THE SIMULATION OF BUILDING ELECTRICAL POWER FLOWS

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ABSTRACT

Recent developments in photovoltaic components, small-scale combined heat and power systems and ducted wind turbines have opened up the possibility for an embedded generation approach to building design. In conjunction with demand reduction measures, embedded generation encourages a new design paradigm based on matching local supply to demand.

For building simulation programs to offer effective support for embedded systems design, they must be capable of modelling the electrical energy flows in a fully integrated manner.

This paper describes recent work to develop an electrical power flow model and ensure its interoperability with the other technical domains found within building simulation. Specifically, the paper describes:

a) an electrical network solver for d.c., a.c. and hybrid circuits;

b) the thermal/electrical models of power producing and consuming equipment;

c) the nature of the interactions between the thermal, lighting, air flow and electrical power flow domains;

d) aspects relating to the conflation of the power flow model with the other technical domains as found within the ESP-r system.

The paper concludes with a case study of the integrated thermal, lighting, air flow and power flow modelling approach when applied to a building/ HVAC problem with an embedded generation component in the form of a photovoltaic facade operating in hybrid mode.

Keywords:Integrated building simulation, electrical power flow modelling, urban scale embedded generation.

INTRODUCTION

Modern building technologies such as small-scale combined heat and power (CHP), photovoltaic (PV) facades and ducted wind turbines have fostered the concept of *embedded generation* in building design, where some or all of the heat and power demands are produced locally from renewable energy sources.

To achieve the optimum energy potential of these systems they must be designed as an integral part of the building such that the energy output is matched to the building's energy demands. A twin-track approach can be adopted: the building's energy demand profile is altered using demand-side measures and the embedded energy systems are designed to match the modified profile. The inclusion of energy production in the design process increases the complexity:

- Embedded energy systems interact with all the constituent parts of a building: fabric, plant and control, lighting and occupants. Effective design requires that these interactions be accounted for at the analysis stage.
- The transient effects of climate, occupation and control action will affect the operation of the embedded system and must be considered.
- Embedded energy systems straddle thermal and electrical energy flow-paths; the couplings between these two domains must be considered in detail.

An appropriate means by which the complexity of an embedded energy system can be analysed is through computer simulation using an integrated energy model, incorporating both the embedded energy system and the building(s) it serves. Clearly, the model must be able to determine the electrical flows occurring within the building, allowing the performance of all the energy subsystems to be gauged in a realistic manner.

In the field of building simulation, the modelling of the building's electrical system has tended to be overlooked. To facilitate the integrated modelling of thermal *and* electrical energy flows within building simulation, a means of modelling electrical systems is described in this paper. This has been incorporated within the ESP-r system.†

† It should be noted that though the following sections

The fundamental concepts of ESP-r have been described by Clarke [1]:

"... ESP-r will accept some building/plant description in terms of three-dimensional geometry, construction, usage and control. This continuous system is then made discrete by division into many small, but finite volumes of space. These finite volumes then represent the various regions of the building within and between which energy can flow."

In the current work, the use of finite control volumes is extended to the electrical system.

MODELLING ELECTRICAL SYSTEMS

The purpose of the electrical power system model is to determine the distributed power flows and state variables (voltage and current). Within ESP-r a nodes and arcs approach is used to model plant and fluid flow: this approach was extended to the electrical system. Essentially, the electrical network can be thought of as a collection of nodes, which represent the junctions between conducting elements and/or points where power is added or extracted. The nodes are connected by electrical conductors such as cables and lines.

Application of Kirchoff's current law to each node in the network forms the basis for the solution of the electrical energy flows. This law states that the sum of the current at any point (or node) in an electrical system is zero:‡

$$\sum_{j=1}^{n} I_{i,j} = 0. \qquad (1)$$

At each node current can either be injected from power sources, drawn by loads or transmitted to/from other connected nodes via conductors. Separating these currents gives

$$\sum_{j=1}^{nG} I_{Gi,j} - \sum_{j=1}^{nT} I_{Li,j} + \sum_{j=1}^{nL} I_{Ti,j} = 0. \qquad (2)$$

The currents drawn from and injected into the nodes can be considered as boundary conditions in the solution of the electrical network. These boundary conditions form the pivot points in the coupling of the electrical system with the other technical domains.

