

Singular Spectrum Analysis for identifying structural nonlinearity using free-decay responses. Application for delamination detection and diagnosis in composite laminates.

D. Garcia¹, I. Trendafilova¹

¹ University of Strathclyde, Department of Mechanical and Aerospace Engineering
75 Montrose street, Glasgow G1 1XJ, United Kingdom
e-mail: david.garcia@strath.ac.uk

Abstract

This investigation considers a methodology for analysis of the vibratory response of composite laminate structures which is based on Singular Spectrum Analysis. Composite laminate structures generally demonstrate nonlinear dynamic behaviour as a result of their intrinsic material nonlinear nature. Since the nonlinearities on a number of occasions induce relatively small changes in the vibratory response which are difficult to identify, the raw measured dynamic responses have to be subjected to certain pre-treatment before it can be used for purposes of nonlinearity and damage analysis. To approach this problem this work investigates the effect of some key signal and transformation parameters, such as signal length and sampling frequency as well as the SSA window length on the performance of the methodology itself. The selection of these parameters has a direct influence on the damage sensitivity and the accuracy of the methodology. The variation of these parameters can produce radical changes on the clustering effect of the methodology and it is demonstrated that this might affect the results interpretation.

1 Introduction

The growth of the engineering challenges in different industry sectors requires the development of proper Structural Health Monitoring (SHM) methodologies to inspect the integrity and the health of structures. The rapid developments in this area are due to the real engineering problems related to safe design, manufacturing, maintenance and safe operation of the technical infrastructure. A similar way, introduction of new materials for engineering infrastructures gives an additional importance to SHM. In particular the use of composite materials is growing as they are continuously replacing traditional ones. Composites are generally implemented in structures with high performance requirements and their nature depends on the material composition. The interaction between their components makes their analysis still more difficult. The different properties of their components generate interface failures that often result delamination. Delamination in composite structures affects adversely the system's performance while in the same time they can lose up to 60% of their stiffness and still remain visibly unchanged. Accordingly the development of proper SHM methodologies becomes a must for such materials and structures.

Vibration based SHM (VSHM) methods are one of the most widely used methods as all structures and machinery vibrate. There are different VSHM methodologies but these can be generally divided in two main categories, model based and non-model based ones. In the first category, the methods use a certain model of the structure's dynamic in order to assess its health and integrity state. On the contrary non-model based

methods do not assume any model or linearity and they are purely based on the measured structural vibratory response. The methods developed and being developed for composite structures are primarily non-model based one due to the fact such materials are difficult to model precisely. In practice most of the purely data-driven methodologies make use of data analysis and utilize different statistical methods and characteristics to extract features which characterize the health and integrity of a structure [1]. In this paper a method based on Singular Spectrum Analysis (SSA) is introduced. SSA is a technique applied for time series analysis which incorporates multivariate analysis, dynamical systems analysis and signal processing [2]. SSA uses Principal Component Analysis [3, 4] as a technique for the analysis of auto-correlated non-independent time series [?]. SSA is applied for diverse applications ranging mathematics and physics [6] to weather forecasting [7], financial mathematics [8], market research and social sciences [9]. The aim of SSA is to decompose the original signal using a small number of independent and much more interpretable components which can be used for trend identification, detection of oscillatory components, periodicity extraction, signal smoothing, noise reduction and detection of structural changes in time series [2].

SSA is applied in this study to develop a methodology for damage and delamination assessment in composite laminate structures. The developed methodology is able to detect structure changes in the vibratory response of the composite laminated structures. In general, damage and delamination introduce small differences in the vibratory response of the structure which are difficult to detect and process for the purposes of damage assessment. Differently from traditional spectrum analysis, SSA is able to uncover rotational periodicities at any frequency. Thus, in a certain sense it can be applied for the purpose of modal analysis for non-linearly vibrating structures. In this paper, a method based on SSA decomposition was developed to address the problem for delamination assessment in composite structures [10]. SSA is highly sensitive to changes in a dynamical system therefore it can be used as a powerful tool for nonlinearities detection including damage and/or delamination. However, this sensitivity is highly dependant on the choice of the signal parameters. Significant changes in the time series structure can be detected for any reasonable choice of parameters [11]. To detect small changes in noisy series careful tuning of some of the key parameters is required. To approach this problem this work investigates the effect of some key signal and transformation parameters, such as signal length and sampling interval as well as the window length on the precision and the performance of the methodology. The proper selection of these parameters is demonstrated to have direct influence on the sensitivity to damage and the accuracy of the methodology. The variation of these parameters can produce radical changes on the clustering effect of the methodology and it might affect the results interpretation. This study approaches the above mentioned problems for two cases: a simple nonlinear 2-DoF spring-damper-mass system and for a real experiment performed for composite laminate beams.

