TEAM 2013, *Sep. 9 - 12, 2013, Keelung, Taiwan*

The radiation problem of vessels advancing in waves by using a new radiation condition

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Abstract

A 3-D panel method code based on the Rankine source approach has been developed to calculate the hydrodynamic properties of floating objects in waves with or without forward speed. To solve the very low forward speed problem, a new radiation condition has been used, which takes double Doppler shift into consideration. Meanwhile, the second derivative of the potential function on the free surface will cause a singularity problem. In the numerical study, the source points, which are originally placed on the undisturbed free-surface, are shifted in a horizontal distance above the undisturbed surface. In order to verify the new radiation condition, a Wigley III hull has been modelled by using a panel code based this new radiation condition. The hydrodynamic coefficients of the vessel with zero forward speed, low forward speed and medium forward speed are calculated separately. Comparing the present results with experimental data, it can be concluded that the new radiation condition is effective to predict the hydrodynamic properties of vessels travelling or stationary in waves.

Key words: 3-D panel method; Rankine source; radiation condition; Doppler shift;

1 INTRODUCTION

The forward speed problem of marine structures has drawn intensive attention for many decades. There are a considerable number of publications concerning the prediction of hydrodynamic responses of vessels travelling in waves. Based upon the Green function employed in the boundary integral formulation, these papers could be divided into two categories. In the first category, the translating and pulsating sources are only distributed on the wetted body surface. This method utilizes a Green function that satisfies the Kelvin free surface conditions, as well as the radiation condition [1, 2]. It is an effective method for the zero forward problem, but if the vessel is travelling with forward speed, this method still has some limitations. Firstly, it could not account for the near-field flow condition. Although some researchers [3, 4] extended it to include the near-field free surface, the so-called irregular frequency still cannot be avoided. And it will bring singularity to the coefficient matrix equation. Secondly, it is impossible for the Green function to account for the interaction between the steady and unsteady flow.

The second category is called Rankine source approach, which uses a very simple Green function in the boundary integral formulation. This method requires the sources distributed not only on the body surface, but also on the free surface and control surface. Therefore, a flexible choice of free-surface conditions can be realized in these methods. The coupled behaviour between steady and unsteady wave potential could be expressed in a direct formula. Meanwhile, the nonlinearity on the free surface could also be added in the boundary condition. However, there are still some limitations for the extensively use of Rankine source approach. First of all, the Rankine source method requires much more panels. It will considerably increase the computation time, especially when the matrix equation is full range matrix. Besides, the Rankine source method requires a suitable radiation boundary condition to account for the scattered wave in current. A very popular radiation condition for the forward speed problem, which is so-called upstream radiation condition, was proposed by Nakos [5]. The free surface was truncated at some upstream points, and two boundary conditions were imposed at these points to ensure the consistency of the upstream truncation of the free surface. Another method to deal with the radiation condition is to move the source points on the free surface at some distance downstream [6, 7]. The results from these two methods show very good agreement with published experimental data when the Brard number τ (τ =uω/g) is greater than 0.25, since they are both based on the assumption that there is no scattered wave travelling ahead of the vessel. However, when the forward speed of the vessel is very low, the Brard number will smaller than 0.25. When this case occurs, these radiation conditions could no longer be valid.

Low forward speed problem could occur when the ships are travelling in harbour area or between tugs and vessels during escorting or manoeuvring and berthing operations as well as during ship-to-ship operations for cargo transfers during oil and gas offloading operations. In these cases, a new radiation should be proposed. Das and Cheung (2012) [8] provided an alternate solution to the boundary-value problem for forward speeds above and below the group velocity of the scattered waves. They corrected the Sommerfeld radiation condition by taking into account the Doppler shift of the scattered waves at the control surface that truncates the infinite fluid domain. In the present study, this new radiation condition is adapted to complement of the boundary-value problem. A Wigley hull with different forward speeds will be studied. The results obtained are compared with Journee's [9] experimental data, which could verify the effectiveness of the new radiation condition.

