SCIENTIFIC MISSION TO A SOLAR POLAR ORBIT USING SOLAR SAIL PROPULSION

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Solar sail propelled missions to a polar orbit of the Sun offer unique science opportunities. Previous proposals have recommended the use of a 2-phase transfer to reach a solar polar orbit, however a 3-phase transfer has since been shown to offer a significant reduction in the transfer time at the expense of higher thermal stresses. The 3-phase transfer involves spiralling in close to the Sun, performing a rapid inclination increase, and spiralling back out to the final target orbit. A general perturbation solution for such a transfer has been defined which offers significant advantages over the numerically optimised solutions currently available. The insights provided by this analytical solution are used here to rapidly generate a holistic understanding of the mission architecture options available and hence how the mission and system design could be traded. A number of potential science missions are identified which could benefit uniquely from the use of such an orbit. These require that a solar latitude of 60° be achieved within 5 years before proceeding to a true polar orbit. A comparison between the use of the 2- and 3-phase transfer options identify that in real terms, the 3-phase transfer will reach a polar orbit approximately 1 year ahead of the 2-phase transfer. In addition, the increased efficiency of the transfer would allow for an increase in the allowable payload mass; with up to an extra 33kg payload potential predicted. Further work should allow for the mission and system design to be traded; for example to investigate the implications of increased thermal system mass (due to a reduction in the minimum solar approach distance) against reduced transfer time or sail size.

I. INTRODUCTION

To date, observation of the Sun has primarily taken place from Earth and in more recent years using space-based platforms. Due to the inclination of the Earth with respect to the Sun's equator, terrestrial observations have only been capable of observing the Sun within ± 7.25° of the solar equator. With the exception of Ulysses, space-based observations have also been restricted to within ± 32° of the solar equator. The Ulysses mission used a Jupiter gravity assist to achieve an inclination of ± 80.2° during its observation of the Sun between 1992 and 2008. However, the highly elliptical orbit of Ulysses meant that it returned to the Sun only every six years. In addition, it only provided field and particle observations of the heliosphere but no images. Solar Orbiter, due to launch in 2017, will allow space based observations only within ± 36° of the solar equator due to the high change in velocity required to increase inclination. Thus, a mission which could observe the solar poles closely from a high inclination, enabling the construction of a complete three dimensional image of the Sun, is highly desirable.

Solar Sail Propulsion

A solar sail is a form of spacecraft propulsion which uses sunlight to generate a propulsive force. It consists of a very light structure with a large highly reflective surface area. Photons reach the sail in the form of sunlight and reflect off, exerting a small but measurable force on the sail. While this provides a very low thrust, over time the effect accumulates to produce high acceleration. The sail can be oriented to produce thrust in the required direction and in this way the orbit of the spacecraft can be altered.

The primary advantage of solar sailing is that it requires no additional propellant to be carried on board. This makes it ideal for missions requiring high energy manoeuvres, such as the large inclination change required by the proposed solar polar mission. In addition, the effectiveness of the solar sail increases close to the Sun as evidenced by

$$F = \frac{F_0 (\cos\theta)^2}{R^2} \qquad [1]$$

A study carried out by the University of Strathclyde and the University of Glasgow in conjunction with the European Space Agency investigated the feasibility of a solar polar observation mission using solar sail technology[1]. This initial investigation found that the difficulties inherent in providing the high ΔV required to achieve a high inclination orbit around the Sun could be overcome using solar sail propulsion in place of standard chemical or electric propulsion. However, further advancements in solar sail technology would be required.

II. MISSION ARCHITECTURE

Previous studies have proposed the use of a 2-phase transfer to reach a solar polar orbit, consisting of an initial spiral phase in which the orbit semi-major axis is reduced to the target value, and a second phase, known as the cranking phase, in which the inclination is rapidly increased[2] while the semi-major axis remains constant.