The paper is organized as follows.

At §2, the delamination assessment methodology is detailed. Firstly a short description of the steps followed by the suggested SSA based in the methodology is described and secondly its application for damage assessment is introduced.

§3 is devoted to the study of some signal and another parameters and how they can affect the results and the performance of the methodology. The methodology is applied for the 2-DoF system and the effect of the parameters in question is then analyzed.

At §4, the methodology is applied for the data measured from the experimental test. Five composite laminated beams were manufactured. One of the beams is considered as a Healthy beam (non-delaminated), and the other four beams are divided within four different delaminated scenarios. Subsequently the effect of the chosen key parameters is analyzed.

2 Damage and delamination assessment methodology

Multiple realisations of the data recorded (accelerations) were arranged into vectors $\mathbf{x}^i = (\mathbf{x}_1^i, \mathbf{x}_2^i, \dots, \mathbf{x}_j^i, \dots, \mathbf{x}_N^i)'$ where $i = 1, 2, \dots, M$ is the number of realisations and $j = 1, 2, \dots, N$ is the number of components in each signal.

Each signal was transformed into the frequency domain. In this way, the spectral data matrix $\mathbf{Z} = (\mathbf{z}^1, \mathbf{z}^2, \dots, \mathbf{z}^i, \dots, \mathbf{z}^M)$ was obtained with all vectors arranged in columns.

The next step is to embed the vibratory responses.

Given a window with length W ($1 < W \leq \frac{N}{2}$), the W -frequency-lagged vectors arranged in columns are used to define the trajectory matrix. These vectors are padded with zeros to keep the same vector length. The embedding matrix $\tilde{\mathbf{Z}}$ is the representation of the system in a succession of overlapping vectors of the time series by W points.

At the next step, the covariance matrix of the matrix $\tilde{\mathbf{Z}}$ was obtained following the Equation 1 below, where $N' = \frac{N}{2}$.

$$\mathbf{C}_Z = \frac{\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}}{N'} \quad (1)$$

The eigenvalues λ_k and the eigenvectors ρ_k of \mathbf{C}_Z were obtained according to the following expression.

$$\mathbf{C}_Z \rho_k = \lambda_k \rho_k \quad (2)$$

The eigenvalues λ_k were then arranged in the diagonal matrix $\mathbf{\Lambda}_Z$ in decreasing order and the matrix \mathbf{E}_Z contains their corresponding eigenvectors ρ_k written as columns. The \mathbf{E}_Z vectors are called Empirical Orthogonal Functions (EOFs) and they contain the data as a decomposition into orthogonal basis. The eigenvalues define the partial variance of each eigenvectors, therefore the total sum of all of these variances gives the total variance of $\tilde{\mathbf{Z}}$.

$$\mathbf{E}_Z' \mathbf{C}_Z \mathbf{E}_Z = \mathbf{\Lambda}_Z \quad (3)$$

The projection of the measured data $\tilde{\mathbf{Z}}$ onto the matrix \mathbf{E}_Z yields the corresponding Principal Components (PCs) matrix $\mathbf{A} = \tilde{\mathbf{Z}}\mathbf{E}_Z$.

A matrix which contains the projection of the PCs onto the new space was created to reconstruct the signal. The Reconstructed Components (RCs) were obtained according to Equation (4). For a given set of indices \mathcal{K} corresponding to a set of PCs, the RCs were obtained by projecting the corresponding PCs onto the EOFs.

$$R_{m,n}^k = \frac{1}{W} \sum_{w=1}^W A_{n-w}^k E_{m,w}^k \quad (4)$$

where k -eigenvectors give the k^{th} RC at n -frequency between $n = 1 \dots N'$ for each m -channel ($m = 1 \dots M$) which was embedded in w -lagged vectors ($w = 1, 2, \dots, W$).

The oscillatory responses of the system were decomposed using a certain number of RCs. Each of RCs contain a certain percentage of the variance provided by the EOFs. The projection of the original data onto the RCs can be modelled as a single point. The points into the new space created by the orthogonal basis (EOFs) defines the coordinates of the projection of the original data \mathbf{Z} onto the RCs. These coordinates represent the vibratory responses as a point. The information contained within these projections was further utilised as pattern recognition features for classification purposes.

The components containing more of the variance of the initial signal contain more information about the signal itself. Therefore, these RC's are expected to contain most of the information about the vibratory system and the changes in it.

