ARTICLE IN PRESS
Systems & Control Letters ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stabilization of hybrid stochastic differential equations by feedback control based on discrete-time state observations[☆]

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ARTICLE INFO

Article history:
Received 30 November 2013
Received in revised form
5 April 2014
Accepted 12 August 2014
Available online xxxx

Keywords:
Brownian motion
Markov chain
Mean-square exponential stability
Feedback control
Discrete-time state observation

ABSTRACT

Recently, Mao (2013) discusses the mean-square exponential stabilization of continuous-time hybrid stochastic differential equations by *feedback controls based on discrete-time state observations*. Mao (2013) also obtains an upper bound on the duration τ between two consecutive state observations. However, it is due to the general technique used there that the bound on τ is not very sharp. In this paper, we will consider a couple of important classes of hybrid SDEs. Making full use of their special features, we will be able to establish a better bound on τ. Our new theory enables us to observe the system state less frequently (so costs less) but still to be able to design the feedback control based on the discrete-time state observations to stabilize the given hybrid SDEs in the sense of mean-square exponential stability.

1. Introduction

Hybrid stochastic differential equations (SDEs) (also known as SDEs with Markovian switching) have been used to model many practical systems where they may experience abrupt changes in their structure and parameters. One of the important issues in the study of hybrid systems is the automatic control, with consequent emphasis being placed on the asymptotic analysis of stability [1–19]. In particular, [20,21] are two of most cited papers (Google citations 447 and 269, respectively) while [22] is the first book in this area (Google citation 496).

Recently, Mao [23] investigates the following stabilization problem by a feedback control based on the discrete-time state observations: consider an unstable hybrid SDE

$$dx(t) = f(x(t), r(t), t)dt + g(x(t), r(t), t)dw(t), \tag{1}$$

where $x(t) \in R^n$ is the state, $w(t) = (w_1(t), \dots, w_m(t))^T$ is an m-dimensional Brownian motion, $r(t)$ is a Markov chain (please see Section 2 for the formal definitions) which represents the system mode, and the SDE is in the Itô sense. The aim is to design a feedback control $u(x([t/\tau]\tau), r(t), t)$ in the drift part so that the controlled system

$$\begin{aligned} dx(t) = {} & \big(f(x(t), r(t), t) + u(x([t/\tau]\tau), r(t), t)\big)dt \\ & + g(x(t), r(t), t)dw(t) \end{aligned} \tag{2}$$

becomes stable, where $\tau > 0$ is a constant and $[t/\tau]$ is the integer part of t/τ. The key feature here is that the feedback control $u(x([t/\tau]\tau), r(t), t)$ is designed based on the discrete-time observations of the state $x(t)$ at times $0, \tau, 2\tau, \dots$. This is significantly different from the stabilization by a continuous-time (regular) feedback control $u(x(t), r(t), t)$, based on the current state, where the aim is to design $u(x(t), r(t), t)$ in order for the controlled system

$$\begin{aligned} dx(t) = {} & \big(f(x(t), r(t), t) + u(x(t), r(t), t)\big)dt \\ & + g(x(t), r(t), t)dw(t) \end{aligned} \tag{3}$$

to be stable. In fact, the regular feedback control requires the continuous observations of the state $x(t)$ for all $t \geq 0$, while the feedback control $u(x([t/\tau]\tau), r(t), t)$ needs only the discrete-time observations of the state $x(t)$ at times $0, \tau, 2\tau, \dots$. The latter is

☆ This paper was not presented at any IFAC meeting.
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http://dx.doi.org/10.1016/j.sysconle.2014.08.011
0167-6911/

clearly more realistic and costs less in practice. To the best knowledge of the authors, Mao [23] is the first paper that studies this stabilization problem by feedback controls based on the discrete-time state observations in the area of SDEs, although the corresponding problem for the deterministic differential equations has been studied by many authors (see e.g. [24–28]).

Mao [23] shows that if continuous-time controlled SDE (3) is mean-square exponentially stable, then so is the discrete-time-state feedback controlled system (2) provided that τ is sufficiently small. This is of course a very general result. However, it is due to the general technique used there that the bound on τ is not very sharp. In this paper, we will consider a couple of important classes of hybrid SDEs. Making full use of their special features, we will be able to establish a better bound on τ.

Mathematically speaking, the key technique in Mao [23] is to compare the discrete-time-state feedback controlled system (2) with the continuous-time controlled SDE (3) and then prove the stability of system (2) by making use of the stability of SDE (3). However, in this paper, we will work directly on the discrete-time-state feedback controlled system (2) itself. To cope with the mixture of the continuous-time state $x(t)$ and the discrete-time state $x([t/\tau]\tau)$ in the system, we have developed some new techniques. Let us begin to develop these new techniques and to establish our new theory.

2. Notation and stabilization problem

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Let $w(t) = (w_1(t), \dots, w_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. If A is a vector or matrix, its transpose is denoted by A^T. If $x \in R^n$, then $|x|$ is its Euclidean norm. If A is a matrix, we let $|A| = \sqrt{\text{trace}(A^TA)}$ be its trace norm and $\|A\| = \max\{|Ax| : |x| = 1\}$ be the operator norm. If A is a symmetric matrix ($A = A^T$), denote by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ its smallest and largest eigenvalues, respectively. By $A \le 0$ and $A < 0$, we mean A is non-positive and negative definite, respectively. Denote by $L^2_{\mathcal{F}_t}(R^n)$ the family of all $\mathcal{F}_t$-measurable R^n-valued random variables ξ such that $\mathbb{E}|\xi|^2 < \infty$, where $\mathbb{E}$ is the expectation with respect to the probability measure $\mathbb{P}$. If both a, b are real numbers, then $a \vee b = \min\{a, b\}$ and $a \wedge b = \max\{a, b\}$. Let $r(t)$, $t \ge 0$, be a right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \dots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \ne j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \ge 0$ is the transition rate from i to j if $i \ne j$ while

$$\gamma_{ii} = -\sum_{j\ne i} \gamma_{ij}.$$

We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $w(\cdot)$. It is known that almost all sample paths of $r(t)$ are constant except for a finite number of simple jumps in any finite subinterval of $R_+ (:= [0, \infty))$. We stress that almost all sample paths of $r(t)$ are right continuous.