The current transmitted between nodes can be re-written in terms of the nodal voltage difference and the connecting component admittance (the inverse of the impedance which characterises the component, e.g. cable, transformer, etc.):

$$I_{Ti,j} = (V_i - V_j)Y_{i,j}. \qquad (3)$$

Substituting equation (3) into equation (2) and rearranging terms gives the following general equation for each node in the electrical network.

$$\sum_{j=1}^{nT}(V_i - V_j)Y_{i,j} = \sum_{j=1}^{nL} I_{Li,j} - \sum_{j=1}^{nG} I_{Gi,j}. \qquad (4)$$

While equation (4) offers a suitable means of determining the nodal voltages of the network, the solution is only achievable if the boundary conditions are defined in terms of current flows. However, in the field of energy simulation, loads would typically be defined in terms of a power demand. Similarly, the output of electrical sources would also be defined in terms of power. It is therefore appropriate to define the electrical network boundary conditions as power flows.

At any node i in the network, the complex (apparent) power flow is given by:

$$S_i = V_i I'_i. \qquad (6)$$

The complex power at each node is obtained by multiplying the conjugate current (equation (4)) by the nodal voltage:

$$V_i\left[\sum_{j=1}^{nT}(V_i - V_j)Y_{i,j}\right]' = \sum_{j=1}^{nL} S_{Li,j} - \sum_{j=1}^{nG} S_{Gi,j}. \qquad (7)$$

Rearranging this equation gives

$$V_i\left[\sum_{j=1}^{nT} V_j \mathbf{Y}_{i,j}\right]' = \sum_{j=1}^{nL} S_{Li,j} - \sum_{j=1}^{nG} S_{Gi,j} \qquad (8)$$

where $\mathbf{Y}_{i,i} = \sum_{j=1}^{nT} Y_{i,j}$ and $\mathbf{Y}_{i,j} = -Y_{i,j}$.

This equation can be split into its real and reactive parts:

$$\left[\sum_{j=1}^{nT} V_i(V_j \mathbf{Y}_{i,j})\right](\cos\theta_i - \theta_j - \gamma_{i,j}) = \sum_{j=1}^{nL} P_{Li,j} - \sum_{j=1}^{nG} P_{Gi,j}, \qquad (9a)$$

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describe the integration of the thermal and electrical domains within ESP-r, the integration method is generic enough to be applied to any simulation tool.

‡ Note that the emphasis here is on a.c. power flow, with all variables being complex phasor quantities, e.g. $I = I(cos\theta + j\sin\theta)$. Note however that the real parts of the equations are applicable to d.c. power flow.

and

$$\left[\sum_{j=1}^{nT} V_i (V_j \mathbf{Y}_{i,j})\right](\sin\theta_i - \theta_j - \gamma_{i,j}) = \sum_{j=1}^{nL} Q_{L\,i,j} - \sum_{j=1}^{nG} Q_{G\,i,j}. \quad (9b)$$

NETWORK SOLUTION

At each node i in the network there are $2n$ non-linear equations, of the general form given by equations (9a) and (9b). There are also $2n$ unknowns: the voltage magnitude at each node V_i and the phase angle θ_i; the voltage at each node being $V_i(\cos\theta_i + j\sin\theta_i)$.

This system of $2n$ equations is solved using a Newton-Raphson method, giving the voltage at each node at a particular time step. The network power balance, in matrix equation notation, is as follows (note that $P_{i_{i=1\ldots n}}$ represents equation (9a)† implemented at some node i in the network; similarly $Q_{i_{i=1\ldots n}}$ represents equation (9b) implemented at the same node.