3 Some parameters and their effect on the method

3.1 The 2 – DoF system

As was mentioned, the above methodology is first applied to 2-DoF system defined by the Equation 5.

$$[\mathbf{M}]\ddot{\mathbf{x}} + [\mathbf{C}]\dot{\mathbf{x}} + [\mathbf{K}]\mathbf{x} + \mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}) = 0 \quad (5)$$

where $[\mathbf{M}]$, $[\mathbf{C}]$, $[\mathbf{K}]$ are constant coefficients mass, damping and stiffness matrices respectively defined by the Equation (6). The function $\mathbf{f}(\dot{\mathbf{x}}, \mathbf{x})$ provides a quadratic coupling between masses and it is defined by Equation (7).

$$[\mathbf{M}] = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad [\mathbf{C}] = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{pmatrix} \quad [\mathbf{K}] = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{pmatrix} \quad (6)$$

$$\mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}) = \begin{Bmatrix} -k_n(x_2 + x_1)^2 \\ k_n(x_2 + x_1)^2 \end{Bmatrix} \quad (7)$$

The following initial conditions and parameter values used $x_0^{(1)} = x_0^{(2)} = 0$, $\dot{x}_0^{(1)} = 0$, $\dot{x}_0^{(2)} = 1$ m/s^2 , $k_1 = k_2 = k_3 = 2000$ N/m , $k_n = 10000$ N/m , $c_1 = c_2 = c_3 = 6$ Nm/s and $m_1 = m_2 = 5$ kg . These values correspond to the baseline healthy state of the system. An initial velocity was applied to m_2 to simulate an impulse. The system describes a free-decay response and the acceleration was recorded by each instant of time to generate the data to introduce in the methodology.

Damage was introduced in the 2-DoF system by introducing different levels of stiffness reduction which simulate different damage levels. Reduction of 10%, 20% and 30% are applied to the nonlinear stiffness k_n . Multiple realizations were generated by the introduction of white noise at 20dB to the obtained accelerations.

The system is used as a model to analyze the effect of the parameters in the accuracy of the method described in §2. The accuracy of the method is judged by the detection of the stiffness reduction in k_n . In the next two sections the effect of signal length, the sampling frequency interval (Δf) and the window length (W) are studied and analyzed.

3.2 Effect of the sampling frequency and the signal length

The following paragraphs discuss the principal considerations for selecting the sampling frequency interval Δf and the signal length for a successful delamination detection.

In order to study the effect on the delamination detection caused by these parameters, some values must be fixed. The window length was fixed at $W = 7$ and the maximum frequency range was fixed at 400 Hz .

Observe that increasing the signal length by decreasing the sampling frequency interval Δf gives additional information to the singular values which are able to detect more variations within the embedded signals or within the window dimension. This can be shown in the comparison between Figure 2 and Figure 3. It can be observed that with the reduction of Δf , the resolution of the signal in frequency domain improves and the variations are better detected. However, an excessive reduction of Δf does not vary significantly the results of the clustering effect, they can even affect the results adversely (see Figure 1(c)). From these observations, it can be judged that the resolution of the signal has an influence on the results obtained by the methodology described in §2.

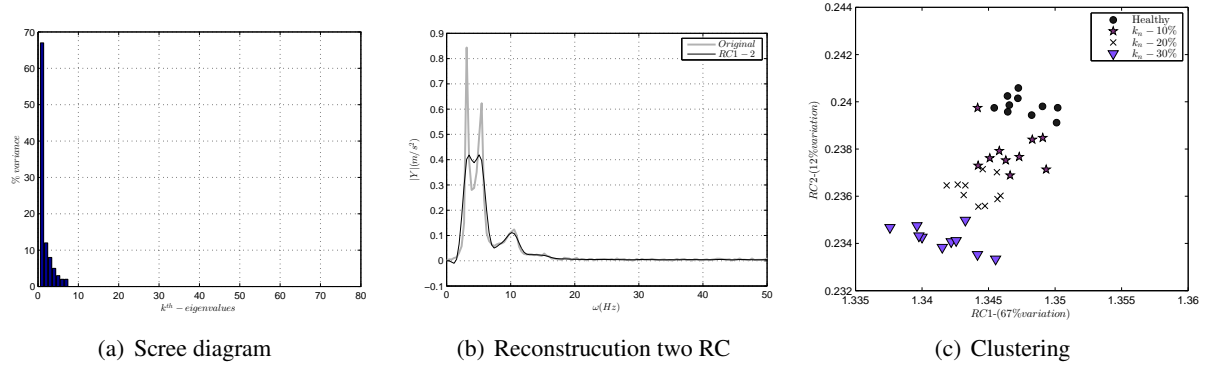


Figure 1: For a fixed $W = 7$ the effects on the variation of sampling frequency. The resolution of the signal was recorded 2.56 s long and sampled at 400 Hz. For the visualization issue, the ω scale was reduced until 50 Hz.