It has since been identified that using a 3-phase transfer may offer significant time savings over the 2-phase approach[3]. This involves an initial spiral phase to a cranking orbit which is closer to the Sun than the target orbit. Here the inclination can be rapidly increased while maintaining a constant semi-major axis. The third phase then involves spiralling back out to the final target orbit.

The definition of a phase used here is a section of the transfer trajectory throughout which the solar sail is held at a constant angle; that is, the solar sail angle in Phase 1 differs from the sail angle in Phase 2, which differs from the sail angle in Phase 3. Both the 2- and 3-phase transfers assume a transfer from 1AU in the Earth's ecliptic plane and arriving in a target orbit inclined at 82.75° to the ecliptic. While in the case of both transfers the majority of the inclination change takes place in the cranking phase, it is possible for some inclination change to be performed during the spiral phases as well.

It is of note that whilst the 3-phase transfer may be faster than the 2-phase transfer, it also necessitates a closer solar approach resulting in higher thermal stresses on the spacecraft. As such, it is advantageous to be able to perform a trade study to determine whether the advantage of using the 3-phase transfer is outweighed by the increased system requirements.

For the 2-phase transfer, it is straightforward to perform a system trade as an analytical, general perturbation solution exists[4]. In the case of the 3-phase transfer, numerically optimized solutions for the 3-phase transfer are available[5] but they do not allow for a complete understanding of the optimal structure of the trajectory to be gained. However, by extending the existing general perturbation solution that has previously been derived for the 2-phase transfer, it has been shown to be possible to use a general perturbation solution to describe the optimal trajectory for a 3-phase transfer from the Earth to a solar polar orbit using solar sail propulsion[6]. The insights provided by the general perturbation solution can be used to rapidly generate a holistic understanding of the architectural options available and hence how the mission and system design could be traded.

III. GENERAL PERTURBATION SOLUTION

The general perturbation solution for the 3-phase transfer is based on the Lagrange-Gauss variational equations adapted for a spacecraft using solar sail propulsion. Expressions for the transfer time for each phase are derived and summed up to produce an expression for the total transfer time for the 3-phase manoeuver. This is expressed in terms of the sail lightness number β, the semi-major axis a at each transition point, the augmented sail clock angle γ for each manoeuvre phase, the initial and final inclinations i, the gravitational constant for the central body μ, and the sail pitch angle α. The sail lightness number, augmented sail clock angle and sail pitch angle are all as previously defined by Macdonald[7]. This final expression is of the form,

$$
\begin{aligned}
t = \frac{1}{\beta}\Big[\mathsf{A}\,\frac{1}{\cos\gamma_1} + \mathsf{B}\,\frac{1}{\cos\gamma_3}\,a_2^{3/2} \\
+ a_1^{3/2}\Big(\mathsf{B}\Big\{-\frac{1}{\cos\gamma_1} - \frac{1}{\cos\gamma_3} - \log\Big[\frac{a_2^{3/2}}{a_1^{3/2}}\Big]\tan\gamma_3\Big\} \\
+ \mathsf{C}i_3 + \tan\gamma_1\big(\mathsf{B}\,\log[a_1^{3/2}] - \mathsf{D}\big)\Big)\Big]
\end{aligned}
\qquad [2]
$$

where A, B, C and D are all constants determined by the orbit parameters.

Solution Method

From this solution it is possible to determine the precise transfer trajectory which will result in the minimum transfer time. Although equation [2] shows the solution to be analytically described, the number of variables means that the 3-phase trajectory general perturbation solution cannot be analytically solved. Instead, a constrained non-linear optimisation, with constraints set on the minimum cranking and target orbit semi-major axes, is used to determine the combination of variables which will provide the minimum total transfer time.

Findings of General Perturbation Solution

By imposing limits on the minimum values of the cranking and target semi-major axes, a_1 and a_2, and defining the target inclination i_3, the optimum sail angles for each phase can be identified for which the total transfer time is minimised. The existence of this minimum point is evidenced in Fig. 1. When compared with existing analyses, these results showed that the 3-phase transfer offered a 30% reduction in overall trip time when compared with the 2-phase transfer.