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta P_n \\ \cdots \\ \Delta Q_1 \\ \Delta Q_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta Q_n \end{bmatrix} = - \begin{bmatrix} \frac{\partial P_1}{\partial V_1} \cdots & \cdots \frac{\partial P_1}{\partial V_n} & \frac{\partial P_1}{\partial \theta_1} \cdots & \cdots \frac{\partial P_1}{\partial \theta_n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial P_n}{\partial V_1} \cdots & \cdots \frac{\partial P_n}{\partial V_n} & \frac{\partial P_n}{\partial \theta_1} \cdots & \cdots \frac{\partial P_n}{\partial \theta_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial Q_1}{\partial V_1} \cdots & \cdots \frac{\partial Q_1}{\partial V_n} & \frac{\partial Q_1}{\partial \theta_1} \cdots & \cdots \frac{\partial Q_1}{\partial \theta_n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial Q_{n_k}}{\partial V_1} \cdots & \cdots \frac{\partial Q_{n_k}}{\partial V_n} & \frac{\partial Q_n}{\partial \theta_1} \cdots & \cdots \frac{\partial Q_n}{\partial \theta_n} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta V_n \\ \cdots \\ \Delta \theta_1 \\ \Delta \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta \theta_n \end{bmatrix} \quad (10)$$

All other network variables (current, transmitted power, imported power) can be calculated once the nodal voltages are known.

The form of electrical network solution detailed above is commonly referred to as a *load flow solution* and is described in many texts including Stagg and El-Abiad [2] and Gross [3]. The solution method and network model are suitable for the simulation of all types of electrical systems: single and multi-phase a.c. and d.c. It must be stressed that all the network variables are phasor values and the solution obtained is not transient; it is a snap-shot of the electrical system state at the particular simulation time increment.

It is noted that that many tools for use in electrical circuit and analysis already exist, (i.e. EMTDC ∗, MATLAB†, ESACAP, ‡ etc.). However, these programs are generally used in transient circuit simulations using small sub-second time steps. In an integrated building simulation, solution frequency ranges from a few seconds up to an hour. The load flow solution described above is suitable for integration within the temporal framework of a building simulation and provides information on longer time constant electrical power flow phenomena related to climatic variation, plant and lighting control and occupant interaction (e.g. voltage variations, supply and demand levels, phase-balances, grid import, etc.).

INTEGRATION INTO BUILDING SIMULATION

To facilitate integrated thermal/electrical building energy simulations, the electrical network must be coupled with the rest of the building model and solved with the other energy subsystems. This is achieved through the connection of power consuming or generating components to nodes in the electrical network (Figure 1). These supply the power flow boundary conditions, necessary for the solution of the electrical network. Note that using power flows as boundary conditions has the advantage that explicit circuit models of loads and sources do not need to be developed for the simulation.

The components, which supply or draw power from the network also exist as entities in other areas of the building model, typically as components in the plant network (fans, pumps and heaters), elements of the construction (e.g. a PV facade) or in the zones (lighting and small power loads). To provide the required power flow information, these components must convert thermal or fluid flow phenomena into real and reactive power flows and vice-versa. Examples include: photovoltaic components converting solar radiation falling on the building facade to electrical power, or pumps

† To formulate the Newton-Raphson solution all the terms are moved to the left hand side to give an expression of the form $f(V_i, \theta_i) = 0$.

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‡ STANSIM Research, Denmark

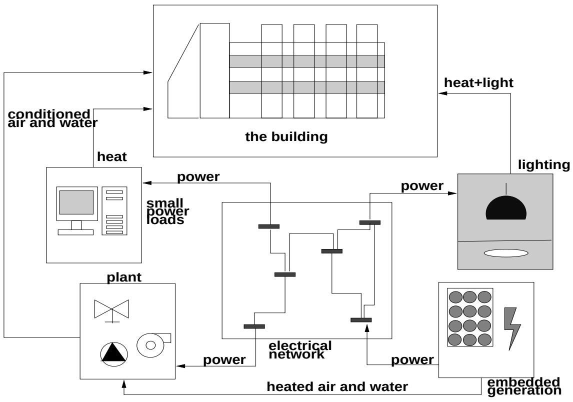

Figure 1: Network integration.

consuming electrical energy to transport hot water within a heating system. These *hybrid* components [4] simultaneously affect the thermal and electrical constituents of the integrated model.

The use of hybrid components to provide the boundary conditions for the solution of the electrical network strongly couples its solution with the thermal/flow processes occurring in other parts of the model. The electrical network therefore needs to be solved simultaneously with the other energy subsystems.