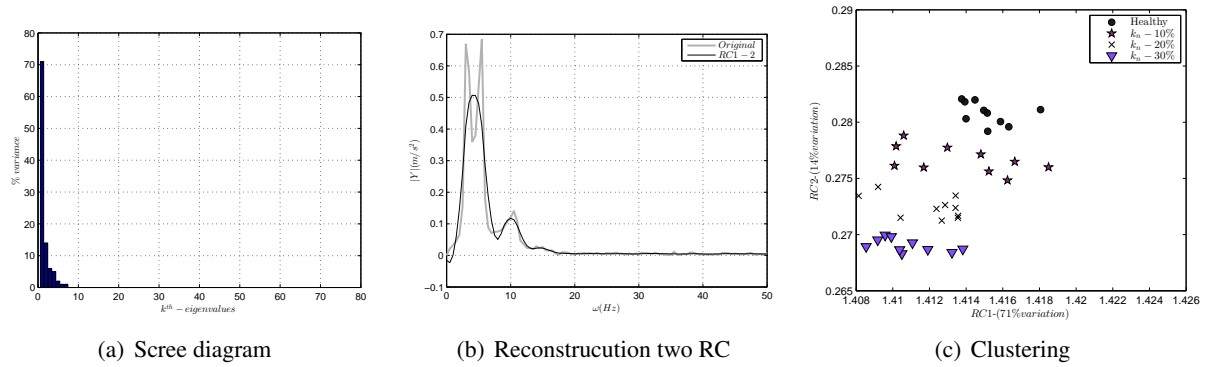


Figure 2: For a fixed $W = 7$ the effects on the variation of sampling frequency. The resolution of the signal was recorded 2 s long and sampled at 400 Hz. For the visualization issue, the ω scale was reduced until 50 Hz.

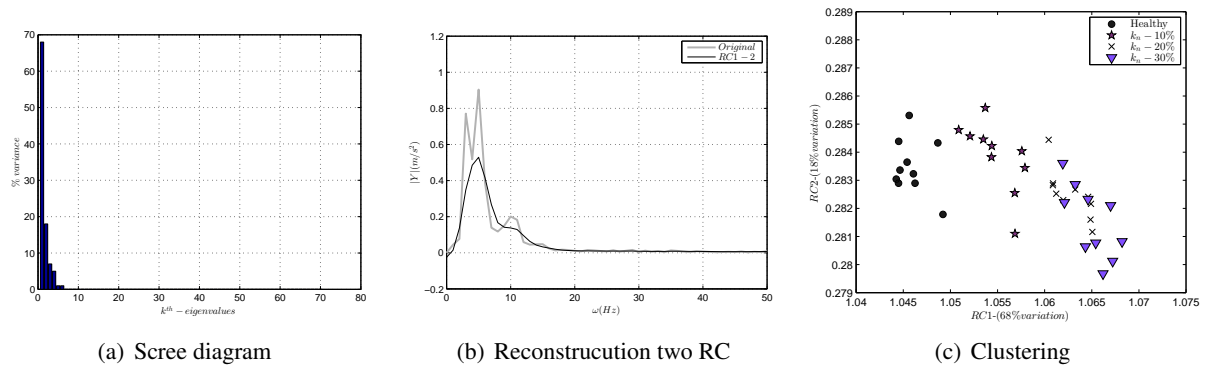
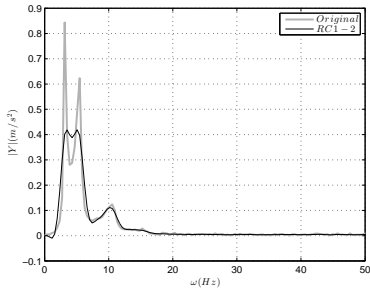


Figure 3: For a fixed $W = 7$ the effects on the variation of sampling frequency. The resolution of the signal was recorded 1 s long and sampled at 400 Hz. For the visualization issue, the ω scale was reduced until 50 Hz.

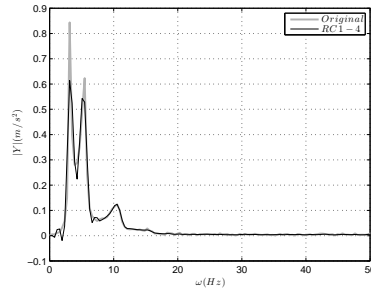
3.3 Effect of the window length W

The following paragraphs discuss the principal considerations for selecting the window length W for a successful delamination detection.