Consider an n-dimensional linear hybrid SDE

$$dx(t) = A(r(t))x(t)dt + \sum_{k=1}^m B_k(r(t))x(t)dw_k(t) \tag{4}$$

on $t \ge 0$, with initial data $x(0) = x_0 \in L^2_{\mathcal{F}_0}(R^n)$. Here $A,\ B_k : S \to R^{n\times n}$ and we will often write $A(i) = A_i$ and $B_k(i) = B_{ki}$. Suppose that this given equation is unstable and we are required to design a feedback control $u(x(\delta(t)), r(t))$ based on the discrete-time state observations in the drift part so that the controlled SDE

$$dx(t) = [A(r(t))x(t) + u(x(\delta(t)), r(t))]dt + \sum_{k=1}^m B_k(r(t))x(t)dw_k(t) \tag{5}$$

will be mean-square exponentially stable, where u is a mapping from $R^n \times S$ to R^n, $\tau > 0$ and

$$\delta(t) = [t/\tau]\tau \quad \text{for } t \ge 0, \tag{6}$$

in which $[t/\tau]$ is the integer part of t/τ. We repeat that the feedback control $u(x(\delta(t)), r(t))$ is designed based on the discrete-time state observations $x(0), x(\tau), x(2\tau), \dots,$ though the given hybrid SDE (4) is of continuous time. As the given SDE (4) is linear, it is natural to use a linear feedback control. One of the most common linear feedback controls is the structure control of the form $u(x, i) = F(i)G(i)x$, where F and G are mappings from S to $R^{n\times l}$ and $R^{l\times n}$, respectively, and one of them is given while the other needs to be designed. These two cases are known as:

- State feedback: design $F(\cdot)$ when $G(\cdot)$ is given.
- Output injection: design $G(\cdot)$ when $F(\cdot)$ is given.

Again, we will often write $F(i) = F_i$ and $G(i) = G_i$. As a result, controlled system (5) becomes

$$dx(t) = [A(r(t))x(t) + F(r(t))G(r(t))x(\delta(t))]dt + \sum_{k=1}^m B_k(r(t))x(t)dw_k(t). \tag{7}$$

We observe that Eq. (7) is in fact a stochastic differential delay equation (SDDE) with a bounded variable delay. Indeed, if we define the bounded variable delay $\zeta : [0, \infty) \to [0, \tau]$ by

$$\zeta(t) = t - v\tau \quad \text{for } v\tau \le t < t(v+1)\tau, \tag{8}$$

and $v = 0, 1, 2, \dots,$ then Eq. (7) can be written as

$$dx(t) = [A(r(t))x(t) + F(r(t))G(r(t))x(t - \zeta(t))]dt + \sum_{k=1}^m B_k(r(t))x(t)dw_k(t). \tag{9}$$

It is therefore known (see e.g. [22]) that Eq. (7) has a unique solution $x(t)$ such that $\mathbb{E}|x(t)|^2 < \infty$ for all $t \ge 0$.

3. Main results

In this section, we will first write $F(r(t))G(r(t)) = D(r(t))$ and establish the stability theory for the following hybrid SDE

$$dx(t) = [A(r(t))x(t) + D(r(t))x(\delta(t))]dt + \sum_{k=1}^m B_k(r(t))x(t)dw_k(t). \tag{10}$$

We will then design either $G(\cdot)$ given $F(\cdot)$ or $F(\cdot)$ given $G(\cdot)$ in order for controlled SDE (7) to be stable.

3.1. Stability of SDE (10)

Let us begin with a useful lemma.

Lemma 3.1. *Set*

$$M_A = \max_{i\in S} \|A_i\|^2, \qquad M_D = \max_{i\in S} \|D_i\|^2,$$

$$M_B = \max_{i\in S} \sum_{k=1}^m \|B_{ki}\|^2,$$

Please cite this article in press as: X. Mao, et al., Stabilization of hybrid stochastic differential equations by feedback control based on discrete-time state observations, Systems & Control Letters (2014), http://dx.doi.org/10.1016/j.sysconle.2014.08.011

and define

$$K(\tau) = [6\tau(\tau M_A + M_B) + 3\tau^2 M_D]e^{6\tau(\tau M_A + M_B)} \tag{11}$$

for $\tau > 0$. If τ is sufficiently small for $2K(\tau) < 1$, then the solution $x(t)$ of SDE (10) satisfies

$$\mathbb{E}|x(t) - x(\delta(t))|^2 \le \frac{2K(\tau)}{1 - 2K(\tau)}\mathbb{E}|x(t)|^2 \tag{12}$$

for all $t \ge 0$.

Proof. Fix any integer $v \ge 0$. For $t \in [v\tau, (v+1)\tau)$, we have $\delta(t) = v\tau$. It follows from (10) that

$$\begin{aligned} x(t) - x(\delta(t)) = x(t) - x(v\tau) &= \int_{v\tau}^{t} [A(r(s))x(s) + D(r(s))x(v\tau)]ds \\ &\quad + \sum_{k=1}^{m} \int_{v\tau}^{t} B_k(r(s))x(s)dw_k(s). \end{aligned}$$

We can then derive

$$\begin{aligned} &\mathbb{E}|x(t) - x(\delta(t))|^2 \\ &\le 3(\tau M_A + M_B)\int_{v\tau}^{t} \mathbb{E}|x(s)|^2 ds + 3\tau^2 M_D \mathbb{E}|x(k\tau)|^2 \\ &\le 6(\tau M_A + M_B)\int_{v\tau}^{t} \mathbb{E}|x(s) - x(\delta(s))|^2 ds \\ &\quad + [6\tau(\tau M_A + M_B) + 3\tau^2 M_D]\mathbb{E}|x(v\tau)|^2. \end{aligned}$$

