Signal amplification and control in optical cavities with off-axis feedback

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We consider a large class of optical cavities and gain media with an off-axis external feedback which introduces a two-point nonlocality. This nonlocality moves the lasing threshold and opens large windows of control parameters where weak light spots can be strongly amplified while the background radiation remains very low. Furthermore, transverse phase and group velocities of a signal can be independently tuned and this enables to steer it non mechanically, to control its spatial chirping and to split it into two counter-propagating ones.

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In this letter we study the effects introduced by a two-point nonlocality [1] on a broad class of nonlinear equations with both diffusion and diffraction. Systems modelled by this type of equations can be experimentally realised in optics by cavities with an off-axis external feedback, which is the spatial analogous of a feedback with temporal delay [2]. Off-axis feedback has been subject of theoretical and experimental study in liquid crystals light valves [3], Kerr-like media [4, 5] and generic nonlinear systems with diffusive coupling [6]. Here instead we consider a broad class of *optical cavities* using a general formalism that can be applied to gases, solid state and semiconductor media with fast decay of the polarization, including media with negative refractive index and devices with soft apertures. The simultaneous presence of diffusive and diffractive terms appears in universal Ginzburg-Landau equations describing the behaviour of any spatially extended system near the onset of oscillations, such as, for instance, reaction-diffusion systems and lasers [7]. These equations describe also properties of systems with time-delayed feedback and no spatial degrees of freedom when the delay is order of magnitude larger than the other time scales [8], with the slow time formally taking the role of the spatial variable.

We show that the inclusion of a two-point nonlocality generalises these equations introducing new regimes and is a powerful way to amplify, characterise and control perturbations, either external or intrinsic to the system. In particular, nonlocality changes the nature of the first instability, which without nonlocality leads to a spatially extended, lasing state. With nonlocality, on the contrary, there are large windows of control parameters where small localized signals can be strongly amplified while the background radiation in other region of the system remains very low. Furthermore, the signal moves across the cavity with transverse phase and group velocities that are easily managed to have the same or opposite signs. It is indeed possible, *without altering the mechanical alignment* of the set-up, to control signals motion, tuning continuously the group velocity so that a localized perturbation is steered either towards or against the off-set direction and can even be *split* into two counter-propagating components. The tunability of the phase velocities allows to control the spatial chirping of light signals independently from the direction of steering. These unusual properties open new possibilities for light control and can underpin applications in optical communications, imaging and micromanipulation.

In the following we analyse how the first threshold depends on nonlocality, diffusion and diffraction, determine the nature of the instability, find a second threshold and derive the equations for the phase and group velocity of localized perturbations. We consider optical systems described by non-dimensional equations of the type

$$\begin{aligned} \partial_t E &= g_1(|E|^2, N; \mu)E + e^{i\delta}\partial^2_{xx}E + re^{i\phi}E_{\Delta x}, \\ \partial_t N &= g_2(|E|^2, N, \partial^2_{xx}N; \mu), \end{aligned} \tag{1}$$

where E is the slowly-varying amplitude of the electric field, N is the population inversion and μ is a control parameter. We consider here one transverse dimension x as nonlocality changes only the spatial dependence of the dispersion along the direction of the shift. Time and space are scaled with field decay and with the square root of the modulus of the Laplacian coefficient. Our analysis encompasses devices with diffusion that is due to Fourier filtering by intracavity soft apertures [9] or to elimination of the fast variables [10], as well as media with positive or negative refractive index [11]. δ gives the relative strength of diffusion and diffraction, with $\delta \in (0, \pi/2)$ for positive refractive indexes and $\delta \in (-\pi/2, 0)$ for negative indexes, corresponding to left-handed materials. The term $re^{i\phi}E_{\Delta x}$ represents nonlocal coupling of the field E in a point x with the field $E_{\Delta x}$ in a point $x + \Delta x$ and is the consequence of an off-axis, single-passage feedback loop. This is characterised by an amplitude $0 < r < 1$ and a phase shift ϕ accumulated by the fast component of the electric field in the external loop. We assume here that the temporal delay of the feedback is negligible compared to the time scales of E and N. The generic complex functions $g_{1,2}$ allow us to describe all class B lasers, including semiconductor. The following analysis immediately applies also to (i) the simpler case of systems in which the variable N can be eliminated (class A) and (ii) a more general class of equations in which the feedback term is nonlinear [12].