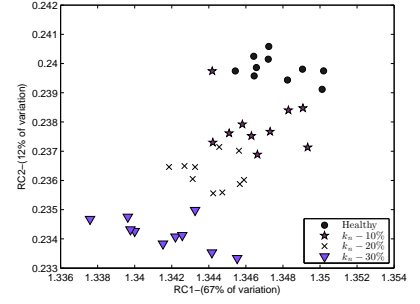
The window length has a particular significance on the singular spectrum analysis and eventually in the form of the reconstructed signal [6]. The selection of the proper window length depends on the problem at hand



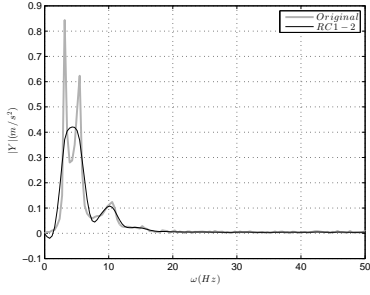
(a) Reconstruction two RC $W = 7$



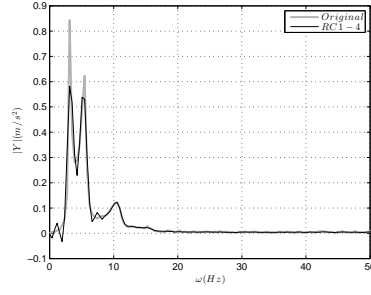
(b) Reconstruction four RC $W = 7$



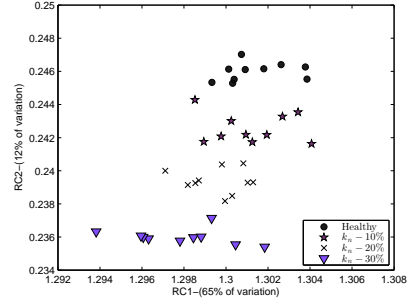
(c) Clustering $W = 7$



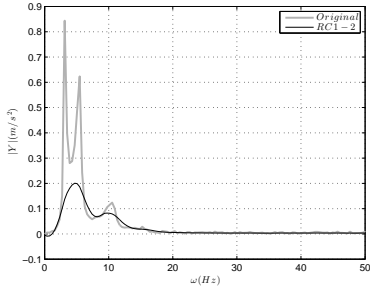
(d) Reconstruction two RC $W = 8$



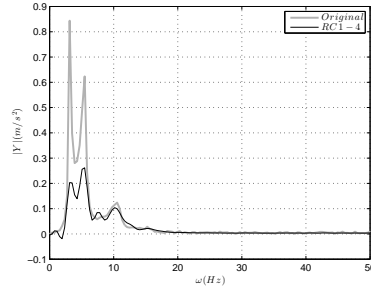
(e) Reconstruction four RC $W = 8$



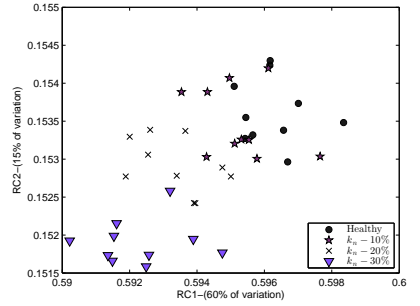
(f) Clustering $W = 8$



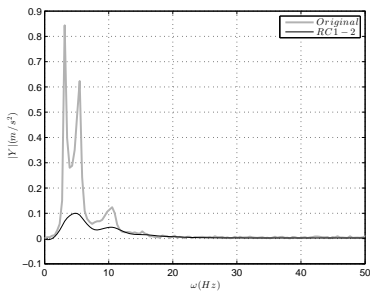
(g) Reconstruction two RC $W = 25$



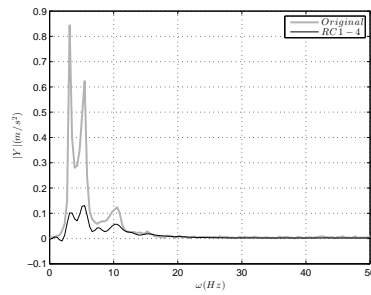
(h) Reconstruction four RC $W = 25$



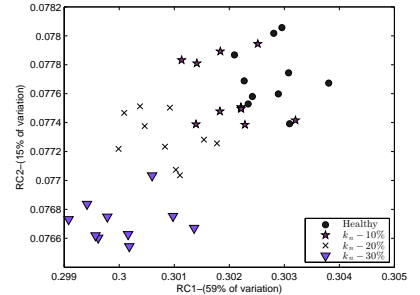
(i) Clustering $W = 25$



(j) Reconstruction two RC $W = 50$



(k) Reconstruction four RC $W = 50$



(l) Clustering $W = 50$

Figure 4: Effects on the variation of W for $W = 7$, $W = 8$, $W = 25$ and $W = 50$ in the analytical model. The resolution of the signal was fixed at $2.56 s$ long and sampled at $400 Hz$. For the visualization issue, the ω scale was reduced until $50 Hz$.

and on the preliminary information about the time series [12]. The main principle for selecting a proper window length is to find the value which produces separable and independent principal components. This is important for the proper reconstruction of the signal. It is more beneficial to use the minimum possible number of principal components for the signal reconstruction.