The well-known Gronwall inequality shows

$$\mathbb{E}|x(t) - x(\delta(t))|^2 \le K(\tau)\mathbb{E}|x(v\tau)|^2.$$

Consequently

$$\mathbb{E}|x(t) - x(\delta(t))|^2 \le 2K(\tau)\Big(\mathbb{E}|x(t) - x(\delta(t))|^2 + \mathbb{E}|x(t)|^2\Big).$$

This implies that (12) holds for $t \in [v\tau, (v+1)\tau)$. But $v \ge 0$ is arbitrary so desired assertion (12) must hold for all $t \ge 0$. The proof is complete. □

Theorem 3.2. *Assume that there are symmetric positive-definite matrices $Q(i) = Q_i$ $(i \in S)$ such that*

$$\begin{aligned} \bar{Q}(i) = \bar{Q}_i &:= Q_i(A_i + D_i) + (A_i + D_i)^T Q_i \\ &\quad + \sum_{k=1}^{m} B_{ki}^T Q_i B_{ki} + \sum_{j=1}^{N} \gamma_{ij} Q_j \end{aligned} \tag{13}$$

are all negative-definite matrices. Set

$$-\lambda := \max_{i\in S} \lambda_{\max}(\bar{Q}_i) \quad \text{and} \quad M_{QD} = \max_{i\in S} \|Q_i D_i\|^2$$

(and of course $\lambda > 0$). If τ is sufficiently small for $\lambda > 2\lambda_\tau$, where

$$\lambda_\tau := \sqrt{\frac{2M_{QD}K(\tau)}{1 - 2K(\tau)}}, \tag{14}$$

then the solution of SDE (10) satisfies

$$\mathbb{E}|x(t)|^2 \le \frac{\lambda_M}{\lambda_m}\mathbb{E}|x_0|^2 e^{-\theta t}, \quad \forall t \ge 0, \tag{15}$$

where $\lambda_M = \max_{i\in S}\lambda_{\max}(Q_i)$, $\lambda_m = \min_{i\in S}\lambda_{\min}(Q_i)$, $K(\tau)$ has been defined in Lemma 3.1 and

$$\theta = \frac{\lambda - 2\lambda_\tau}{\lambda_M}. \tag{16}$$

In other words, SDE (10) is exponentially stable in mean square.

Proof. Applying the generalized Itô formula (see e.g. [22, Theorem 1.14 on page 48]) to $x^T(t)Q(r(t))x(t)$ we get

$$\begin{aligned} &d[x^T(t)Q(r(t))x(t)] \\ &= \Big(2x^T(t)Q(r(t))[A(r(t))x(t) + D(r(t))x(\delta(t))] \\ &\quad + \sum_{k=1}^{m} x^T(t)B_k^T(r(t))Q(r(t))B_k(r(t))x(t) \\ &\quad + \sum_{j=1}^{N} \gamma_{r(t),j}x^T(t)Q_j x(t)\Big)dt + dM_1(t) \\ &= \Big(x^T(t)\bar{Q}(r(t))x(t) \\ &\quad - 2x^T(t)Q(r(t))D(r(t))(x(t) - x(\delta(t)))\Big)dt + dM_1(t). \end{aligned}$$

Here $M_1(t)$ and the following $M_2(t)$ are martingales with $M_1(0) = M_2(0) = 0$. Their forms are not used so are not specified here as we will take expectations later and their means are zero. Applying the generalized Itô formula now to $e^{\theta t}x^T(t)Q(r(t))x(t)$, we then have

$$\begin{aligned} &d[e^{\theta t}x^T(t)Q(r(t))x(t)] \\ &= e^{\theta t}\Big(\theta x^T(t)Q(r(t))x(t) + x^T(t)\bar{Q}(r(t))x(t) \\ &\quad -2x^T(t)Q(r(t))D(r(t))(x(t) - x(\delta(t)))\Big)dt + dM_2(t). \end{aligned}$$

This implies

$$\begin{aligned} \lambda_m e^{\theta t}\mathbb{E}|x(t)|^2 &\le \lambda_M \mathbb{E}|x_0|^2 + \int_0^t (\theta\lambda_M - \lambda)e^{\theta s}\mathbb{E}|x(s)|^2 ds \\ &\quad + \int_0^t 2e^{\theta s}\sqrt{M_{QD}}\,\mathbb{E}(|x(s)||x(s) - x(\delta(s))|)ds. \end{aligned} \tag{17}$$

But, by Lemma 3.1, we have

$$\begin{aligned} &2\sqrt{M_{QD}}\,\mathbb{E}(|x(s)||x(s) - x(\delta(s))|) \\ &\le \lambda_\tau \mathbb{E}|x(s)|^2 + \frac{M_{QD}}{\lambda_\tau}\mathbb{E}|x(s) - x(\delta(s))|^2 \\ &\le \lambda_\tau \mathbb{E}|x(s)|^2 + \frac{M_{QD}}{\lambda_\tau}\,\frac{2K(\tau)}{1 - 2K(\tau)}\,\mathbb{E}|x(t)|^2 \\ &= 2\lambda_\tau \mathbb{E}|x(s)|^2. \end{aligned} \tag{18}$$

Substituting this into (17) yields

$$\lambda_m e^{\theta t}\mathbb{E}|x(t)|^2 \le \lambda_M \mathbb{E}|x_0|^2 + \int_0^t (\theta\lambda_M + 2\lambda_\tau - \lambda)e^{\theta s}\mathbb{E}|x(s)|^2 ds.$$

But, by (16), $\theta\lambda_M + 2\lambda_\tau - \lambda = 0$. Thus

$$\lambda_m e^{\theta t}\mathbb{E}|x(t)|^2 \le \lambda_M \mathbb{E}|x_0|^2,$$

which implies desired assertion (15). The proof is complete. □

3.2. State feedback: design $F(\cdot)$ when $G(\cdot)$ is given

We can now begin to consider the case of state feedback. In this case, $G(\cdot)$ is given so our aim is to design $F(\cdot)$ such that controlled SDE (7) is exponentially stable in mean square. One technique used frequently in the study of stability of linear SDEs is the method of linear matrix inequalities (LMIs) (see e.g. [29–32,10]), although there are other methods (see e.g. the survey paper [33]). We will use the technique of LMIs to design $F(\cdot)$ in this section.

