reduce the threat presented by students’ encounter with failure and difficulties by “being human” (Mason, 2002: 57–58) and sharing our own experiences. Rather than presenting mathematicians as uniquely gifted geniuses, we can
demonstrate that we are human beings who meet difficulties and struggle through them — both making and correcting errors on the way.

Tackling problems live on the visualiser or chalkboard, rather than writing up a solution from notes or memory, naturally involves us in bringing multiple resources to bear on problems and in dealing with difficulties and errors. Once this pattern is established, even common errors that students make can plausibly be presented as errors that one has made oneself in the past (whether this is true or not). If not overused, this ploy can generate a real air of relief in the classroom. The disadvantage of live performance is that it involves risk: Körner (2013) likens lecturing both to a conjuring trick and to a high-wire act. Some students will interpret any mistakes we make as evidence of poor preparation or incompetence, and it needs some self-confidence to shrug off their criticism. It is certainly safer not to expose our own weaknesses in class, but it may be necessary to do so if we are serious about inviting students into a real experience of mathematics.

4.1.5 Establishing the purpose of lectures

The final challenge identified above was that students readily misunderstand the purpose of lectures and how they relate to other activities. This manifests itself in passive behaviour during lectures, and in a tendency to forget that homework and tutorials are necessary to develop an understanding that merely begins with lectures.

Like Plutarch, we can start to address this misunderstanding by explaining to our students what lectures are for and how to get the most out of them. Alcock (2012: §9) provides a good example; it might be worth going one step further and providing an overview of the entire lecture–homework–tutorial cycle, a pattern that seems so obvious to us that it may be invisible to students. Just explaining to students how to use lectures will certainly not be enough, however: we need to find ways to ease them into a new way of learning. The difficulties here are to introduce familiar elements without compromising the strengths of lectures, and to provide a smooth transition from strongly directed to more independent learning.

Every guide for new lecturers (e.g. Mason, 2002: chapter 2; Cox, 2011: chapter 3) contains suggestions of how to combine lectures with complementary activities. An increasingly popular approach is to set short questions and poll the class on them using a classroom response system (King and Robinson, 2009); a two-stage process (Nicol and Boyle, 2003), in which students first work and vote on questions individually and then revisit them in small groups, can be particularly effective. Such exercises help to vary the pace of a lecture and provide immediate feedback on students’ difficulties, though they can degenerate into another “tyranny of examples”.

An especially thorny question is whether students should be required to take notes in lectures or have full notes supplied online. The more that is supplied to students, the more accurate the information that they have and the less of their attention has to be devoted to copying from the board. Equally, the more that is supplied, the more likely students are to try, unsuccessfully, to manage without lectures (Cretchley, 2005; Inglis et al., 2011; Trenholm et al., 2012), and the less attention they will pay when they are there. Mason (2002: 64) gives a nice summary of the possible compromises: for many courses, some form of partial notes, which may state the theorems or the questions but leave the proofs or solutions to be constructed in class, seem to strike a reasonable balance.

Other practical issues with notes are less obvious. One is that many students arrive at university unable to read mathematical texts (Robinson et al., 2010: §5.1; Shepherd et al., 2012), so supplying written notes merely replaces a note-taking problem with a reading problem. Another is that contemporary students, used to keyboards, often write much more slowly than
those of earlier generations, while computer slideshows tempt us to present material far too fast. The traditional way to control pacing is to write by hand everything that students are expected to copy: on a visualiser, it is helpful also to use a pen that slows one’s writing down. (No pace, though, will be slow enough to satisfy some students, and it may be necessary to resist their protests.)

4.1.6 Using the lecturer’s authority

Finally and very generally, a crucial feature of lecturing is that it lends the lecturer an artificial social authority, as the figure on whom attention is focussed. This authority means that a lecturer’s behaviour can do a lot to shape the contract with new students. The points raised above concerning the nature of mathematics are one reflection of this: if we teach mainly set procedures we reinforce the procedural contract inherited from school; if we teach genuine mathematics we may start to renegotiate this contract.