A general aspect for the selection of W is that longer window length will provide more detailed decomposition. According to this statement, the best detailed decomposition is obtained for $W \simeq \frac{N}{2}$. Despite the previous statement, a large window length will require more components but it can in the same time introduce more noise in the reconstruction [13, 14]. In this case, it is worth testing different window lengths starting from large values of W until the proper effect is achieved.

In order to study the effect on the delamination detection caused by the window length, the signals were recorded for 2.56 s and sampled at 400 Hz.

As it is shown in Figure 4, large window length results in a smoother the signal and distributes the information about the dynamical system over more principal components. Then, the first two principal components does not provide a good reconstruction (see Figure 4(g) and Figure 4(j)). The delamination assessment is based on the data projection onto the first two principal components as it is described in §2. In the Figure 4 can be observed that the variation of W alters the results. For instance, in this particular case, the reconstruction which uses four principal components provides a very good reconstruction of the original signal (see Figure 4(b) and Figure 4(e)).

Large values of W does not provide good results in the clustering effect as it is shown in Figure 4(l). Therefore, for better classification, more principal components should be used for the reconstruction.

4 Experimental case study

Five composite laminated beams were manufactured. The specifications of the beams are 10-layered carbon woven laminate multipreg *E722* resin with the following dimensions: 980 x 42 x 2.5 mm. In four of the beams, delamination was introduced by the inclusion of a Teflon sheet. The non-adherent property of the Teflon provides a controlled region where the interlayer adhesion does not occur. The five beams are defined as: *B1*–Non-delaminated beam (Healthy), *B2*–Delamination in the middle lengthwise between 5th – 6th layer and 50mm length, *B3*–Delamination in the middle lengthwise between 5th – 6th layer and 80mm length, *B4*–Delamination on the left side (at 220mm from the edge) between 5th – 6th layer and 50mm length and *B5*–Delamination on the left side (at 220mm from the edge) between 2th – 3th layer and 50mm length.

The beams were fully-fixed at both ends with a free length between the supports of 900mm. The acceleration for the case of free-decay responses was recorded for the specimens.

According to the conclusions from §3.2 a reduced Δf and long signals were selected to increase the resolution of the signal. The frequency length of the signal was large enough to involve the first five eigenfrequencies of the beam. Therefore, the resolution parameters chosen to record the free-decay responses were 1.6 s and sampled at 640 Hz.

The selection of the window length W required more detailed analysis. To find the right parameter of W the conclusions in §3.3 were followed. Large W does not give a good reconstruction of the signal. An example of the different cases is given in Figure 5.

The analysis was based on the conclusions described in previous sections and therefore the best window length for this particular case was $W = 7$, as it can be demonstrated in Figure 6. The results observed clearly demonstrate the potential of the methodology for delamination detection and localization. As a result of this analysis a distinguishable and well-separated clustering effect is obtained.