According to Theorem 3.2, it is sufficient if we can design $G(\cdot)$, namely G_i for $i \in S$, so that we can further find positive-definite symmetric matrices Q_i $(i \in S)$ in order for

$$Q_i(A_i + F_iG_i) + (A_i + F_iG_i)^T Q_i + \sum_{k=1}^m B_{ki}^T Q_i B_{ki} + \sum_{j=1}^N \gamma_{ij} Q_j < 0, \quad i \in S. \tag{19}$$

We observe that the above matrix inequalities are not linear in Q_i and F_i's. However, if we set $Y_i = Q_iF_i$, then they become the following LMIs

$$Q_iA_i + Y_iG_i + A_i^T Q_i + G_i^T Y_i^T + \sum_{k=1}^m B_{ki}^T Q_i B_{ki} + \sum_{j=1}^N \gamma_{ij} Q_j < 0, \quad i \in S. \tag{20}$$

If these LMIs have their solutions $Q_i = Q_i^T > 0$ and Y_i $(i \in S)$, then, setting $F_i = Q_i^{-1}Y_i$, we have (19). Applying Theorem 3.2, we hence obtain the following corollary.

Corollary 3.3. *Assume that the LMIs in (20) have their solutions $Q_i = Q_i^T > 0$ and Y_i. Set $F_i = Q_i^{-1}Y_i$ and $D_i = F_iG_i$. Then the conclusion of Theorem 3.2 holds. In other words, controlled SDE (7) will be exponentially stable in mean square if we set $F_i = Q_i^{-1}Y_i$ and make sure $\tau > 0$ be sufficiently small for $\lambda > 2\lambda_\tau$.*

3.3. Output injection: design $G(\cdot)$ when $F(\cdot)$ is given

Let us now consider the case of output injection. In this case, $F(\cdot)$ is given and our aim is to design $G(\cdot)$ so that controlled SDE (7) is exponentially stable in mean square. Once again, based on Theorem 3.2, it is sufficient if we can design $F(\cdot)$, namely F_i for $i \in S$, so that we can further find positive-definite symmetric matrices Q_i $(i \in S)$ in order for matrix inequalities (19) to hold. Multiplying Q_i^{-1} from left and then from right, and writing $Q_i^{-1} = X_i$, we see that matrix inequalities (19) are equivalent to the following matrix inequalities

$$A_iX_i + F_iG_iX_i + X_iA_i^T + X_iG_i^T F_i^T + \sum_{k=1}^m X_iB_{ki}^T X_i^{-1} B_{ki} X_i + \sum_{j=1}^N \gamma_{ij} X_i X_j^{-1} X_i < 0, \quad i \in S. \tag{21}$$

By setting $G_iX_i = Y_i$, these matrix inequalities become

$$A_iX_i + F_iY_i + X_iA_i^T + Y_i^T F_i^T + \gamma_{ii}X_i + \sum_{k=1}^m X_iB_{ki}^T X_i^{-1} B_{ki} X_i + \sum_{j \neq i}^N \gamma_{ij} X_i X_j^{-1} X_i < 0, \quad i \in S. \tag{22}$$

By the well-known Schur complements (see [22, Theorem 2.8 on page 64]), we see these matrix inequalities are equivalent to the following LMIs

$$\begin{bmatrix} M_{i1} & M_{i2} & M_{i3} \\ M_{i2}^T & -M_{i4} & 0 \\ M_{i3}^T & 0 & -M_{i5} \end{bmatrix} < 0, \quad i \in S, \tag{23}$$

where

$$M_{i1} = A_iX_i + F_iY_i + X_iA_i^T + Y_i^T F_i^T + \gamma_{ii}X_i,$$
$$M_{i2} = [X_iB_{1i}^T, \ldots, X_iB_{mi}^T],$$
$$M_{i3} = [\sqrt{\gamma_{i1}}X_i, \ldots, \sqrt{\gamma_{i(i-1)}}X_i, \sqrt{\gamma_{i(i+1)}}X_i, \ldots, \sqrt{\gamma_{iN}}X_i],$$
$$M_{i4} = \mathrm{diag}[X_i, \ldots, X_i],$$
$$M_{i5} = \mathrm{diag}[X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N].$$

In other words, if the LMIs in (23) have their solutions $X_i = X_i^T > 0$ and Y_i $(i \in S)$, then, setting $Q_i = X_i^{-1}$ and $G_i = Y_iX_i^{-1}$, we have (19). Applying Theorem 3.2, we hence obtain the following corollary.

Corollary 3.4. *Assume that the LMIs in (23) have their solutions $X_i = X_i^T > 0$ and Y_i $(i \in S)$. Set $Q_i = X_i^{-1}$ and $G_i = Y_iX_i^{-1}$. Then the conclusion of Theorem 3.2 holds. In other words, controlled SDE (7) will be exponentially stable in mean square if we set $G_i = Y_iX_i^{-1}$ and make sure $\tau > 0$ be sufficiently small for $\lambda > 2\lambda_\tau$.*

4. Stabilization of nonlinear hybrid SDEs

Let us now discuss a more general nonlinear problem. Assume that the underlying system is now described by a nonlinear hybrid SDE

$$dx(t) = f(x(t), r(t), t)dt + g(x(t), r(t), t)dw(t) \tag{24}$$

on $t \geq 0$ with the initial data $x(0) = x_0 \in L^2_{\mathcal{F}_0}(R^n)$. Here, $f : R^n \times S \times R_+ \to R^n$ and $g : R^n \times S \times R_+ \to R^{n \times m}$. Assume that both f and g are locally Lipschitz continuous and obey the linear growth condition (see e.g. [22]). We also assume that $f(0, i, t) = 0$ and $g(0, i, t) = 0$ for all $i \in S$ and $t \geq 0$ so that $x = 0$ is an equilibrium point for (24).