Table 1 shows the optimised transfer times for trajectories from a circular 1AU orbit within the ecliptic plane to a circular orbit inclined at 82.75°, at a range of target solar radii, and for a variety of cranking orbit semi-major axes.

Comparison with Existing Solutions

Comparison of the general perturbation solution described with the existing numerical solution proposed by Leipold[8] for a 3-phase transfer to a solar polar orbit has also been performed[9]. The results of this analysis show that the general perturbation solution offers a 10% reduction in the total transfer time. This is achieved by performing a portion of the plane-change manoeuver in the first and third phases, rather than performing the entire plane change during the cranking phase as is proposed by Leipold. The results of this comparison are shown in Table 2.

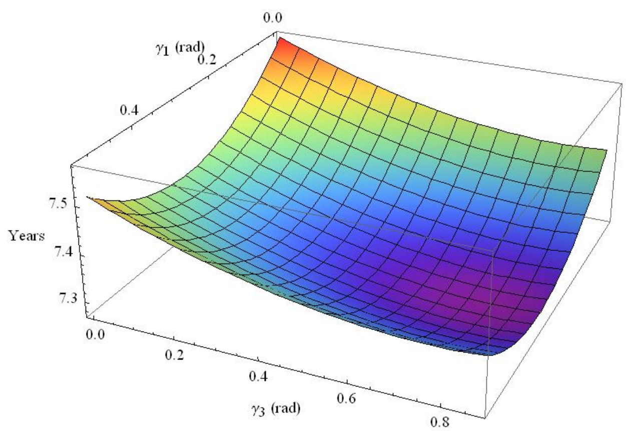

Fig. 1: Minimum total transfer time versus sail angle for the first and third phases with a_1=0.25AU, a_2=0.4AU, i_3=82.75°

Target Orbit Radius (AU)	Cranking Orbit Radius (AU)	Total Duration to 82.75° (years)
0.63	0.25	$\frac{0.278274}{\beta}$
	0.30	$\frac{0.300639}{\beta}$
	0.35	$\frac{0.325463}{\beta}$
	0.4	$\frac{0.352856}{\beta}$
0.48	0.25	$\frac{0.255542}{\beta}$
	0.30	$\frac{0.278646}{\beta}$
	0.35	$\frac{0.304664}{\beta}$
	0.4	$\frac{0.333877}{\beta}$
0.40	0.25	$\frac{0.245221}{\beta}$
	0.30	$\frac{0.269168}{\beta}$
	0.35	$\frac{0.296594}{\beta}$
	0.4	$\frac{0.328460}{\beta}$

Table 1: Results for optimal 3-Phase transfer

	Time for Phase 1	Time for Phase 2	Time for Phase 3	Total Transfer Time
Numerical Solution	540 days	755 days	55 days	3.7 years
General Perturbation Solution	534 days	603 days	86 days	3.35 years

Table 2: Results of Comparison of General Perturbation Solution with Existing Numerical Solution

IV. MISSION APPLICATIONS

As previously mentioned, a polar orbit of the Sun with short revisit times and a long view of the poles is highly desirable for scientific research. Three scientific missions in particular have been proposed for a solar polar mission that could be feasibly carried out using solar sail propulsion[10].

The first mission proposes to observe latitude variations of irradiance of the Sun. This can be performed with a payload mass of 5kg. The second proposal aims to study the internal structure of the Sun and the solar dynamo and can be carried out with a 15kg payload. The final proposed payload is in the region of 40kg and aims to study the heliophysics of the Sun, as well as the solar dynamo.

