An investigation in vibration modeling and vibration-based monitoring for composite laminates

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Abstract

This paper introduces a novel methodology for health monitoring which is applied here for composite plates. In the present study we mainly concentrate on numerical results for validating the suggested methodology. The methodology suggested is principally based on a variation of singular spectrum analysis as applied in the frequency domain. For the purposes of modeling and simulation of the response of composite laminate plates we use finite element modeling and namely a specific Finite Element modeling (FEM) scheme, which we found to be most appropriate with regards to damage diagnosis. The results validate the methodology suggested for the case of composite plates and they demonstrate good agreement with the simulated results in terms of delamination detection and localization.

1   Introduction

The group of materials known as "composites" is extremely large and widely used in many engineering structures. Composite laminates, in particular, are made by stacking composite layers in a sequence depending on the final usage of the laminate. Delamination is the most common failure mode for laminates, and it can be due to impacts, fatigue loading, excessive strain and so on [1]. Delamination is a particularly dangerous failure mode, because it takes place and growths under the surface without being visible from the outside, causing matrix breakage and leading to changes in physical properties of the whole laminate such as reduction in stiffness and strength. This aspect suggested researchers to use physical properties of the laminate to monitor its health. Structural Health Monitoring (SHM) techniques must use non-destructive procedures, and should be easy to use, since sometimes the structure can be not easy to access and difficult to inspect. For these considerations, modal parameters (e.g. natural frequencies) are particularly suitable for the purpose, since they can be determined by a simple vibration test, which is totally non-destructive, they give global information of the structure, and can be precisely enough to give a sufficient amount of information.

Vibration based Structural health Monitoring (VSHM) techniques have already been investigated by some researchers. In [2], Frieden et al. investigated fiber reinforced polymer plates and the change of eigenfrequencies due to impact damage for different impact energies. They demonstrated that most of the frequency changes can be explained by a delamination type of damage, whereas the total delamination surface has an affine relation to the absorbed impact energy. A homogenized damage model, including two damage factors, allows predicting the change of natural frequencies for a known damage size. Wei et al. [3] presented a study on active detection of delamination for multi-layer composites using a combination of modal analysis

and wavelet transform by means of analysis on energy spectrum of wavelet packet decomposition even small delamination can be detected using the measured dynamic response signals. In a previous paper of some of the authors of this work [4] it was determined how the first six harmonics frequencies of a composite plate change due to the presence of delamination. Authors showed that harmonics can be successfully used to investigate the presence, the location and the dimensions of the delamination in a composite beam.

This study looks at a purely data driven methodology which is applied for the purposes of delamination detection and localization. In this case it is applied for composite plates and it is validated in here using simulated vibratory responses. We use finite element modeling to simulate the transient response of composite laminate plates and in this particular case we use a specific FEM scheme which we previously found to be the most suitable according to a number of criteria including the precision of the simulation. The simulated responses are further noise contaminated to simulate a real situation before the method is applied.

It should be noted that a number of research papers consider the problem for simulating the vibratory behavior of composite structures, especially beams and a few of them are focused on plates. A number of works can be found dealing with modeling of delaminated composite laminate structures [5, 6, 7, 8, 9, 10]. Even if some methods can relatively precisely model the presence of delamination, little research is available on the topic of delamination detection through dynamic modeling of laminate [11, 12, 13]. It should be noted that most of the model based and updating methodologies are difficult to apply for laminate structures especially because it is rather difficult to precisely model the vibratory behavior of laminates structures. Thus it should be kept in mind that the question for precise simulation of the dynamic behavior is still an open one and it still attracts a lot of research. From such view point most of the methods suggested for VSHM of laminate structures are primarily non-model based [14] . This study also suggests a completely non-model based methodology for damage assessment, which is applied here for the purposes of delamination detection and localization in composite plates. Some of the author of this work previously developed the methodology [15], which is here extended for laminate plates. It is based on principal component analysis, which is applied in the frequency domain. The frequency domain acceleration signals measured on a structure are first embedded in a matrix using a window length much smaller than their initial dimension. After that they are projected on a new space using the eigen-decomposition of the obtained embedding matrix. Only a certain number of principal components containing most of variance of the initial signal are then used to represent each vibration signal. This projection into a new space principally reduces the dimension of the initially measured signal-vectors but it also enhances the clusterization of the new variables from different categories vs signals from healthy and delaminated structures. The rest of the paper is organized as follows. Firstly the delamination assessment procedure is introduced as developed for the case of composite plates. Then the paper concentrates on the simulation of the vibratory behavior of composite plates and on the validation and especially the precision validation of the method used in here. The study goes on investigating a specific problem for delamination assessment in composite plates subjected to free vibration with delamination in different locations according to the thickness and the position within the plate. The results are then discussed and compared to the real/expected ones. The paper finishes with some conclusions regarding the methodology and this specific application.