Suppose that the given SDE (24) is unstable and we are required to design a linear feedback control $F(r(t))G(r(t))x(\delta(t))$ based on the discrete-time state observations in the drift part so that the controlled system

$$dx(t) = [f(x(t), r(t), t) + F(r(t))G(r(t))x(\delta(t))]dt + g(x(t), r(t), t)dw(t) \tag{25}$$

will be mean-square exponentially stable. Recalling the definition of ζ by (8), we see that SDE (25) can be written as an SDDE

$$dx(t) = [f(x(t), r(t), t) + F(r(t))G(r(t))x(t - \zeta(t))]dt + g(x(t), r(t), t)dw(t). \tag{26}$$

It is therefore known (see e.g. [22]) that Eq. (25) has a unique solution $x(t)$ such that $\mathbb{E}|x(t)|^2 < \infty$ for all $t \geq 0$.

Given that we use a linear control to stabilize a nonlinear system, it is natural to impose some conditions on the nonlinear coefficients f and g.

Assumption 4.1. For each $i \in S$, there is a pair of symmetric $n \times n$-matrices Q_i and $\hat{Q}_i$ with Q_i being positive-definite such that

$$2x^T Q_i f(x, i, t) + g^T(x, i, t) Q_i g(x, i, t) \leq x^T \hat{Q}_i x$$

for all $(x, i, t) \in R^n \times S \times R_+$.

Assumption 4.2. There is a pair of positive constants δ_1 and δ_2 such that

$$|f(x, i, t)|^2 \leq \delta_1 |x|^2 \quad \text{and} \quad |g(x, i, t)|^2 \leq \delta_2 |x|^2$$

for all $(x, i, t) \in R^n \times S \times R_+$.

Let us first present a useful lemma.

Lemma 4.3. *Let Assumption 4.2 hold. Set*

$$\delta_3 = \max_{i \in S} \sum_{k=1}^m \|F_iG_i\|^2,$$

and define

$$H(\tau) = [6\tau(\tau\delta_1 + \delta_2) + 3\tau^2\delta_3]e^{6\tau(\tau\delta_1 + \delta_2)} \tag{27}$$

for $\tau > 0$. If τ is sufficiently small for $2H(\tau) < 1$, then the solution $x(t)$ of SDE (25) satisfies

$$\mathbb{E}|x(t) - x(\delta(t))|^2 \leq \frac{2H(\tau)}{1 - 2H(\tau)} \mathbb{E}|x(t)|^2 \tag{28}$$

for all $t \geq 0$.

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This lemma can be proved in the same way as Lemma 3.1 was proved so we omit the proof.

Theorem 4.4. *Let Assumptions 4.1 and 4.2 hold. Assume that the following LMIs*

$$U_i := \hat{Q}_i + Q_iF_iG_i + G_i^TF_i^TQ_i + \sum_{j=1}^N \gamma_{ij}Q_j < 0, \quad i \in S, \tag{29}$$

have their solutions F_i ($i \in S$) in the case of feedback control (i.e. G_i's are given) or their solutions G_i in the case of output injection (i.e. F_i's are given). Set

$$-\gamma := \max_{i\in S} \lambda_{\max}(U_i) \quad and \quad \delta_4 = \max_{i\in S} \|Q_iF_iG_i\|^2.$$

If τ is sufficiently small for $\gamma > 2\gamma_\tau$, where

$$\gamma_\tau := \sqrt{\frac{2\delta_4 H(\tau)}{1-2H(\tau)}}, \tag{30}$$

then the solution of SDE (25) satisfies

$$\mathbb{E}|x(t)|^2 \le \frac{\lambda_M}{\lambda_m}\mathbb{E}|x_0|^2 e^{-\theta t}, \quad \forall t \ge 0, \tag{31}$$

where $\lambda_M = \max_{i\in S}\lambda_{\max}(Q_i)$, $\lambda_m = \min_{i\in S}\lambda_{\min}(Q_i)$, $H(\tau)$ has been defined in Lemma 4.3 and

$$\theta = \frac{\gamma - 2\gamma_\tau}{\lambda_M}. \tag{32}$$

Proof. This theorem can be proved in a similar way as Theorem 3.2 was proved so we only give the key steps. Applying the generalized Itô formula to $x^T(t)Q(r(t))x(t)$ we get

$$d[x^T(t)Q(r(t))x(t)] = \Big(x^T(t)U(r(t))x(t) - 2x^T(t)Q(r(t))F(r(t))G(r(t))(x(t) - x(\delta(t)))\Big)dt + dM_3(t),$$

where $M_3(t)$ is a martingale with $M_3(0) = 0$. Applying the generalized Itô formula further to $e^{\theta t}x^T(t)Q(r(t))x(t)$, we can then obtain

$$\lambda_m e^{\theta t}\mathbb{E}|x(t)|^2 \le \lambda_M\mathbb{E}|x_0|^2 + \int_0^t (\theta\lambda_M - \gamma)e^{\theta s}\mathbb{E}|x(s)|^2 ds + \int_0^t 2e^{\theta s}\sqrt{\delta_4}\,\mathbb{E}(|x(s)||x(s) - x(\delta(s))|)ds. \tag{33}$$

But, by Lemma 4.3, we can show

$$2\sqrt{\delta_4}\,\mathbb{E}(|x(s)||x(s) - x(\delta(s))|) \le 2\gamma_\tau\mathbb{E}|x(s)|^2. \tag{34}$$

Substituting this into (33) yields

$$\lambda_m e^{\theta t}\mathbb{E}|x(t)|^2 \le \lambda_M\mathbb{E}|x_0|^2,$$

which implies desired assertion (31). The proof is complete. □

To apply Theorem 4.4, we need two steps:

1 we first need to look for the $2N$ matrices Q_i and $\hat{Q}_i$ for Assumption 4.1 to hold;
2 we then need to solve the LMIs in (29) for their solutions F_i (or G_i).