ESP-r solves its integrated model, consisting of the building fabric and associated subsystems (plant, air flow, CFD, electrical network, etc.) using a *modular simultaneous solution process* [5]. This enables each energy subsystem to be solved by the most appropriate and efficient solver. The solvers are linked together to create a unified solution process for the entire model. The exact construction of the solution process is variable. Different combinations of solvers may be used at any one time, their number and type depending upon the characteristics of the particular model. The framework upon which the particular model solution process is built is provided by a numerical controller which determines the structure of the solution process in terms of the solvers that are used and their temporal coupling.

To enable the integration of the electrical network ESP-r's numerical controller was modified. The positioning of the new solver within the overall solution process is dictated by the fact that the power flow boundary conditions must be known before the electrical network solution can proceed. The electrical network must also be solved at a frequency capable of capturing the important effects relating to power flow: the switching of power consuming equipment such as lighting, variations in solar radiation on PV materials, etc. Therefore, within ESP-r, the solution of the electrical network proceeds apace with the most frequently solved subsystem of the building model.

COMPONENT MODELS

Component models are critical to the functioning of the integrated thermal/electrical simulation: they provide the means of coupling subsystems and the boundary conditions for the solution of the electrical network; and they describe the conductors which link the nodes in the electrical network.

The inter-nodal components are represented as admittances in the power flow balance equations. Conductor models are described by Kelly [6]. Consider the example of a short, single phase conductor (Figure 2).

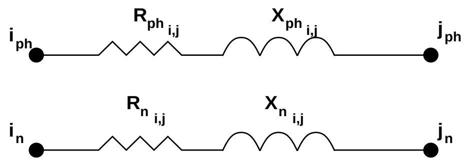

Figure 2: Single phase conductor.

The basic equation for current flowing through this conductor is

$$\begin{bmatrix} Z_{ph} & 0 \\ 0 & Z_n \end{bmatrix} \begin{bmatrix} I_{ph} \\ I_n \end{bmatrix} = \begin{bmatrix} V_{a\,ph} - V_{b\,ph} \\ V_{an} - V_{bn} \end{bmatrix}. \quad (11)$$

The model can be simplified by referencing the voltages to the neutral, achieved by subtracting the equation for the current flow through the neutral from the current flow through the phase conductor:

$$\begin{bmatrix} Z_{ph} & -Z_n \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_{ph} \\ I_n \end{bmatrix} = \begin{bmatrix} (V_{i\,ph} - V_{in}) - (V_{j\,ph} - V_{jn}) \\ 0 \end{bmatrix} \quad (12)$$

Applying Kirchhoff's current law to the model gives $I_n = -I_{ph}$. This allows the elimination of the neutral current from equation (12):

$$\begin{bmatrix} Z_{ph} + Z_n \end{bmatrix} \begin{bmatrix} I_{ph} \end{bmatrix} = \begin{bmatrix} (V_{i\,ph-n}) - (V_{j\,ph-n}) \end{bmatrix}. \quad (13)$$

Rearranging equation (13) gives an expression for the phase current:

$$\begin{bmatrix} I_{ph} \end{bmatrix} = \begin{bmatrix} Y_{ph} \end{bmatrix} \begin{bmatrix} (V_{i\,ph-n}) - (V_{j\,ph-n}) \end{bmatrix} \quad (14)$$

where

$$Y_{ph} = \begin{bmatrix} Z_{ph} + Z_n \end{bmatrix}^{-1}. \quad (15)$$

Finally, equation (14) can be converted to an expression for power flow from i to j:

$$V_a \begin{bmatrix} I_{ph} \end{bmatrix}' = V_a \begin{bmatrix} Y_{ph} \end{bmatrix}' \begin{bmatrix} (V_{a\,ph-n}) - (V_{b\,ph-n}) \end{bmatrix}'. \quad (16)$$

The admittance Y_{ph} is the value used in the network equations (9a) and (9b). It characterises the single-phase conductor linking nodes *a* and *b*. Similar impedance models of d.c. conductors, multi-phase (mutually coupled) conductors and transformers have been developed for use within ESP-r [6].