2   Delamination Assessment Methodology

The methodology used here is based on Principal Component Analysis (PCA) which is a statistical procedure that uses an orthogonal transformation to convert a set of multivariate observations, in this particular case acceleration and strain signals, into a set of linearly correlated variables. However, PCA generally assumes that the data components are independent, but in the case of time series, the values are generally non-independent, and thus an extension of PCA called SSA provides a better alternative [14]. SSA is PCA applied to lagged versions of a single time series variable. MSSA is the natural extension of the SSA for multivariate systems. The method suggested here applies SSA in the frequency domain. It follows the following four steps [15]:

- Data collection: the variables measured, which are acceleration and strain signals in this particular application, are arranged into N- dimensional vectors $x = (x_1, x_2, ..., x_N)$. Due to the different magnitudes in the measurements the values are first standardized. The time domain vectors are then transformed into frequency domain signal-vectors with length N': $z = (z_1, z_2, ..., z'_N)$ with $N' = \frac{N}{2}$;

- Embedding: Given a window with length W ($1 < W \leq \frac{N}{2}$), the $W-$frequency-lagged vectors arranged in columns are used to define the trajectory matrix $\widetilde{Z}$. These vectors are padded with zeros to keep the same vector length. The embedding matrix $\widetilde{Z}$ is the representation of the system in a succession of overlapping vectors of the time series by W points.

- the Empirical Orthogonal Functions (EOFs), which represent the principal directions of the system, are calculated by the eigen-decomposition of $\frac{\widetilde{Z}'\widetilde{Z}}{N'}$, which is equivalent to a Singular Value Decomposition (SVD). The eigen-decomposition yields $k$eigenvalues and $k$eigenvectors which define orthonormal basis of the decomposition of $\widetilde{Z}$;

- Reconstruction: The EOFs represent the data as a decomposition of the orthogonal basis functions with a certain percentage of variance of the original signal corresponding to each EOF. The data in $\widetilde{Z}$ is projected onto the subspace $\mathcal{L}^k$ built by the EOFs and the corresponding PCs [17];

- Clustering: the oscillatory responses of the system can be reconstructed using a certain number of the new variables, which are for this reason called reconstructive components (RCs) Each of the RCs contains a certain percentage of variance of the initial variables provided by the EOFs. The projection of the original data onto the new RCs can be modeled as a single point projection [18]. The points into the new space created by the orthogonal basis (EOFs) give the coordinates of the projection of the original data Z onto the RCs. These coordinates characterize the vibratory responses of a point. The information contained within these projections was further utilised in this study to build features, which are consequently used for pattern recognition/classification purposes. The components containing more of the variance of the initial signal contain more information. Therefore, the first several components are expected to contain most of the information about the vibratory system as well as about any changes in it..

3 Modeling and testing the vibratory behavior of composite plates with delamination

The methodology described in §2 was applied for two study cases: first for composite laminated plates modeled in finite elements by ANSYS and secondly for real composite laminated plates. In both cases different delaminated scenarios were considered. In the first case four plates were modeled, one non-delaminated and three delaminated ones with different delaminations. In the second case, four composite laminated plates were manufactured following the same delamination configurations as in the FEM case. A comparison and validation of the numerical results was performed using the experimental results.

3.1 Finite element model

Non-delaminated and delaminated composite laminate plates were modeled by using SOLID186 elements from ANSYS library. Cantilever square plates with 150 mm side and 8 woven layers (1.6 mm total thickness), are used. The following material characteristics were used: $E_1 = E_2 = 59\ GPa$, $E_3 = 9\ GPa$, $G_{12} = G_{13} = G_{23} = 7.17\ GPa$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$ and $\rho = 1500\ kg/m^3$. Four plates were modeled, named: (H)-Healthy (non-delaminated), $(D1)$-Delamination 1, $(D2)$-Delamination 2, $(D3)$-Delamination 3 (see Figure 1). The delamination size was 40 mm x 40 mm.