There are available computer softwares e.g. Matlab for step 2 so in the remaining part of this section we will develop some ideas for step 1. To make our ideas more clear, *we will only consider the case of feedback control, but the same ideas work for the case of output injection*.

In theory, it is flexible to use $2N$ matrices Q_i and $\hat{Q}_i$ in Assumption 4.1. But, in practice, it means more work to be done in finding these $2N$ matrices. It is in this spirit that we introduce a stronger assumption.

Assumption 4.5. There are $N+1$ symmetric $n \times n$-matrices Z and Z_i ($i \in S$) with $Z > 0$ such that

$$2x^TZf(x,i,t) + g^T(x,i,t)Zg(x,i,t) \le x^TZ_ix$$

for all $(x,i,t) \in R^n \times S \times R_+$.

Under this assumption, if we let $Q_i = q_iZ$ and $\hat{Q}_i = q_iZ_i$ for some positive numbers q_i, then Assumption 4.1 holds. Moreover, the LMIs in (29) become

$$q_iZ_i + q_iZF_iG_i + q_iG_i^TF_i^TZ + \sum_{j=1}^N \gamma_{ij}q_jZ < 0, \quad i \in S.$$

If we set $Y_i := q_iF_i$, then these become the following LMIs in q_i and Y_i:

$$q_iZ_i + ZY_iG_i + G_i^TY_i^TZ + \sum_{j=1}^N \gamma_{ij}q_jZ < 0, \quad i \in S. \tag{35}$$

We hence have the following corollary.

Corollary 4.6. *Let Assumptions 4.2 and 4.5 hold. Assume that the LMIs in (35) have their solutions $q_i > 0$ and Y_i ($i \in S$). Then Theorem 4.4 holds by setting $Q_i = q_iZ$, $\hat{Q}_i = q_iZ_i$ and $F_i = q_i^{-1}Y_i$. In other words, controlled SDE (25) will be exponentially stable in mean square if we set $F_i = q_i^{-1}Y_i$ and make sure $\tau > 0$ be sufficiently small for $\gamma > 2\gamma_\tau$.*

An even simpler (but in fact stronger) condition is as follows.

Assumption 4.7. There are constants z_i ($i \in S$) such that

$$2x^Tf(x,i,t) + |g(x,i,t)|^2 \le z_i|x|^2$$

for all $(x,i,t) \in R^n \times S \times R_+$.

Under this assumption, if we let $Q_i = q_iI$ and $\hat{Q}_i = q_iz_iI$ for some positive numbers q_i, where I is the $n \times n$ identity matrix, then Assumption 4.1 holds. Moreover, the LMIs in (29) become

$$q_iz_iI + q_iF_iG_i + q_iG_i^TF_i^T + \sum_{j=1}^N \gamma_{ij}q_jI < 0, \quad i \in S.$$

If we set $Y_i := q_iF_i$, then these become the following LMIs in q_i and Y_i:

$$q_iz_iI + Y_iG_i + G_i^TY_i^T + \sum_{j=1}^N \gamma_{ij}q_jI < 0, \quad i \in S. \tag{36}$$

We hence have another corollary.

Corollary 4.8. *Let Assumptions 4.2 and 4.7 hold. Assume that the LMIs in (36) have their solutions $q_i > 0$ and Y_i ($i \in S$). Then Theorem 4.4 holds by setting $Q_i = q_iI$, $\hat{Q}_i = q_iz_iI$ and $F_i = q_i^{-1}Y_i$. In other words, controlled SDE (25) will be exponentially stable in mean square if we set $F_i = q_i^{-1}Y_i$ and make sure $\tau > 0$ be sufficiently small for $\gamma > 2\gamma_\tau$.*

5. Examples

Let us now discuss some examples to illustrate our theory.

Example 5.1. Let us first consider the same example as discussed in Mao [23], namely the linear hybrid SDE

$$dx(t) = A(r(t))x(t)dt + B(r(t))x(t)dw(t) \tag{37}$$

on $t \ge t_0$. Here $w(t)$ is a scalar Brownian motion; $r(t)$ is a Markov chain on the state space $S = \{1, 2\}$ with the generator

$$\Gamma = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix};$$

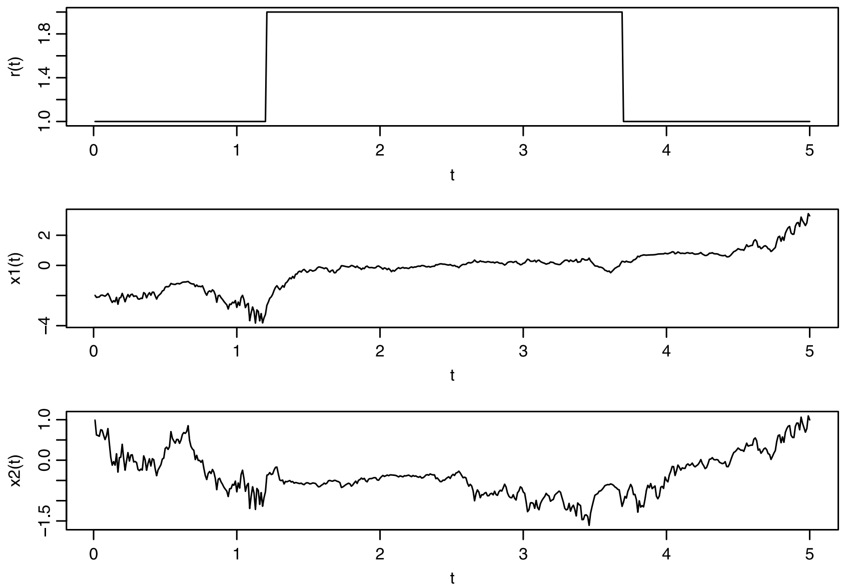

Fig. 5.1. Computer simulation of the paths of $r(t)$, $x_1(t)$ and $x_2(t)$ for the hybrid SDE (37) using the Euler–Maruyama method with step size 10^{-6} and initial values $r(0) = 1$, $x_1(0) = -2$ and $x_2(0) = 1$.