2 MATHEMATICAL FORMULATIONS OF THE POTENTIALS

2.1 Boundary value problem

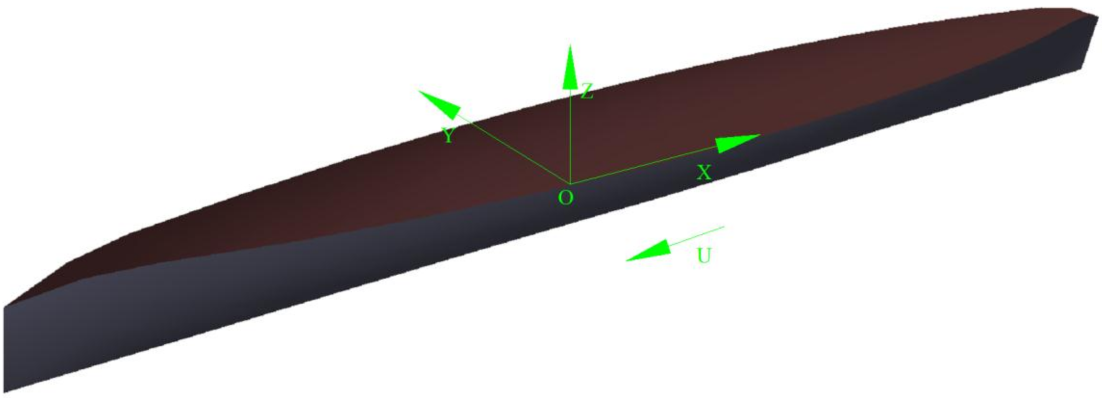

Fig.1 *An example vessels and coordinate system*

Fig.1 shows a vessel travelling with a constant forward speed ***U*** in a Cartesian coordinate system O-XYZ, which is moving together with the body. The origin is located on the still water and axis Z points upward. X and Y axis is on the geometric centroid of the water-plane. Based on the potential flow theory, the radiation potential could be expressed as:

$$\Phi(x,y,z,t)=\mathrm{Re}\sum_{j=1}^{6}\eta_j\varphi_j(x,y.z)e^{-i\omega t} \tag{1}$$

where η_j is the motion amplitude in six degree of freedom and φ_j is the corresponding radiation potential. Based on the assumption of irrotational motion and an incompressible fluid, the total velocity potential exists which satisfies the Laplace equation in the whole fluid domain, together with the body and free surface boundary conditions which could be written in equations (2), (3) respectively.

$$\frac{\partial \varphi_j}{\partial \mathbf{n}} = U\mathbf{m}_j - i\omega_e \mathbf{n}_j \qquad (j=1, 2, \dots, 6) \quad \text{on body surface } S_B \tag{2}$$

$$g\frac{\partial \varphi_j}{\partial z} - \omega_e{}^2 \varphi_j + 2i\omega_e U \frac{\partial \varphi_j}{\partial x} + U^2 \frac{\partial^2 \varphi_j}{\partial x^2} = 0 \quad \text{on free surface } S_F \tag{3}$$

where ω_e is the encounter frequency, $\omega_e = \omega_0 - Uk\cos\beta$. $k = \frac{\omega_0^2}{g}$ is the wave number, ω_0 and β represent angular frequency of incoming waves and incident angle respectively.

$\mathbf{n} = (n_1, n_2, n_3)$ is the unit normal vector directed inward on body surface S_B.

$(n_4, n_5, n_6) = \mathbf{X} \times \mathbf{n}$,

$\mathbf{X}$ is the position vector on S_B.

$$(m_1, m_2, m_3) = -(\mathbf{n} \cdot \nabla)\nabla(\varphi_s - Ux),$$
$$(m_4, m_5, m_6) = -(\mathbf{n} \cdot \nabla)[\mathbf{X} \times \nabla(\varphi_s - Ux)],$$

φ_s represents the steady potential. In the present study, the steady flow is neglected. The m-term could be simplified to

$$m_1 = m_2 = m_3 = m_4 = 0, \quad m_5 = n_3, \quad m_6 = -n_2,$$

In addition to (2) and (3), a suitable radiation condition should be imposed at the control surface to make the boundary value problem integrated. And this will be complemented in section 2.2.