Another aspect is classroom management. As schoolteachers know, this is especially important when dealing with a new class (see e.g. Cox, 2011: §§3.7, 3.9): by arriving promptly, dealing courteously with students and requiring the same behaviour of them (cf. Alcock, 2012: §9.6), lecturers can establish the attitude and the levels of attention that successful learning requires. By using lectures to set or distribute homework and to respond to matters raised in tutorials, one can also make it clear that different elements of the course are expected to reinforce each other, and that all of them are necessary.

The danger of a firm classroom management style is that it can reinforce students’ impression that they are expected to be entirely passive: seen in class but never heard. To mitigate this, a lecturer can use the “mandate to interact” that her status gives her (Yoon et al., 2011), for example explicitly instructing students to discuss a question with their neighbour. It is also wise to lay down some house rules that encourage interaction — such as the rule that “there is no such thing as a stupid question” — reiterating and acting on these until students come to believe them. At the end of the day, different lecturers have different personalities and will handle classes in different styles, but any successful lecturer implicitly uses her authority to shape the expectations of her students.

4.2 The role of institutions

The principal responsibility for lectures lies with lecturers, but our institutions can support or hamper us. Especially in first year, it is important that homework and tutorials are properly resourced, so that students are not forced to rely exclusively on lectures. Similarly, it is necessary to provide properly equipped and suitably designed rooms for lectures. Of course this is desirable for all lectures, but the large and uncertain student numbers in first-year classes tend to breed timetabling problems, while these students are simultaneously much more vulnerable to disruption because their study habits are still being formed. Many lecturers have had the experience of trying to muddle through the first few weeks of term in undersized or ill-equipped rooms; most of us will testify that students who have been introduced to university in this way can take a long time to settle down.

Institutional policies can undermine lectures in unintended ways. It is particularly important that student demands for online material are not answered by a blanket policy of providing everything online, regardless of the damage that this can do (Inglis et al., 2011). Another policy matter concerns student feedback and the attitudes to learning that it encourages. Surveys that treat students as consumers imply, and may reinforce, a sense that their educational contract places all the responsibility on the teachers; surveys that invite students to consider their
own engagement with a course may instil a different expectation. (Multiple-choice questionnaires seem to encourage particularly unreflective responses, as well as providing absolutely no useful suggestions for improvement.)

Perhaps the most important point to recognise is that lecturing to new undergraduates, whether on mathematics degree or service courses, is difficult. Mathematicians sometimes assume that the difficulty of teaching a course depends only on its content, so the most inexperienced staff should teach the most elementary courses. As the discussion above indicates, this is a mistake. To lecture effectively to new students, one needs the confidence to tackle problems live, the performance skills to motivate students without simply playing to the gallery, and the experience to sense and anticipate their technical difficulties. Any department that expects newly-appointed lecturers to possess these qualities has itself to blame should students emerge from first year unprepared for their degrees.

5 Summary

There are both good and bad reasons for lecturing and for attending lectures, and corresponding good and bad uses that staff and students can make of them. Two fundamental errors are to see lectures principally as a way of transmitting the contents of notes or textbooks, and to expect them to be effective in isolation from other learning activities. As lecturers, we sometimes make these errors; evidence suggests that our students are still more likely to make them, especially when their expectations are formed by their experience of strongly directed learning in school. Lectures are thus vulnerable to students’ misconceptions of what mathematical learning involves, and if used unreflectively they may exacerbate these misconceptions.

Nevertheless, lectures have a positive role to play, in which they are not readily replaced by other forms of teaching. In particular, they provide an invaluable opportunity to draw students into the world of university mathematics by modelling the processes of mathematical thinking, demonstrating that mathematics is about creative and logical thought rather than applying memorised algorithms to highly stereotyped problems. When they are properly used and adequately supported, lectures can be a powerful tool for rewriting our students’ expectations and helping them make the transition to university mathematics.

References


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