We consider perturbations $\delta E \propto \exp(\omega t + ikx)$ of the non lasing solution $E_0 = 0$ and N_0 such that $g_2(0, N_0) = 0$. These perturbations have complex dispersion relation

$$\omega = \beta - e^{i\delta}k^2 + re^{i(\phi + k\Delta x)}, \tag{2}$$

with $\beta = g_1(0, N_0; \mu)$ also complex. In the following real and imaginary parts of complex quantities have subindices R and I, respectively. In the limit of vanishing shift $\Delta x = 0$, the laser threshold, given by $\beta_R^{th} = -r\cos(\phi)$, decreases when the feedback interferes constructively with the intracavity field and increases when the interference is destructive. Because the fast relaxation of the polarization implies that the gain bandwidth is very large, all travelling waves have the same gain/loss if there is no diffusion. The effect of diffusion is to filter the high Fourier components so that the most unstable mode is the homogeneous one ($k = 0$) independently from the relative strength of diffusion and diffraction (δ). When $\Delta x \neq 0$, on the other hand, the most unstable mode can have $k \neq 0$. The nonlocality gives rise to a *modulation instability* and allows for the existence of several bands of unstable wavevectors ($\omega_R > 0$) [3].

The off-axis feedback, besides modulation instability to several bands of wavevectors, provides a wide tunability of the properties of the device and enables to control the *first threshold*. Inspection of Eq. (2) shows that the instability threshold can be expressed as a function of four relevant parameters, namely $\phi, \delta, r\Delta x^2$, and $\beta_R \Delta x^2$ (see Figs. 1a-b); therefore increasing the shift size Δx produces on the device the same effect of larger gain β_R and feedback r. As a specific effect of the nonlocality, we find that the relative strength of diffusion and diffraction, δ, also becomes an effective parameter to control the threshold position. Indeed, the lowest gain and feedback thresholds (independently on the feedback phase ϕ) are generally found in the purely diffractive limit ($\delta \sim \pi/2$). The effect of diffusion on the feedback lasing threshold can be appreciated in Fig. 1a: for any not vanishing feedback phase ϕ, the threshold value for the scaled feedback strength $r\Delta x^2$ increases with the diffusion, being independent on the sign of the refractive index (sign of δ). Both β_R and r can be increased to cross the laser threshold as shown in Fig. 1b, and –similarly to the case of perfect alignment– if the feedback is out of phase then stronger gain is required. For fixed values

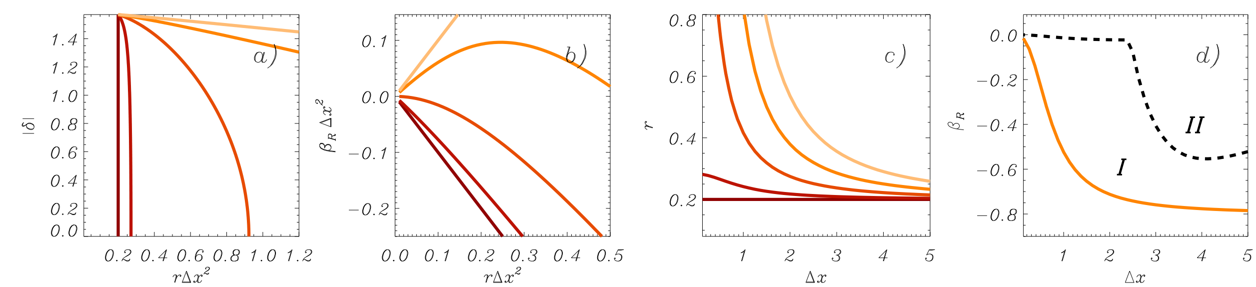

FIG. 1: a) Instability thresholds for $\beta_R \Delta x^2 = -0.2$ and for $\phi = n\pi/4$ with $n = 0, 1, 2, 3, 4$ (from dark to light colors). The lowest threshold is found for $\phi = 0$ and the instability takes place on the right of the lines. b) Thresholds for $\delta = 0.45\pi$ and different feedback ϕ as in (a). c) Thresholds for $\beta_R = -0.2$, $\delta = 0.45\pi$ and different feedback ϕ as in (a). d) First (continuous line) and second (dashed line) thresholds for $\delta = 0.45\pi$, $r = 0.8$ and $\phi = \pi/2$.

either of the gain or of the feedback the nonlocality strongly decreases the threshold values for the gain as well as for the feedback field, as seen in Figs. 1c and d. This can be understood considering that the most unstable mode has $k \neq 0$ so that the effect of the nonlocal coupling is equivalent to a reduction of the feedback dephasing. Consistently with this interpretation, in the case of feedback perfectly in phase with the intracavity field ($\phi = 0$) the threshold is independent on the lateral shift Δx because the most unstable mode is the homogeneous one ($k = 0$).