Once the potential function φ is obtained, the pressure on the body surface SB can be expressed as

$$p_j = \rho(i\omega_e \varphi_j + U\frac{\partial \varphi_j}{\partial x}) \tag{4}$$

ρ is the density of the fluid. The added mass a_{ij} and damping b_{ij} could be obtained by integrating the pressure in Eq. (4) over the wetted body surface S_B.

$$a_{ij} = \frac{1}{\omega_e{}^2} \iint_{S_B} \left(U\frac{\partial \varphi_{Rj}}{\partial x} - \omega_e \varphi_{Ij} \right) n_i ds, \quad \text{i, j= } 1, 2, \dots, 6 \tag{5}$$

$$b_{ij} = -\frac{1}{\omega_e} \iint_{S_B} \left(U\frac{\partial \varphi_{Ij}}{\partial x} + \omega_e \varphi_{Rj} \right) n_i ds, \quad \text{i, j= } 1, 2, \dots, 6 \tag{6}$$

φ_{Rj} is the real part of jth potential, and φ_{Ij} is the imaginary part. The wave elevation could be expressed as

$$\zeta_j = \frac{1}{g}\left(i\omega_e \varphi_j + u_0 \frac{\partial \varphi_j}{\partial x} \right) \tag{7}$$

2.2 Radiation condition

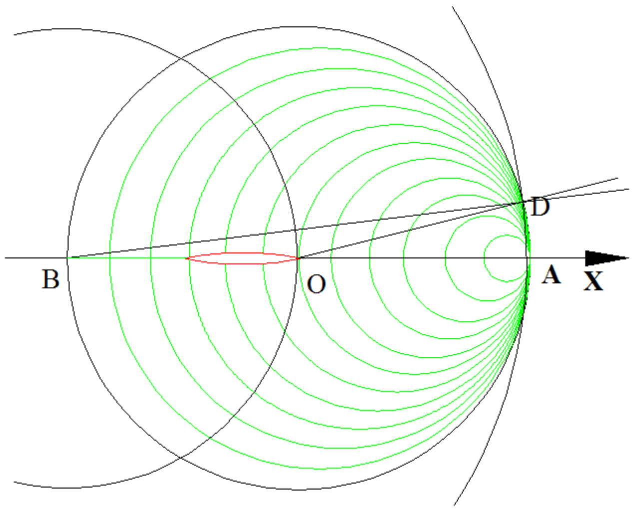

Fig.2 *(a) Sketch of Doppler shift*

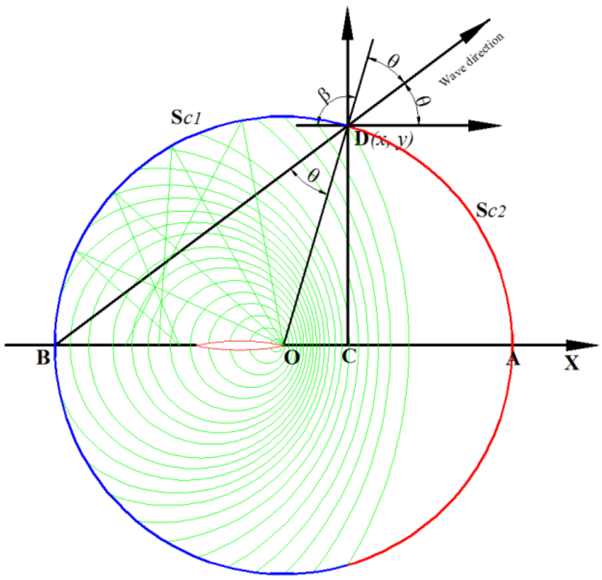

Fig.2 *(b) Radiation condition*

Fig.2 (a) shows the Doppler Shift of scattered wave field by a vessel travelling with constant forward speed u_0 in the positive x direction. When a vessel is moving from point B to point O, the traveling time should be $t=BO/u_0$. During this period of time, the vessel produces the scattered wave all along BO (the first scattered wave should arise at point B). The control surface here is defined as a circle with its centroid on point O and its radius as BO. If u_0 is small, the travelling time t will be very long, which means the scattered wave produced at point B can travel a long distance. When it just reaches point A, the wave group is reaching point O (the wave group velocity is half of scattered wave velocity). In this case, the Brard number τ=0.25 ($\tau = u\omega/g$), $u_0=u_c$ and the scattered wave can reach any points on the control surface. If $\tau<0.25$ and $u_0<u_c$, the wave group will go head of the vessel. Therefore, the upstream radiation condition could not be used any longer. If $\tau>0.25$ and $u_0>u_c$, the scattered wave can only get to point D, as shown in Fig.2 (b). On arc DA, there are no scattered waves reaching here, and it is defined as S_{C2}. To the