and the system matrices are

$$A_1 = \begin{bmatrix} 1 & -1 \\ 1 & -5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -5 & -1 \\ 1 & 1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}.$$

SDE (37) may be regarded as a system which switches between two operation modes, say mode 1 and mode 2, and the switching obeys the law of the Markov chain, where in mode 1, the system evolves according to the SDE

$$dx(t) = A_1 x(t)dt + B_1 x(t)dw(t),$$

while in mode 2, according to the other SDE

$$dx(t) = A_2 x(t)dt + B_2 x(t)dw(t).$$

The computer simulation (Fig. 5.1) shows that this hybrid SDE is not mean-square exponentially stable. (The simulation of the paths is sufficient to illustrate since it is known that the mean-square exponential stability implies the almost sure exponential stability [22].)

Let us now design a discrete-time-state feedback control to stabilize the system. Assume that the controlled hybrid SDE has the form

$$dx(t) = [A(r(t))x(t) + F(r(t))G(r(t))x(\delta(t))]dt \\ + B(r(t))x(t)dw(t), \tag{38}$$

where

$$G_1 = (1, 0), \qquad G_2 = (0, 1).$$

Our aim here is to seek for F_1 and F_2 in $R^{2\times 1}$ and then make sure τ is sufficiently small for this controlled SDE to be exponentially stable in mean square. To apply Corollary 3.3, we first find that the following LMIs

$$\bar{Q}_i := Q_i A_i + Y_i G_i + A_i^T Q_i + G_i^T Y_i^T + B_i^T Q_i B_i \\ + \sum_{j=1}^{2} \gamma_{ij} Q_j < 0, \quad i = 1, 2,$$

have the following set of solutions $Q_1 = Q_2 = I$ (the 2×2 identity matrix) and

$$Y_1 = \begin{bmatrix} -10 \\ 0 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 0 \\ -10 \end{bmatrix},$$

and for these solutions we have

$$\bar{Q}_1 = \begin{bmatrix} -16 & 0 \\ 0 & -8 \end{bmatrix}, \qquad \bar{Q}_2 = \begin{bmatrix} -8 & 0 \\ 0 & -16 \end{bmatrix}.$$

Hence, we have

$$-\lambda = \max_{i=1,2} \lambda_{\max}(\hat{Q}_i) = -8, \qquad M_{YG} = \max_{i=1,2} \|Y_i G_i\|^2 = 100.$$

To determine λ_τ, we compute

$$M_A = 27.42, \qquad M_B = 2, \qquad M_D = 100, \qquad M_{QD} = 100.$$

Hence

$$\lambda_\tau = \sqrt{\frac{200K(\tau)}{1 - 2K(\tau)}},$$

where $K(\tau) = [6\tau(27.42\tau + 2) + 300\tau^2]e^{6\tau(27.42\tau+2)}$. It is easy to show that $\lambda > 2\lambda_\tau$ whenever $\tau < 0.0046$. By Corollary 3.3, if we set $F_1 = Y_1$ and $F_2 = Y_2$ and make sure that $\tau < 0.0046$, then the discrete-time-state feedback controlled hybrid SDE (38) is mean-square exponentially stable. The computer simulation (Fig. 5.2) supports this result clearly. It should be pointed out that it is required for $\tau < 0.0000308$ in Mao [23], while applying our new theory we only need $\tau < 0.0046$. In other words, our new theory has improved the existing result significantly.

Example 5.2. Let us now discuss one more example, where we will not only illustrate our theory but also explain a new concept which may motivate a further research.

Let $r(t)$, $t \geq 0$, be a right-continuous Markov chain on the probability space taking values in the state space $S = \{1, 2\}$ with generator

$$\Gamma = \begin{bmatrix} -\gamma_{12} & \gamma_{12} \\ \gamma_{21} & -\gamma_{21} \end{bmatrix},$$

where both $\gamma_{12} > 0$ and $\gamma_{21} > 0$. Consider an unstable nonlinear hybrid SDE

$$dx(t) = f(x(t), r(t), t)dt + g(x(t), r(t), t)dw(t). \tag{39}$$

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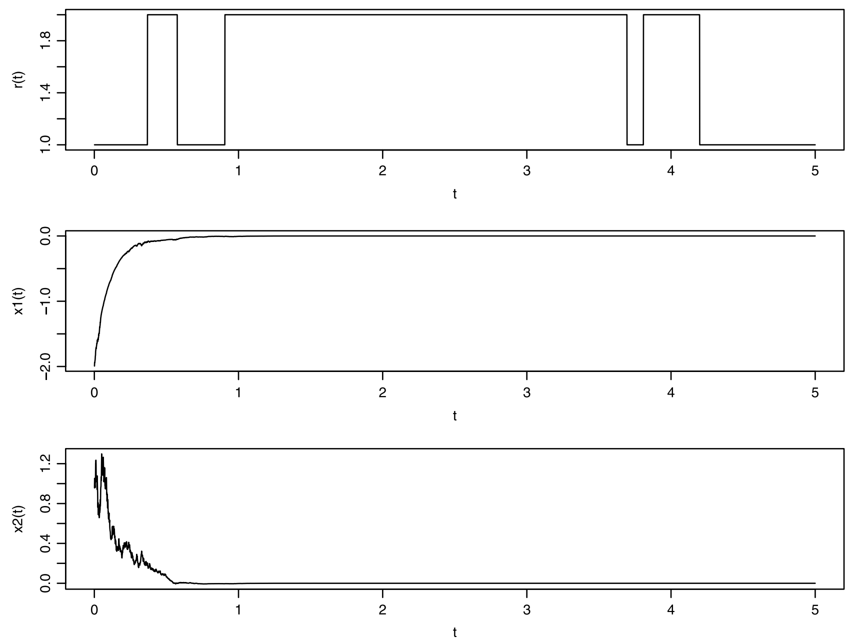

Fig. 5.2. Computer simulation of the paths of $r(t)$, $x_1(t)$ and $x_2(t)$ for the controlled hybrid SDE (38) with $\tau = 10^{-3}$ using the Euler–Maruyama method with step size 10^{-6} and initial values $r(0) = 1$, $x_1(0) = -2$ and $x_2(0) = 1$.