As an example of a hybrid component consider a facade-integrated PV module (Figure 3). This consists of a multi-layered construction (the PV material†, conductors, a binder and a casing), augmented with an equivalent circuit model to deal with the power production of the PV layer.

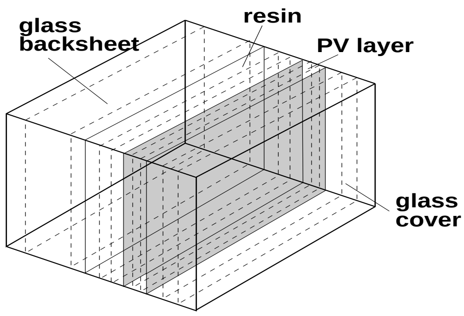

Figure 3: PV multi-layer construction.

In the construction of Figure 3, solar radiation is transmitted to the surface of the PV layer through the glass cover. The calculation of the intensity of solar radiation incident on the glass surface is handled by ESP-r's solar processing algorithm. The incident radiation is a function of the surface geometry, site location, shading and weather parameters prevailing at a particular simulation time step. The control volume flux balance equation for each node in the construction is as follows.

$$\left[\frac{2\rho c_i}{\Delta t} + \frac{k_{i,i+1} + k_{i,i-1}}{\Delta {x_i}^2}\right]{\theta_i}^{t+\Delta t} -$$

$$\left[\frac{k_{i,i+1}{\theta_{i-1}}^{t+\Delta t} + k_{i,i-1}{\theta_{i+1}}^{t+\Delta t}}{\Delta {x_i}^2}\right] - \frac{{\alpha_i}^{t+\Delta t}}{\Delta V}$$

$$= \left[\frac{2\rho c_i}{\Delta t} + \frac{k_{i,i+1} + k_{i,i-1}}{\Delta {x_i}^2}\right]{\theta_i}^{t} +$$

$$\left[\frac{k_{i,i+1}{\theta_{i-1}}^{t} + k_{i,i-1}{\theta_{i+1}}^{t}}{\Delta {x_i}^2}\right] + \frac{{\alpha_i}^{t+\Delta t}}{\Delta V}\,. \qquad (17)$$

† Note that the PV layer in the panel consists of a number of solar cells, which incorporate both the PV material and conducting elements. The individual solar cells, connected together, are considered to be a layer of homogeneous PV material. The ESP-r solar cell model is described by Evans and Kelly [7].

Within the PV layer not all the absorbed solar radiation α_i is converted to heat; a proportion is converted to electrical energy, resulting in a reduced layer temperature. The characteristic equation of the PV material control volumes must therefore be altered to take account of this phenomenon. This is done by altering the final terms on both the left and right hand sides of the nodal heat conduction equation to account for the production of electrical energy:

$$\alpha_i{}' = \alpha_i - q_{ei}\,. \qquad (18)$$

The power production term q_e is calculated by the following equation, derived from a standard one-diode model of a PV cell described by Buresch [8].

$$q_{ei} = nc\left[V_i I_g\left(1 - \exp\left[\frac{eV_i}{\lambda k T_i}\right]\right) - V_i I_{sc}\,\frac{\alpha_i{}'}{\alpha_{iref}{}'}\right] \qquad (19)$$

The temperature T_i is the temperature of the PV material node calculated by ESP-r's conduction model, while the light generated current is calculated as a function of the the solar energy absorbed in the PV layer α_i. Note that the thermal and electrical energy balance equations of the PV model are closely coupled together, with the model translating thermal phenomena (material temperature and absorbed solar radiation) into electrical power output, which is then fed to a node in the electrical network.

Models for a combined heat and power unit, lighting, small power loads and rotational loads have been developed for use in ESP-r simulations [6]. These components perform the same translation function as the PV model: converting thermal and fluid flow phenomena into electrical power flows.

CASE STUDY

To illustrate how the various concepts introduced in this paper can be combined together to from an integrated thermal/electrical model of an embedded energy system, consider the case of a photovotaic facade with heat recovery.