contrary, arc BD is defined as S_{C1}, on which a different radiation condition should be imposed. From Fig.2, the wave direction of a scattered wave reaching point D (x, y), which is produced at B, have been rotated by an angle θ (if there is no forward speed, the wave direction should be along OD. The velocity of the scattered wave is defined as c, $BO/u_0=BD/c$. According to the sine theorem, it can be easily transferred to

$$\frac{u_0}{c} = \frac{\sqrt{x^2+y^2}}{y}\sin\theta \tag{8}$$

The scattered wave velocity at D can be expressed as

$$c = \frac{\omega_s}{k_s} \tag{9}$$

where ω_s is the angular frequency of the scattered waves from a fixed reference point given as

$$\omega_s = \omega_e + u_0 k_s \cos[\tan^{-1}\left(\frac{y}{x}\right) - \theta] = \sqrt{g k_s \tanh k_s d} \tag{10}$$

in which k_s is the local wave number at D(x, y), and d is the water depth. Once the coordinates of any arbitrary point on the control surface are given, the unknowns θ and k_s could be obtained by solving the nonlinear equation system (8)-(10). If one cannot find solutions, these points must be on the control surface S_{c2}. Otherwise, they are on S_{c1}. The radiation condition is defined as two different equations,

$$\frac{\partial \varphi_j}{\partial n} - i k_s \varphi_j \cos\theta = 0 \quad \text{(j=1, 2, … , 6)} \quad \text{on } S_{c1} \tag{11}$$

$$\nabla \varphi_j = 0 \quad \text{(j=1, 2, … , 6)} \quad \text{on } S_{c2} \tag{12}$$

Eq. (10) is an updated Sommerfeld radiation condition with forward speed correction. If the forward speed is zero, $k_s = k$, θ=0 and Eq. (11) could reduce to the Sommerfeld radiation condition

$$\frac{\partial \varphi_j}{\partial n} - i k \varphi_j = 0 \quad \text{(j=1, 2, … , 6)} \quad \text{on } S_c \tag{13}$$

3 VALIDATION AND APPLICATION

In the numerical study, a Wigley III hull is modelled by using the new radiation condition. Journee's [9] experimental results are quoted to verify the efficiency of the present model.

The computational range on the free surface is extended to 2L upstream, 2L downstream and 2L sideways, where L=3m is the length of the vessel. There are 300 elements on the body surface, 7200 on free surface and 1200 on the control surface. Figs.3 (a)-(d) are the hydrodynamic coefficients at F_n=0.3 ($F_n = u_o/\sqrt{gL}$).