Here, f and g are both mappings from $R^n \times S \times R_+$ to R^n. This SDE may be regarded as a system which switches between two operation modes, say mode 1 and mode 2, and the switching obeys the law of the Markov chain, where in mode 1, the system evolves according to the SDE

$$dx(t) = f(x(t), 1, t)dt + g(x(t), 1, t)dw(t),$$

while in mode 2, according to the other SDE

$$dx(t) = f(x(t), 2, t)dt + g(x(t), 2, t)dw(t).$$

Assume that in mode 1, the state $x(t)$ can be observed at discrete times (intermittent time instants) but in mode 2, it is not observable. Therefore, we can design a feedback control based on discrete-time observations of the state in mode 1, but we cannot have a feedback control in mode 2. In terms of mathematics, the controlled SDE is

$$\begin{aligned} dx(t) = {} & [f(x(t), r(t), t) + F(r(t))G(r(t))x(\delta(t))]dt \\ & + g(x(t), r(t), t)dw(t), \end{aligned} \tag{40}$$

where $G_1 = I$, the $n \times n$ identity matrix but $G_2 = 0$. Given $G_2 = 0$ we can simply set $F_2 = 0$. Hence, the stabilization problem becomes: can we find a matrix $F_1 \in R^{n\times n}$ so that the controlled SDE (40) becomes exponentially stable in mean square?

To give a positive answer to the question, we assume that f and g obey Assumptions 4.2 and 4.5, respectively. To apply Corollary 4.6, we only need to look for the solutions $q_1, q_2 > 0$ and $Y_1 \in R^{n\times n}$ to the following LMIs

$$q_1 Z_1 + ZY_1 + Y_1^T Z - \gamma_{12} q_1 Z + \gamma_{12} q_2 Z < 0 \tag{41}$$

and

$$q_2 Z_2 + \gamma_{21} q_1 Z - \gamma_{21} q_2 Z < 0. \tag{42}$$

It is easy to see from (42) that we have to assume

$$Z_2 - \gamma_{21} Z < 0. \tag{43}$$

This means that the rate at which the system switches from the unobservable mode 2 to the observable mode 1 should be sufficiently large. This is reasonable because the system in mode 2 is not controllable while it is controllable (hence stabilizable) in mode 1. Let us now choose $q_1 = 1$. Under condition (43), we can further choose

$$q_2 > \frac{\gamma_{21}\lambda_{\max}(Z)}{\lambda_{\min}(\gamma_{21}Z - Z_2)} \tag{44}$$

for (42) to hold. Finally, we can choose Y_1 to be symmetric for

$$q_1 Z_1 + 2ZY_1 - \gamma_{12} q_1 Z + \gamma_{12} q_2 Z = -I, \tag{45}$$

where I is the $n \times n$ identity matrix. That is, we set

$$Y_1 = 0.5Z^{-1}(-I - q_1 Z_1 + \gamma_{12}(q_1 - q_2)Z), \tag{46}$$

which guarantees (41). Let us summarize what we have so far: under condition (43), we can choose $q_1 = 1$ and q_2 sufficiently large for (44) to hold and then compute Y_1 by (46) and set $F_1 = Y_1$.

To determine τ, we note that $\delta_3 = \delta_4 = \|F_1\|^2$. We then compute

$$-\gamma = \max_{i=1,2} \lambda_{\max}(U_i),$$

where

$$U_1 = -I, \; U_2 = q_2 Z_2 + \gamma_{21}(1 - q_2)Z.$$

Finally, make sure that $\tau > 0$ is sufficiently small for $2\gamma_\tau < \gamma$, where γ_τ can be computed by (27) and (30). Then, by Corollary 4.6, controlled system (40) is exponentially stable in mean square.

6. Conclusions and further comments

In this paper we first show that unstable linear hybrid SDEs can be stabilized by the linear feedback controls based on the discrete-time state observations. We then generalize the theory to a class of nonlinear hybrid SDEs. Making full use of their special features, we have established a better bound on τ and this is supported particularly by Example 5.1. Of course, the bound on τ obtained in this paper is certainly not optimal. It is a challenge to obtain the optimal bound, even in the linear case.

The theory established works well for linear hybrid SDEs or a class of nonlinear hybrid SDEs which satisfy Assumptions 4.1 and

4.2. These assumptions are somehow restrictive. It is useful and interesting to replace these by weaker conditions. Moreover, we assume in this paper that the mode $r(t)$ is available for all time although we only require the state $x(t)$ to be available at discrete times. This is the case, for example, when hybrid SDEs are used to model electric power systems [34] and the evasive target tracking problem [3]. On the other hand, one may consider the case when the mode $r(t)$ is available at discrete times while the state $x(t)$ is available for all time. However, due to the page limit here, we will report these results elsewhere.

Acknowledgments

The authors would like to thank the EPSRC (EP/E009409/1), the Royal Society of London (IE 131408), the Royal Society of Edinburgh (RKES115071), the London Mathematical Society (11219), the Edinburgh Mathematical Society (RKES130172), the National Natural Science Foundation of China (11471071, 61174077), the Natural Science Foundation of Shanghai (14ZR1401200) and the State Administration of Foreign Experts Affairs of China (MS2014DHDX020) for their financial support.

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