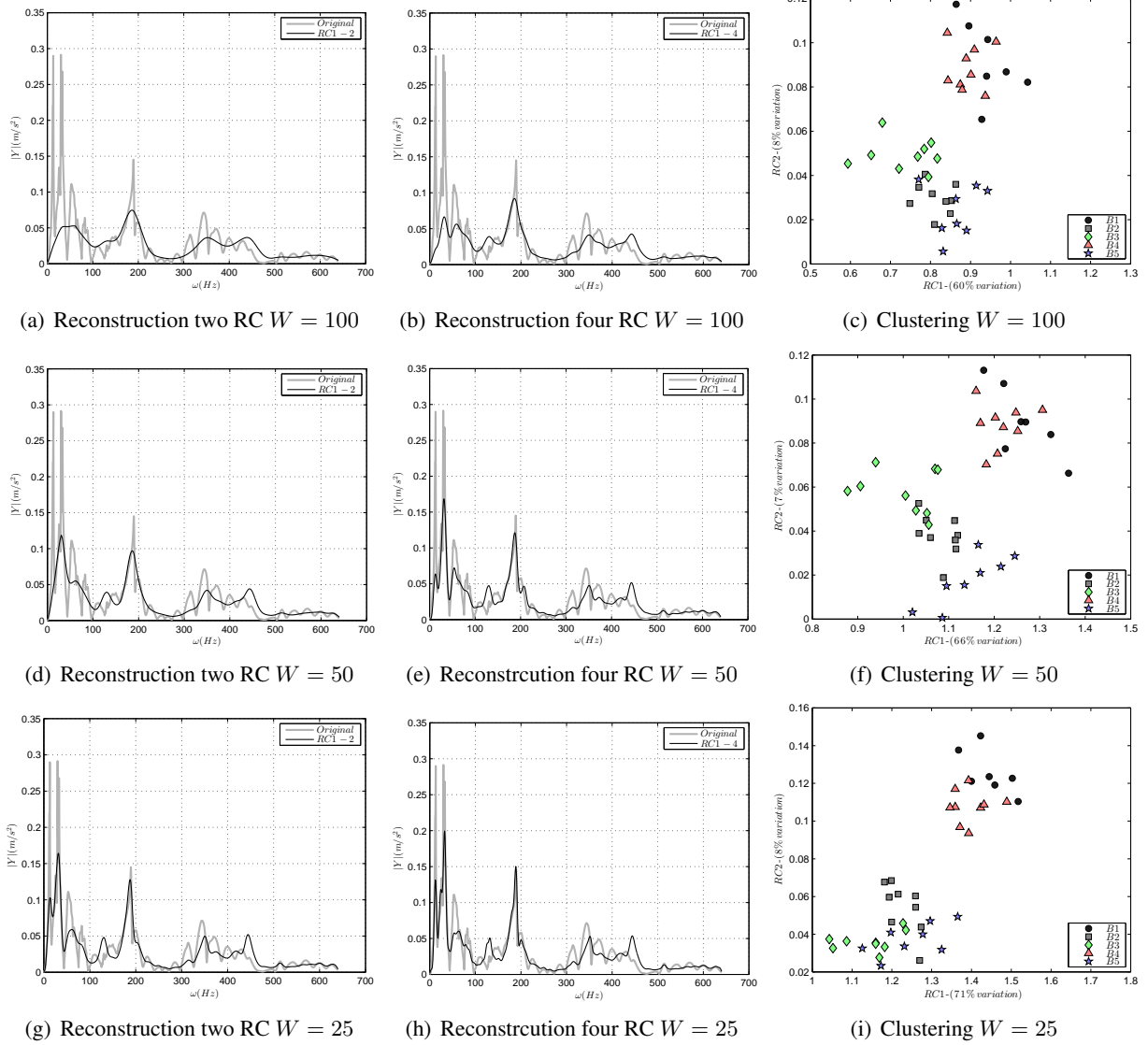
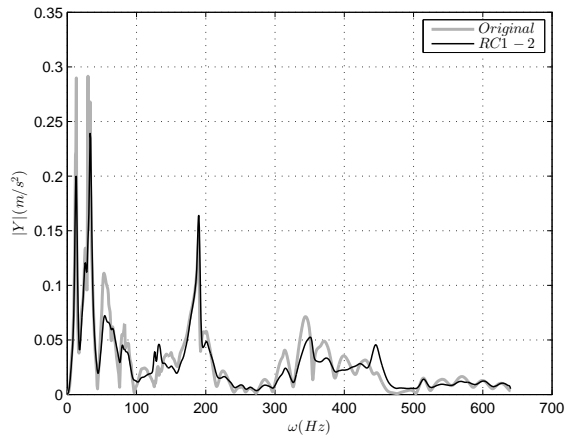
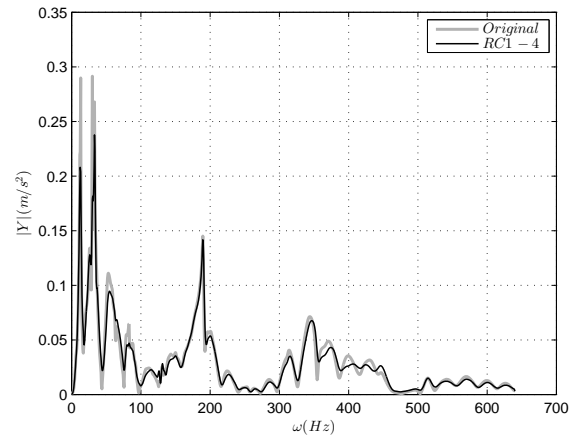