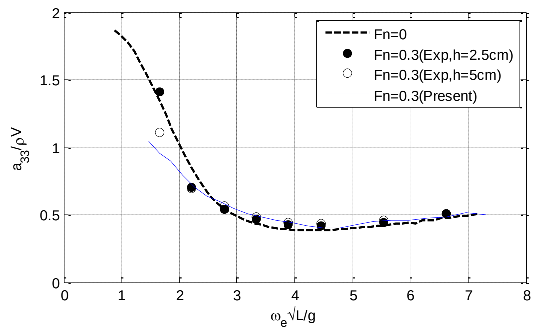

***Fig.3 (a)** Heave added mass (Fn=0.3)*

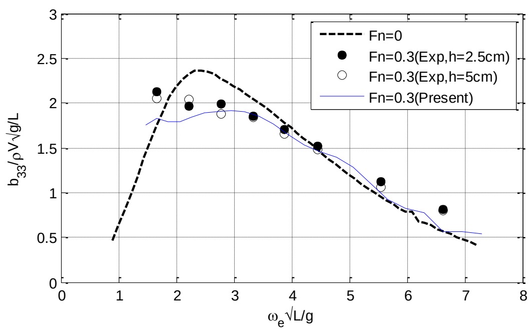

***Fig.3 (b)** Heave damping (Fn=0.3)*

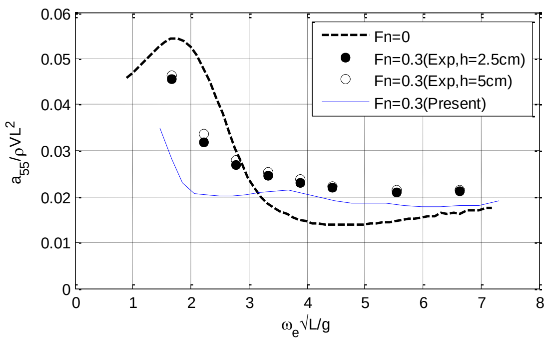

***Fig.3 (c)** Pitch added mass (Fn=0.3)*

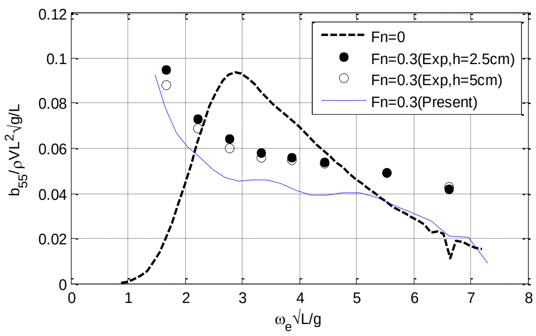

***Fig.3 (d)** Pitch damping (Fn=0.3)*

From Figs.3 (c)-(d), it can be found that the present results are satisfactory as can be seen from the comparison of the predictions with the experimental results of Journee [15]. The difference seen at the low frequency region is more evident. One reason could be the downstream free surface range. From Fig.2 (a), it can be seen that the wave length in the downstream side becomes longer due to the Doppler shift, for this reason, it requires larger free surface range to get accurate results. On the contrast, the wave length in the upstream side is much shorter, which requires smaller panels to resolve the wave field. As the forward speed increases, the Doppler shift becomes more evident. If the panel remains the same, the results would be not as good as that of lower speed, which can be seen in Figs.3 (c)-(d). Besides, the steady potential and the coupled behaviour between the steady and unsteady potentials are neglected, which should bring some influence into the boundary condition, especially when the forward speed is high. Figs.4 (a)-(d) are the radiation wave pattern for a unit heave motion at different Brard number. The upper half of the figures is the wave pattern calculated by Sommerfeld radiation condition, and the lower half is calculated by the new radiation condition, which takes Doppler shift into account. At τ=0, the wave patterns are identical to each other. When the forward speed increases, the reflection from the upstream control surface becomes evident by using Sommerdeld radiation condition. But in the present model, the wave patterns are much regular. At τ=0.245, which is a little smaller than the critical value 0.25, it can be seen in Fig.4 (c) that there are still some wave groups travelling ahead of the vessel. But in Fig.4 (d), when τ=0.257, no wave can go ahead of the vessel in the present model.