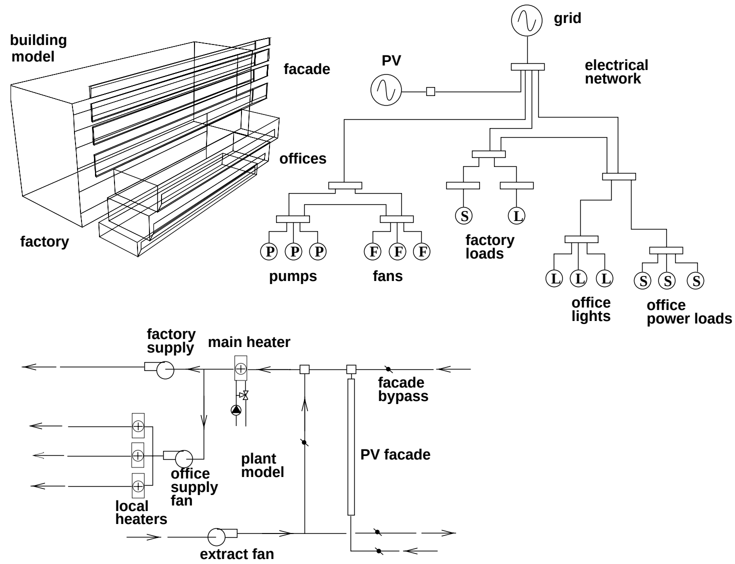

Figure 4: Elsa building model.

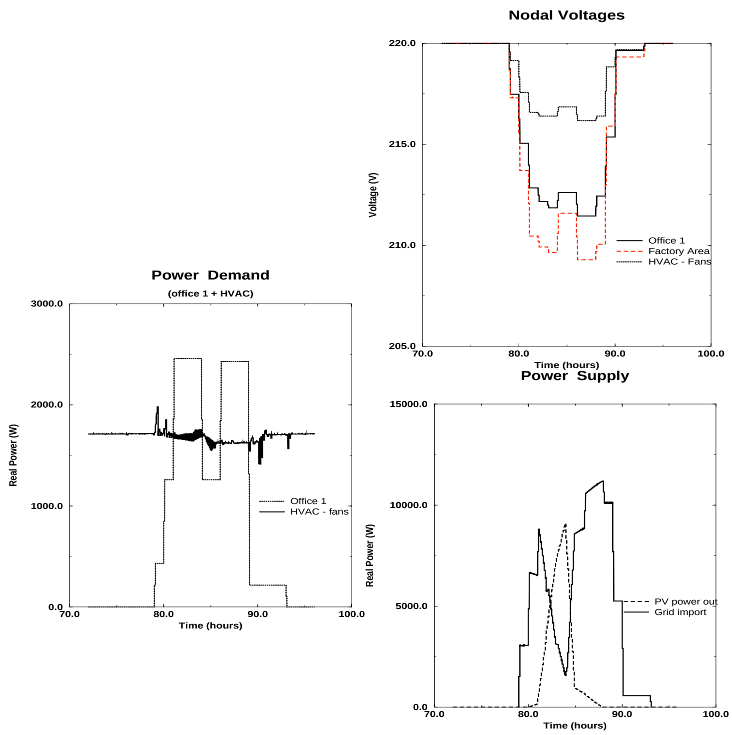

Figure 5: Examples of ESP-r power flow results output.

The problem is loosely based on the ELSA building at the Commission of the European Community's Joint Research Centre, Ispra, Italy. The model comprises a large factory space combined with a three-floor office structure. The south face of the factory accommodates a photovoltaic facade, incorporating crystalline silicon PV modules, with a maximum power output of appoximately 30kW. The facade is also used to pre-heat incoming air to the mechanical ventilation system (Figure 4). The facade is modelled using four stacked thermal zones, while the mechanical ventilation system is modelled using a plant network and an air flow network, which enables the calculation of the air flow through the rear of the facade, within the building interior and throughout the plant network.

The electrical loads and generation sources are integrated into the model:

- Small power and lighting loads are included in each thermal zone and also act as internal heat gains. Both are temporally dependant, with lighting also subject to daylighting control.