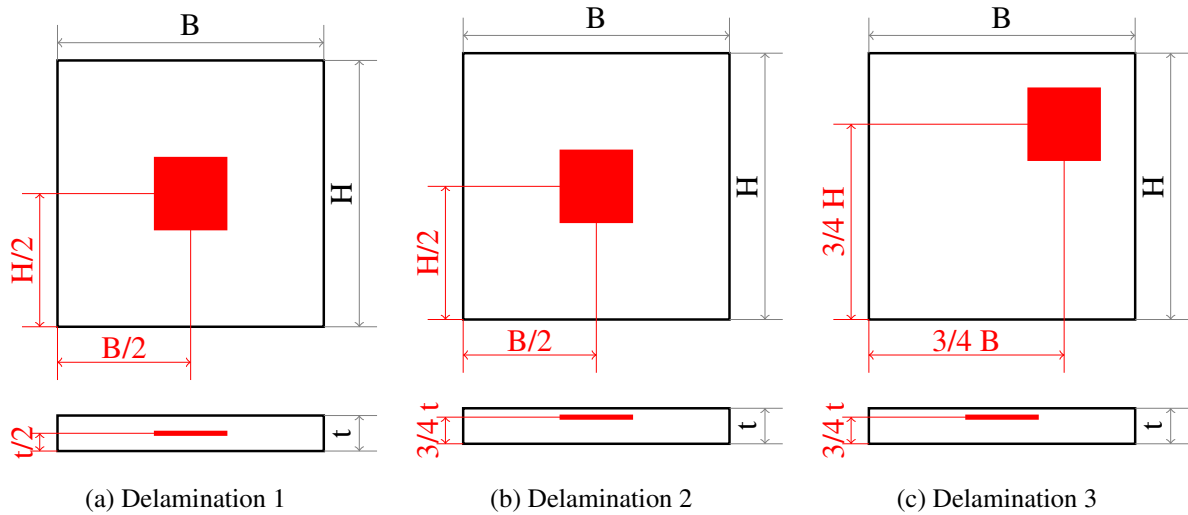(a) Delamination 1   (b) Delamination 2   (c) Delamination 3

Figure 1: Delamination configurations

TARGE170 and CONTA174, have been used as elements for upper and lower (target) surfaces respectively. Transient analysis has been performed to extract the first 5 harmonic frequencies of the structures by applying a 1 N impulse force. Analysis was performed for 1 s and sampled at 2049 HZ.

Vertical displacement was extracted form a certain point of the plate and used to determine the harmonic frequencies of the specimen.

3.2 Experiments

Testing specimens with the same geometry and the same material characteristics as those used for the simulations were used to validate the numerical model. Delamination was introduced by laying down a 15 μm thick Teflon ply of the proper size and in the proper interface during lay up (see Figure 2a); the specimens were cantilevered on one edge (see Figure 2b) and hit by a steel hammer while two strain gauges applied on the upper surface of the plates (see Figure 2c) were used to record the strain.

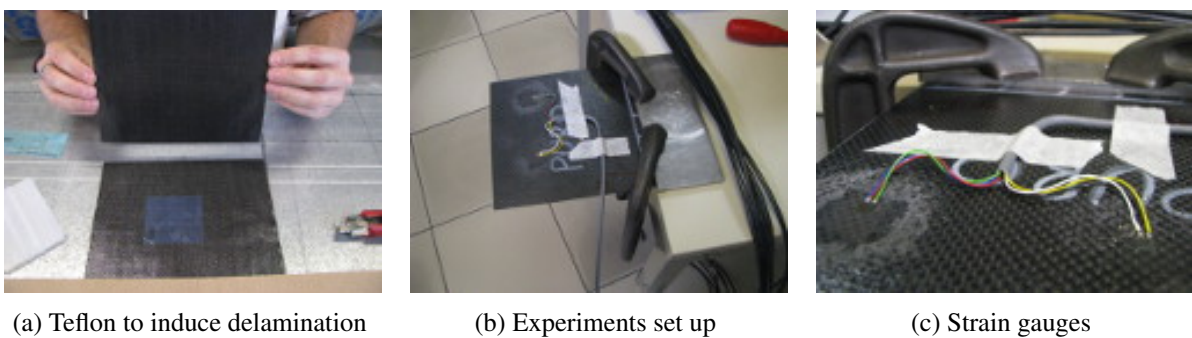(a) Teflon to induce delamination   (b) Experiments set up   (c) Strain gauges

Figure 2: Experimental set-up

The measurement chain consisted of a strain gauge powered by a P-3550 Strain Indicator; the signal was then digitalized by an analogue-to-digital acquisition device. The signal was acquired on a personal computer by a Labview executable program. A sampling frequency of 1.25 kHz and 1.638 s of acquisition time were chosen. The experiments were performed five times per specimen and the results are given in terms of the average of all the tests for each configuration.