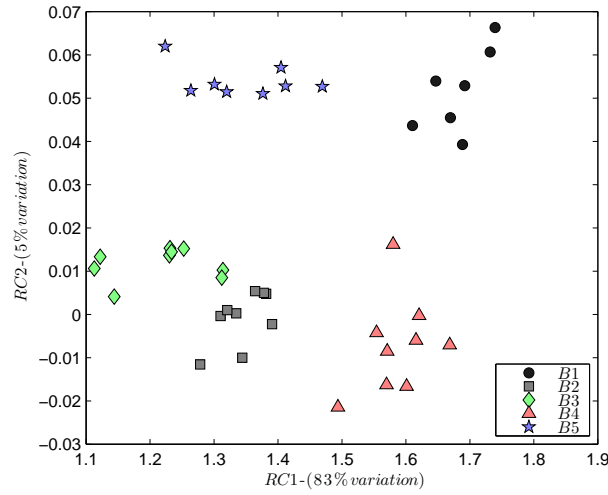
Figure 5: Effects on the variation of W for $W = 100$, $W = 50$ and $W = 25$ in the experimental analysis. The resolution of the signal was fixed at 1.6 s long and sampled at 640 Hz



(a) Reconstruction two RC $W = 7$



(b) Reconstruction four RC $W = 7$



(c) Clustering $W = 7$

Figure 6: Damage assessment with window length $W = 7$. The resolution of the signal was fixed at $1.6 s$ long and sampled at $640 Hz$

5 Conclusions

The aim of this paper is to investigate the effect of some signal parameters on the precision and the performance of a damage and delamination assessment methodology previously suggested by the authors of this study [10]. Some key parameters like the length of the signal, the sampling frequency interval Δf and the window length W have a direct influence on the sensitivity to damage and the accuracy of the methodology. This paper presents the principal aspects for a correct tuning of these parameters and the posterior analysis of the results. The influence of these parameters is first presented for a 2DoF system with a non-linear quadratic stiffness. Secondly, the effect of the parameters is studied for the case of a real composite laminated beams.

References

- [1] Hoon Sohn, Charles R Farrar, Francois M Hemez, Devin D Shunk, Daniel W Stinemates, Brett R Nadler, and Jerry J Czarnecki. *A review of structural health monitoring literature: 1996-2001*. Los Alamos National Laboratory, NM,(2004).
- [2] Nina Golyandina, Vladimir Nekrutkin, and Anatoly A Zhigljavsky. *Analysis of time series structure: SSA and related techniques*. CRC Press, 2010.
- [3] A-M Yan, Gaetan Kerschen, Pascal De Boe, and J-C Golinval. *Structural damage diagnosis under varying environmental conditions part I: a linear analysis*. Mechanical Systems and Signal Processing, 19(4):847864, 2005
- [4] A-M Yan, Gaetan Kerschen, Pascal De Boe, and J-C Golinval. *Structural damage diagnosis under varying environmental conditions part II: local pca for non-linear cases*. Mechanical Systems and Signal Processing, 19(4):847864, 2005
- [5] Ian T Jolliffe. *Principal component analysis*. Volume 487. Springer-Verlag New York, 1986.
- [6] DS Broomhead and Gregory P King. *Extracting qualitative dynamics from experimental data*. Physica D: Nonlinear Phenomena, 20(2):217-236, 1986.
- [7] Robert Vautard and Michael Ghil. *Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series*. Physica D: Nonlinear Phenomena, 35(3):395424, 1989.
- [8] Hossein Hassani and Anatoly Zhigljavsky. *Singular spectrum analysis: methodology and application to economics data*. Journal of Systems Science and Complexity, 22(3):372394, 2009.
- [9] Alexander Basilevsky and Derek PJ Hum. Karhunen-loeve. *Analysis of historical time series with an application to the plantation births in Jamaica*. Journal of the American Statistical Association, 74(366a):284290, 1979.
- [10] David Garcia and Irina Trendalova. *A multivariate data analysis approach towards vibration analysis and vibration-based damage assessment. Application for delamination detection in a composite beam*. Journal of Sound and Vibration, 2014. *In press*.
- [11] V Moskvina and A Zhigljavsky. *Application of the singular spectrum analysis for change-point detection in time series*. Journal of Time Series Analysis, submitted, 2001.
- [12] Hossein Hassani. *Singular spectrum analysis: methodology and comparison* Journal of Data Science 5(2007), 239-257.
- [13] FJ Alonso, JM Del Castillo, and P Pintado. *Application of singular spectrum analysis to the smoothing of raw kinematic signals*. Journal of biomechanics, 38(5):1085-1092, 2005.
- [14] VG Moskvina and AA Zhigljavsky. *Change point detection in time series by means of the singular spectrum analysis*. Journal of biomechanics, 38(5):1085-1092, 2005.