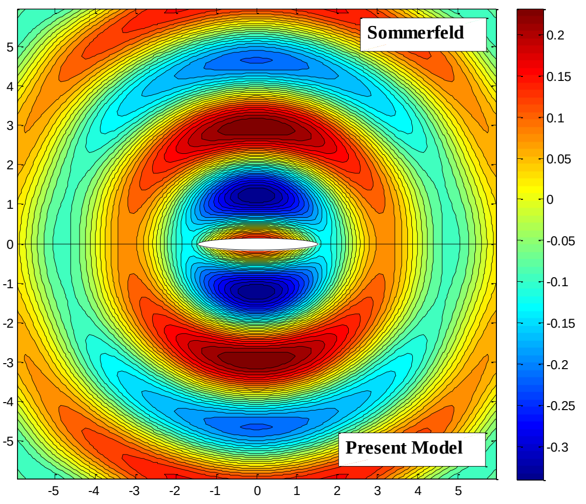

***Fig.4 (a)** Radiation wave (τ=0)*

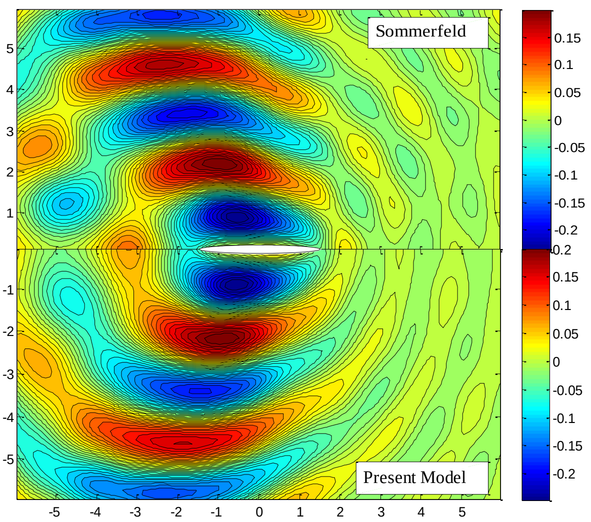

***Fig.4 (b)** Radiation wave (τ=0.2)*

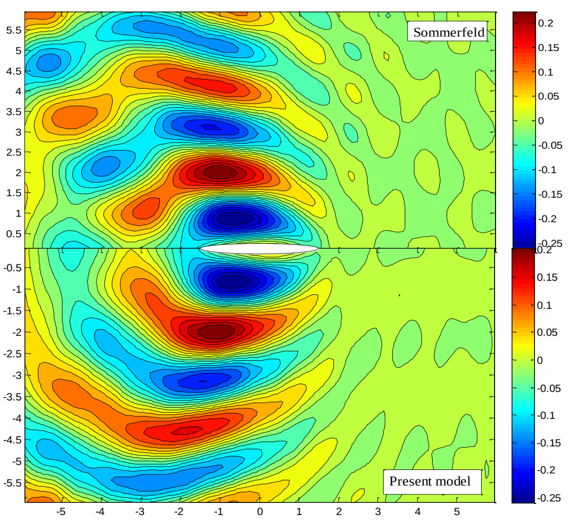

***Fig.4 (c)** Radiation wave (τ=0.245)*

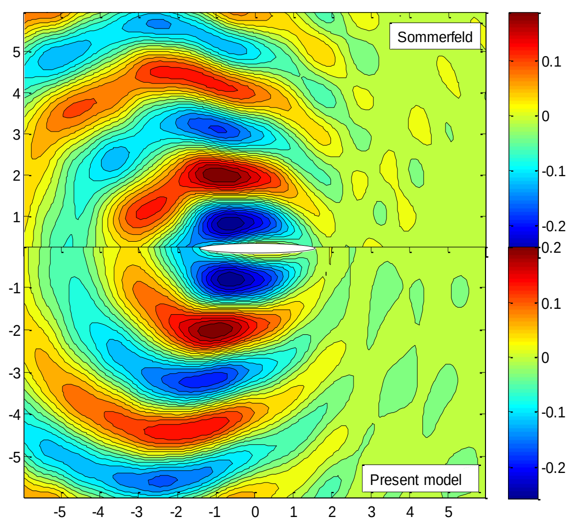

***Fig.4 (d)** Radiation wave (τ=0.257)*

4 CONCLUSIONS

In the present study, a 3-D Rankine source panel code has been developed to predict the hydrodynamic properties of marine vessels stationary or travelling with forward speed. A new radiation condition is used to simulate the vessel with very low forward speed. This new radiation condition corrects the Sommerfeld radiation condition by taking into account of Doppler shift. The numerical results for the problem of Wigley III hull show that the present method is effective to predict the hydrodynamic cofficients of the vessel traveling with any forward speed. The calculation results obtained from the new radiation condition are in good agreement with the results obtained from experimental mesaurements. The numerical results obtained from the present model for the radiation wave pattern of a vessel travelling with a very low forward speed are better than those obtained from Sommerfeld condition when the Brard number τ is around the critical value of 0.25. Therefore, this new radiation condition would be a useful method for the any forward speed problem.

5 ACKNOWLEDGEMENTS

The present work is mainly funded by Lloyd's Register. Authors are grateful to Lloyd’s Register for their support.

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