3.3 FEM validation

The discrete Fourier Transform was used to extract the first five harmonics from the strain and the displacement records during the experiments and simulation respectively. Results for the undamaged and the three delaminated configurations are presented in Table 1.

	Non-damaged			Delamination 1		
Mode	Numerical	Experiments		Numerical	Experiments	
		μ	$\sigma/\mu\%$		μ	$\sigma/\mu\%$
1	74.06	73.59	0.32	74.02	75.89	0.16
2	442.00	450.73	4.04	442.01	468.70	2.26
3	521.98	540.35	5.97	522.26	531.36	2.19
4	853.25	872.36	2.11	853.93	857.91	4.04
5	1206.60	1307.70	6.10	1175.40	1309.98	4.99
	Delamination 2			Delamination 3		
Mode	Numerical	Experiments		Numerical	Experiments	
		μ	$\sigma/\mu\%$		μ	$\sigma/\mu\%$
1	74.03	74.74	0.41	74.13	75.47	0.26
2	441.83	459.22	5.71	439.81	465.98	1.92
3	521.95	528.25	2.94	517.10	549.63	4.16
4	852.08	876.40	1.57	850.54	878.87	0.82
5	1190.20	1314.39	1.10	1203.70	1332.27	1.08

Table 1: Experimental and numerical results. μ: mean value - σ: standard deviation

In Figure 3 the harmonics of undamaged and each delaminated configuration are plotted, showing very good agreement between the numerical and the experimental results. The results show that the model proposed is ac- curate enough to simulate the dynamic behavior of a composite laminate plate, with and without delamination

4 Delamination assessment

The finite elements simulations presented in §3.1 and the experiments shown in §3.2 were used to validate the methodology introduced in §2.

4.1 Application of MSSA to signal extracted from numerical models

During the simulated free-decay responses of each specimen, the acceleration of one node was recorded. The signals were recorded for $1\ s$ and sampled at $2049\ Hz$. Ten realizations for each signal were obtained by adding $20dB$ white noise to the signals extracted by the transient FEM analysis (see §3.1.

The signals were then processed by the methodology in §2 with a window length of $W = 11$ and the results are presented in Figure 4.

The percentage of the variation of the decomposition of the signal is shown in Figure 4c. It can be observed that the first component has the maximum variance up to 80% contained, while the second one has up to 12%, hence the maximum of the variance is contained in the first two components within 92% by the sum of the two.

Figure 4a and 4b present the reconstructed signals by using two and four components respectively. The reconstruction by four components is slightly more accurate than with two components. However, the clustering effect presented by the projection of the original signals onto the two first components is precise enough to