Another effect of the nonlocality concerns the possibility to *tune* transverse phase and group velocities *independently* from one another. This property enables non mechanical steering and spatial chirping of light beams as the high spatial frequencies can accumulate in the left or right side of the beam. We remark that, as for conventional lasers without off-axis feedback [14], phase travelling waves are exact solutions of the model. Phase and group velocities follow from

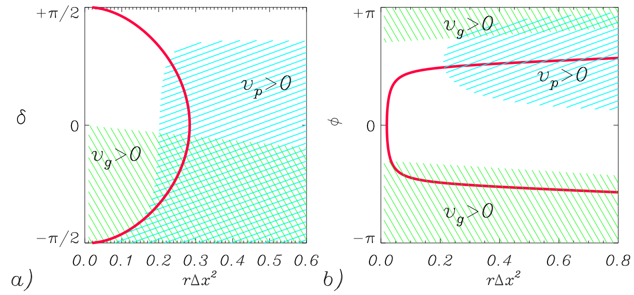

FIG. 2: Sign of velocities for $\beta\Delta x^2 = -0.02 - i0.2$, and for $\phi = \pi/2$ (a) and $\delta = 0.2\pi$ (b). The dashed regions show where phase and group velocities are positive, while the continuous line marks the instability threshold (the system is below threshold on the left sides). Negative values of δ correspond to negative refractive indexes.

Eq. (2):

$$v_p = -\frac{\omega_I(k)}{k} = k\sin\delta - \frac{\beta_I + r\sin(k\Delta x + \phi)}{k} \tag{3}$$

$$v_g = -\partial_k\omega_I = 2k\sin\delta - r\Delta x\cos(k\Delta x + \phi). \tag{4}$$

They can be tuned independently because the parameter β_I enters only in the expression of the phase velocity. Evaluation of the velocities for the critical wavevectors k_c allows us to identify the manifolds in the control parameter space that separate regions in which the group and the phase velocity have the same sign from region in which these velocities have opposite sign. In particular, the group velocity is null for $r\Delta x^2 = -2\delta \pm (4n+1)\pi \mp 2\phi$. As shown in Fig. 2 equal or opposite signs of the velocities can be observed also *at* the instability threshold of the device (continuous line) depending on the values of δ, $r\Delta x^2$ and ϕ. The latter is a promising candidate to tune non mechanically the velocities, for instance by changing the refractive index in the feedback loop.

Whenever the group velocity is non null, one has to determine whether amplified perturbations of the unstable reference state E_0 will drift away (convective instability), or will fill the entire system (absolute instability). The convective regime is the one where the control of localized light signals is possible. The nature of the instability is determined by finding the limit of the Green function of the linearised system of equations for large time. The asymptotic local behaviour of the perturbation is found by generalising the saddle point technique developed in [6, 13] –the details will be reported elsewhere. In Fig. 1d we show an example of thresholds of convective (I) and absolute (II) instabilities; for any choice of parameters there are windows of convective instability before reaching the lasing thresholds. By using the information in Figs. 1-2 and Eqs. (3-4) we can determine linear amplification, direction of propagation and spatial chirping of any light spot in the transverse plane.

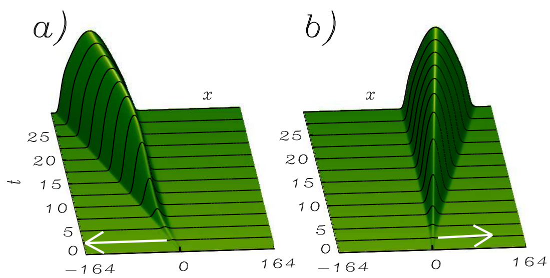

FIG. 3: Spatio-temporal diagram for the field intensity $|E|^2$ starting from a small Gaussian perturbation of the vanishing state E_0, obtained by numerical simulation of Eqs. (5). Parameters: $\mu = 0.98$, $\theta = 0.2$, $\delta = 0.49\pi$, $r = 0.5$, $\Delta x = 1$ (coupling each point with a shifted one on the right) and $\phi = \pi/2$ (a), $\phi = -\pi/2$ (b).

In order to check to what extent the linear analysis we reported predicts the dynamics of the full nonlinear device we consider the standard model for class A lasers, obtained from Eqs. (1) with

$$g_1 = -(1 + i\theta - N)E, \quad N = \mu/(1 + |E|^2)], \tag{5}$$

with the usual parameters θ for the detuning with respect to the medium resonance, and μ for the pump [15]. The dispersion relation for the field perturbations around the homogeneous steady state $E_0 = 0$ are given by Eq. (2) with

$\beta = \mu - 1 - i\theta$. Numerical simulations confirm the predicted thresholds, in agreement with the stability diagrams in Fig. 1. Moreover, the wavenumbers dynamically selected and the velocities are well approximated by those obtained from linear dispersion. In view of applications it is interesting to see the dynamics of local perturbation of the homogeneous state: In Fig. 3 we demonstrate the ability of steering and amplifying beams in the convective region; furthermore, one or both the signs of phase and group velocities can be changed with the proper parameters choice (Figs. 3a-b), consistently with predictions presented in Fig. 2. Numerical simulations also confirm the possibility of chirping; the phase of the field shows indeed a spatial dependent modulation.