- The fans in the plant network draw power from the electrical network and are subject to local, temperature-based control action.
- Photovoltaic components are embedded in the building structure.

The net effect of these hybrid components, subjected to control action, varying climate conditions and occupancy, is to supply time-varying electrical power flows to the electrical network.

The electrical network is modelled as single phase ac. The nodes in the network represent key points in the electrical system: where loads draw power, conductor junctions, points where power is injected into the network form the PV facade and the connection to the grid. The nominal volage for network operation is 220V. The connections between the nodes are represented by single phase conductor models representing electrical cabling.

The power produced by the PV facade is fed into the electrical network after power conditioning in terms of maximum power point tracking and inversion. The fans in the plant network and the lighting and small power loads in the building zones draw power from the network at various points as indicated in Figure 4.

The model is run over a typical spring day (March 4) using the UK climate reference year (Kew 1967). The simulation is conducted at 30 second plant time-steps and 2 minute building time steps. The electrical and air flow networks are solved at the same frequency as the plant network.

Figure 5 shows some of the information which can be extracted from the model, with the emphasis on the electrical data.

CONCLUSIONS AND FUTURE WORK

This paper has described the integration of an electrical power flow model into ESP-r in support of the modelling of embedded energy systems. The electrical system is modelled as a network of connected electrical control volumes. Solution of the network power flows is achieved using a Newton-Raphson solver. The boundary conditions for the solution of the network are provided by hybrid components. These translate thermal and flow phenomena occuring elsewhere in the building model into boundary condition power flows for the electrical network. In this way the thermal and electrical domains are coupled.

The network approach is applicable to any type of electrical system, not just localised power generation. Hence the scope for the use of integrated thermal and electrical systems modelling is large.

The work described here represents only the initial step in the integration of electrical systems modelling within building simulation. The ESP-r capability is currently being improved and expanded upon, with the addition of new component models and user interface facilities.

Work is also underway to address the important issue of power quality in embedded generation. Voltasge, current and power flow data from an ESP-r thermal/electrical simulation can be supplied to the EMTP † tool, which is used to model high frequency electrical phenomena (e.g. harmonic distortion) associated the embedded power sources.

Finally, particular attention is being focused on the possible applications of control. This is perhaps the richest area for exploitation using integrated thermal and electrical modelling. The addition of electrical power flow modelling adds a new range of variables (voltage, power flow, current, etc.) around which new control algorithms can be constructed, particularly in the development and testing of techniques to match local supply to demand. Such algorithms would be responsible for making more efficient use of heat and power, minimising energy usage conflicts (e.g. between artificial lighting, daylighting and cooling), scheduling energy storage, limiting energy demand and controlling energy supply. In effect, the control described here would be a global energy management system, managing both the supply and demand of energy in a building.

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NOMENCLATURE

P - real power power (W);

Q - reactive power power (VAR);

S - apparent power (VA);

T - temperature (K);

V - voltage (V);

Y - admittance (Ω^{-1});

Z - impedance (Ω);

c - specific heat (J/kgK);

e - electron charge (C);

k - Boltzmann constant (-);

q - PV electrical power output (W);

ΔT - timestep (s);

ΔV - control volume (m^3).

Greek characters

θ - phase angle (^{o}) or temperature (^oC);

γ - argument of admittance $\mathbf{Y}$ (^{o})

ρ - density (kgm^{-3});

α - absorbed solar radiation (W).

Subscripts

G - generated quantity;

L - load quantity;

T - transmitted quantity;

c - specific heat;

e - electrical quantity ;

g - light generated quantity;

i - refering to node i ;

j, $i+1$, $i-1$ - adjacent nodes;

i,j - quantity between nodes i and j;

n - number of nodes in a network;

nc - number of solar cells in construction;

ph - phase quantity

$ph\text{-}n$ - phase to neutral quantity;

ref - quantity measured or calculated at PV test reference conditions;

sc - PV module measured short circuit current;

nG - number of connected transmitting elements;

nL - number of connected generator elements;

nT - number of connected load elements.

Superscripts

$'$ - complex conjugate or modified nodal absorption;

t - present simulation time step;

$t + \Delta t$ - future simulation time step.