(a) Non delaminated

(b) Delamination 1

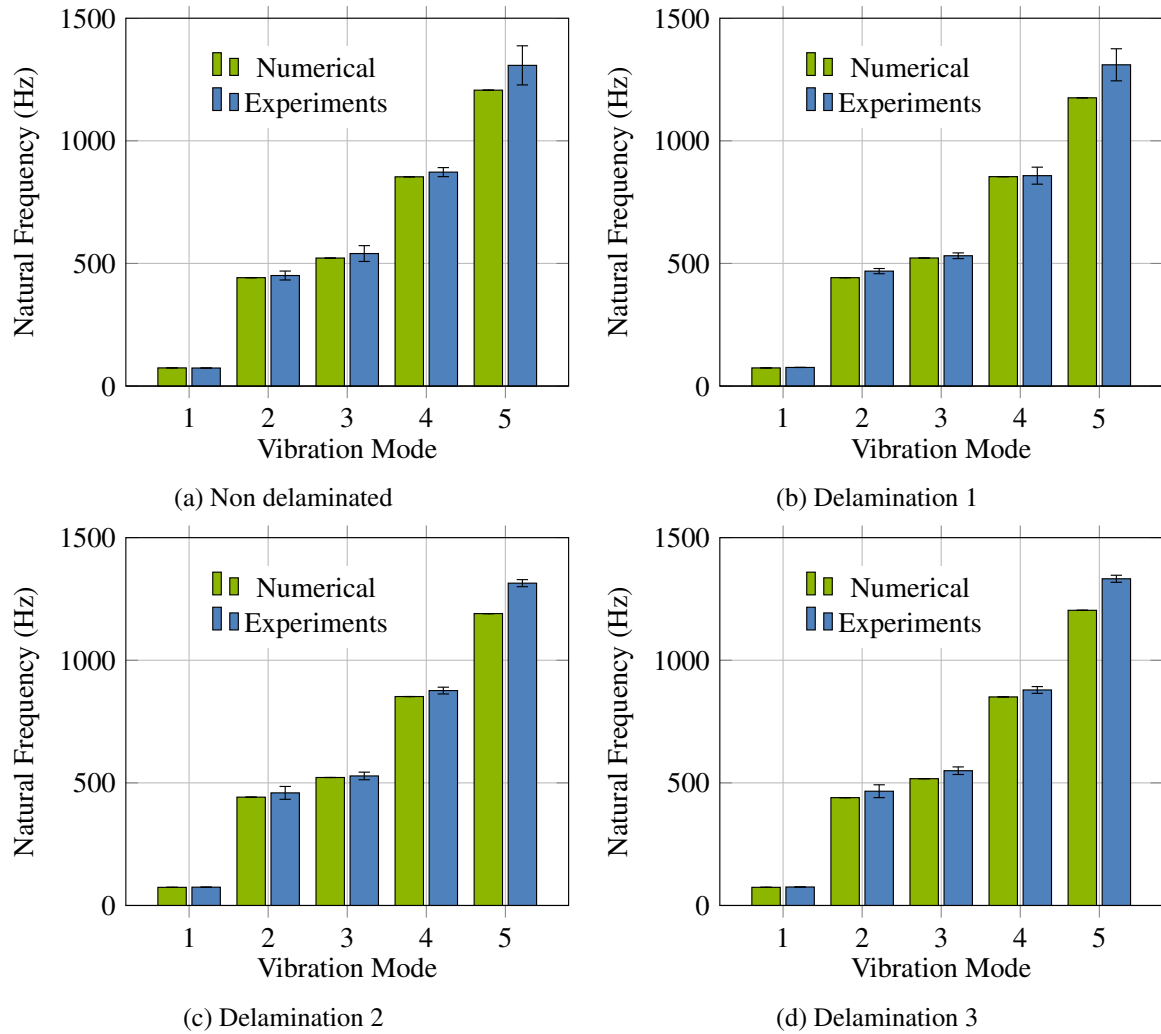(c) Delamination 2

(d) Delamination 3

Figure 3: Numerical and experimental results

identify the different plate configurations. The delamination introduced in the plates are detected and represented with a clear classification, shown in Figure 4d. The four different cluster groups, corresponding to each plate scenario, are clearly differenced.

4.2 Application of MSSA to signal extracted from experiments

The strain values from the free-decay responses of each specimen were recorded by each instant of time. Ten signal realizations were obtained for each of the plate scenarios following the experiment process detailed in §3.2.

The signals were processed by the methodology in §2 with a window length of $W = 11$ and the results obtained are presented in Figure 5.

The percentage of the variation of the decomposition of the signal is shown in Figure 5c. It can be observed that the first component has the maximum of the variance up to 72% contained and the second one up to 12%, hence the maximum of the variance is contained in the first two components within 84% by the sum of the two.