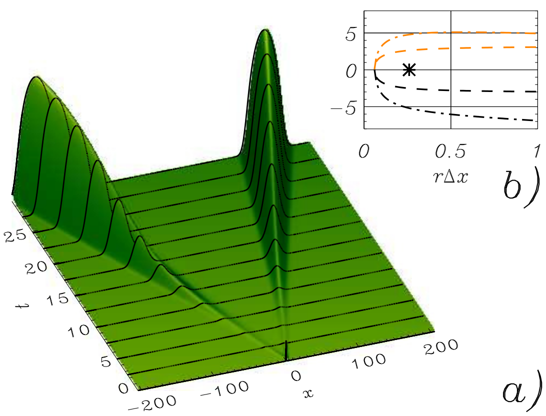

FIG. 4: a) Evolution of a Gaussian perturbation as in Fig. 3 but for $\phi = \pi$. b) Phase (dashed lines) and group (dahsed-dotted lines) velocities. The upper (lower) curves are the velocities for k_c ($-k_c$). For $r\Delta x > 0.26$ (star point) the homogeneous state is unstable.

Special attention needs the case $|\phi| = \pi$ where a small spot of light is amplified and splits in two separate spots travelling in opposite directions as shown in Fig. 4a. Both positive and negative wavevectors with values around the critical ones are selected and then separate moving in opposite regions of the beam area. Our analysis for $|\phi| = \pi$ gives $\omega_R(k) = \omega_R(-k)$ but, in general, $\omega_I(k) \neq \pm\omega_I(-k)$. This is important because in order to see a propagating stripe, for instance $E \propto \cos(kx + \omega t)$, it would be necessary to have an antisymmetric dispersion $\omega_I(k)$ and the simultaneous instability of both positive and negative wavenumbers. This would guarantee that the interfering waves k and $-k$ have the *same* velocities. As shown in Fig. 4b this is not the case for off-axis feedback: the phase and group velocities of opposite waves with critical wavenumbers have *opposite* signs, and in the diffraction limit $\delta \rightarrow \pi/2$ both velocities are odd functions of k_c. Therefore, even if for $\phi = \pi$ both $+k_c$ and $-k_c$ are unstable, from the linear analysis we do not expect intensity stripe patterns above threshold. The existence of exact travelling phase patterns as well as the lack of intensity waves are also known in lasers without feedback [14]. The novelty here is the *prediction of a state in which two waves with wave-vectors* $\pm k$ *travel apart with opposite velocities*. In spite of the definite direction associated to the break of reflection symmetry due to two-point nonlocality, *both* transverse direction of propagation are equally linearly amplified. As shown in Fig. 4a, numerical simulations of the model (5) for $|\phi| = \pi$ fairly agree with these predictions. Even if the linear amplification of both waves has the same strength, one wave is nonlinearly favoured over the other so that a slightly larger intensity and size of the packet are found on one side with respect to the other, depending on the sign of the shift. As a matter of fact, one mode in the far field is more intense of the other, similarly to what is found in systems with drift [16]. We also note that in this case only the Green function correctly characterises the convective or absolute nature of the instability. The standard evaluation of the instability solely in terms of the velocities of the external fronts of a perturbation would erroneously describe the convective instability as absolute. We have seen in fact that here a Gaussian perturbation splits into two wave-packets with the external fronts moving in opposite directions, as is usually the case for absolute instabilities, even if the signal eventually decays between the external fronts.

In conclusion, we have reported a general analysis of the effects of off-axis feedback in a large class of optical cavities and gain media, and shown the threshold dependence on two-point nonlocality, diffusion and positive as well as negative diffraction. The possibility to observe travelling waves at the onset of the instability in media with fast relaxation of the polarization is an important effect of nonlocality, that induces the modulations character of the instability. We have determined the convective and absolute threshold extending our analysis of purely diffusive systems [6]. In presence of nonlocality phase and group velocities of optical fields can be easily tuned to parallel or opposite directions, which enable steering and spatial chirping. Surprisingly, for a particular phase of the feedback loop ($\phi = \pi$) we have found the simultaneous presence of waves travelling apart. The effect is almost symmetrical in the positive and negative directions, even if the off-axis feedback introduces a directional coupling in the transverse plane. The possibility to amplify an initial spot of light, control its velocity and spatial chirping and even split it in two counter-propagating signals makes cavities with off-axis feedback a promising candidate in view of applications in all-optical communications based on the control of light signals, such as optical triggering, switching, routing, delay lines, beam recovery and steering and in manipulation of microparticles. Finally, our theoretical results formally apply to a broad class of devices and similar effects can be observed for localized perturbations of any nonlocal and spatially extended system near the onset of oscillations.

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