In Figure 5a and 5b the reconstructed signals by using two and four components respectively are presented. The reconstruction by four components is slightly more accurate than with two components; the reconstruction matches within the slope in higher frequencies. The clustering effect presented by the projection of the
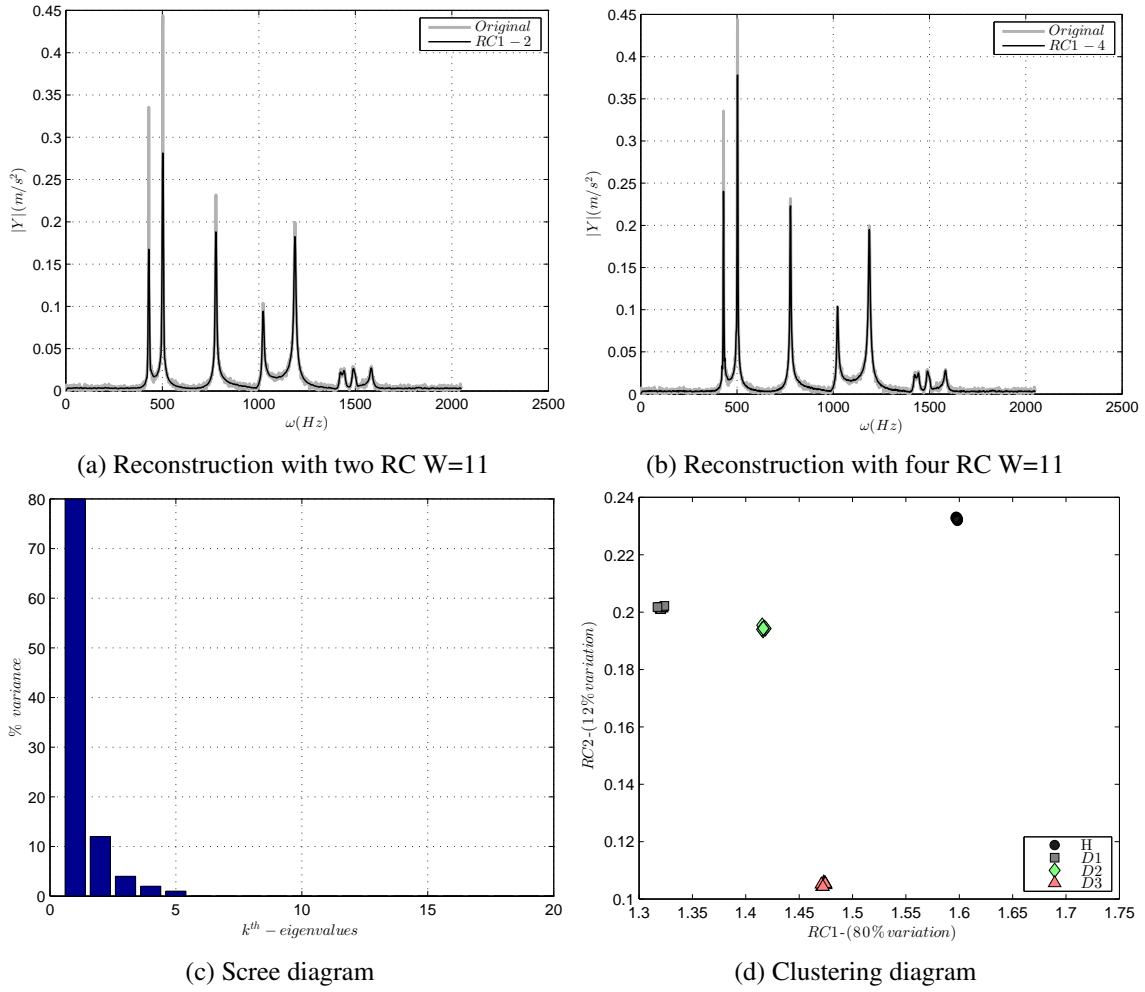
(a) Reconstruction with two RC W=11

(b) Reconstruction with four RC W=11

(c) Scree diagram

(d) Clustering diagram

Figure 4: Results of the methodolgy in the numerial analysis. The resolution of the signal was $1\ s$ sampled at $2049\ Hz$. Window length $W = 11$

original signals onto the two first components is able to identify the delamination scenarios. The delamination introduced in the plates are clearly distinguishable by using only the first two RCs as shown in Figure 5d.

4.3 Comparison between FEM and experiments

In both the cases the maximum of the variance and hence the most relevant information about the signal responses are contained in the first two components. Therefore, the reconstruction of the original signal by two components presents a well-defined approximation of the original signal. By using more components, the signal reconstruction has better approximation.

The clustering effects are clear in both cases, however some of the experimental signals can be considered misclassified. Nevertheless, the clustering effect presented by using only two reconstructed components is clearly achieved. In the FEM case, the clustering effect is much stronger.
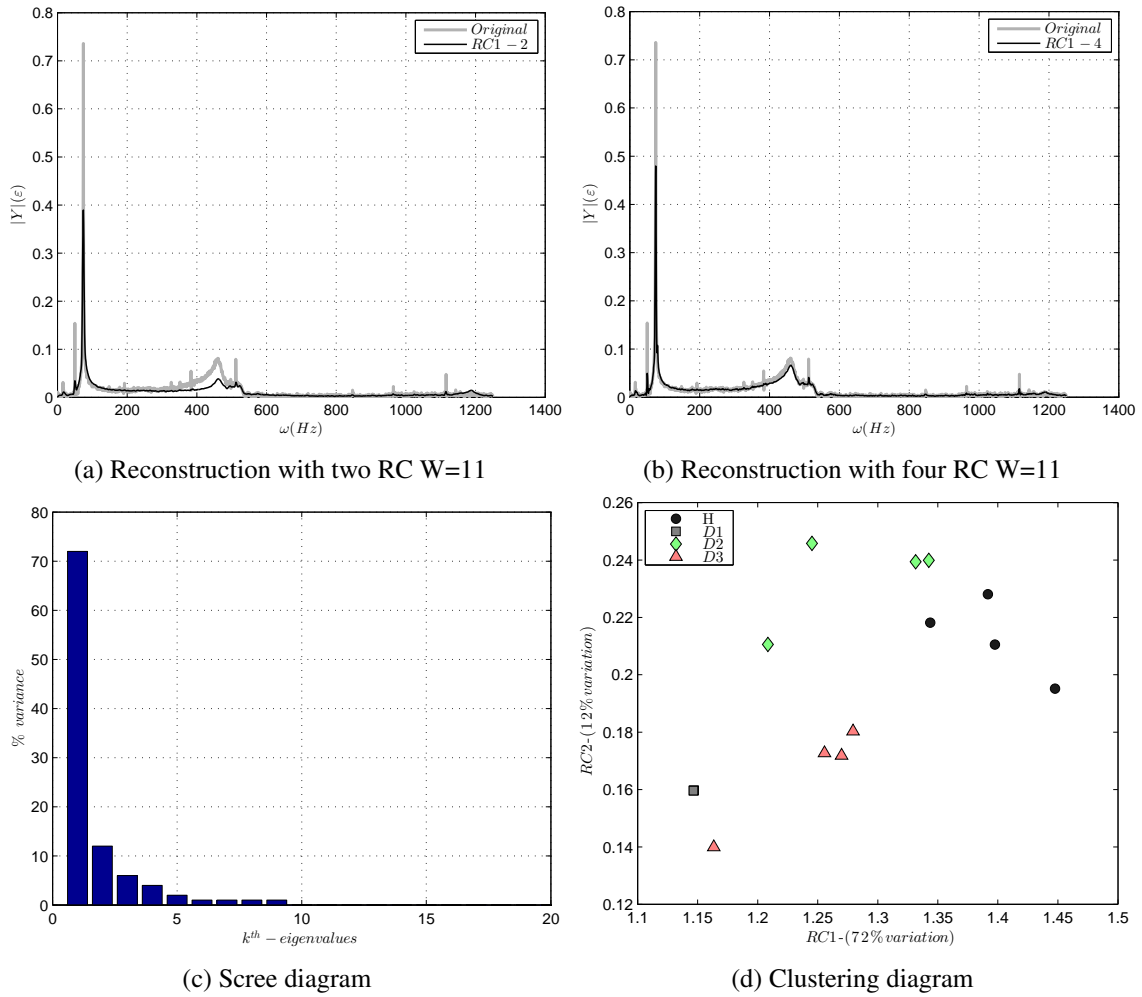
(a) Reconstruction with two RC W=11

(b) Reconstruction with four RC W=11

(c) Scree diagram

(d) Clustering diagram

Figure 5: Results of the methodolgy in the real composite laminated plates. The resolution of the signal was $1.638\ s$ sampled at $= 1250\ Hz$. Window length $W = 11$

5 Conclusions

This paper introduces a novel methodology for delamination assessment in composite laminated plates. The methodology presented utilizes the Multichannel Singular Spectrum Analysis as multivariate decomposition tool of the measured vibration signals. The suggested methodology was first validated using a FE modeling of the composite laminated plates and secondly using experimentally tested composite laminated plates. The results showed a very good agreement and the potential of the methodology is presented. The compression of the data in a certain number of principal components concentrates the information of the small variations of the vibratory responses. Two of these components were used to reconstruct an approximation of the original signal. The projection of the original signal onto the reconstructed ones was used to develop features for pattern recognition aiming at delamination classification. The detection and classification among different composite plate configurations were demonstrated by the clustering effect of the methodology which showed that the delamination detection was clearly achieved.

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