Mission Requirements

To facilitate any of the aforementioned science missions, an orbit inclination of at least 52.75° is required; this corresponds to a solar latitude of 60°. The orbit radius should also be less than 1AU to maximise the quality of the science observations; radii between 0.4AU and 0.6AU provide an adequate view of the poles for long enough to observe a complete rotation of the Sun which is extremely desirable. An Earth resonant orbit is not a requirement for the mission but it would provide continuous spacecraft visibility from Earth. To maximise the science return of such missions, it is required that the spacecraft reach an orbit inclination of 52.75° within 5 years. After this, the inclination can be further increased to reach a polar orbit at an inclination of 82.75° to the ecliptic.

Comparison of 2- and 3-Phase Mission Architectures

Previous studies have analysed such a mission if carried out using a 2-phase transfer[11]. By estimating the mass and performance characteristics of a solar sail which could reasonably be used for such a mission, an upper limit could be imposed on the mass of spacecraft which could be feasibly carried by such a sail. From this, the maximum allowable payload mass could also be estimated as approximately 20% of the platform mass.

As shown in equation [2] and Table 1, the total time taken for the 3-phase transfer can be described in terms of the lightness number β. Thus, it is possible to calculate for a specific trajectory, the lightness number β required to reach an inclination of 52.75° within 5 years. The lightness number is a dimensionless parameter which defines the ratio of the characteristic acceleration of the sail to the Sun's local gravity. The characteristic acceleration is inversely proportional to the sail loading and so can be written as

$$a_c = \frac{8.25A}{m} \qquad [3]$$

where A is the sail area and m is the spacecraft total mass. Hence, by using the same solar sail mass and performance characteristics as were used in the 2-phase study, it is possible to estimate the maximum spacecraft mass that can be carried for a given characteristic acceleration, and hence the maximum allowable payload mass.

Table 3 shows the lightness number and characteristic acceleration required for the spacecraft to reach an inclination of 52.75° within 5 years using both the 2-phase and 3-phase transfer. It can be seen that in all cases the 3-phase transfer requires a lower characteristic acceleration to achieve the same results as the 2-phase option. This implies that by adopting the 3-phase transfer option, the mission could be carried out using a smaller solar sail, or could be capable of carrying a larger mass. In addition, the required characteristic acceleration in the 3-phase case remains relatively constant, regardless of the final target orbit, and is primarily dependent on the cranking orbit. The characteristic acceleration necessary to reach 52.75° within 5 years using the 3-phase transfer is shown in Fig. 2 for a variety of cranking orbits.

It is also of note that even with a smaller characteristic acceleration, using a 3-phase transfer the spacecraft will arrive at the final polar orbit in a much shorter time period than when using a 2-phase transfer, with time savings of approximately 1 year evident in all cases as shown in Table 3.

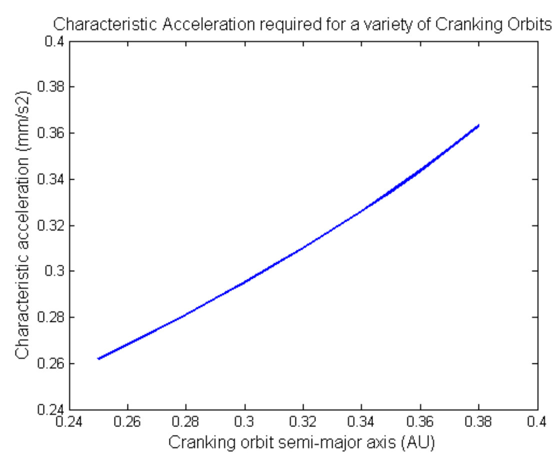

Fig. 2: Characteristic acceleration required for a 3-phase transfer to 52.75° in 5years for a variety of cranking orbits

Architecture Option	2-Phase Transfer A^2	B^2	C^2	3-Phase Transfer A^2	B^2	C^2
Solar pole maximum Observer Zenith Angle	50 deg.	40 deg.	30 deg.	50 deg.	40 deg.	30 deg.
Target solar radius	0.397 au	0.461 au	0.559 au	0.397 au	0.461 au	0.559 au
Cranking solar radius	-	-	-	0.25 au	0.25 au	0.25 au
Required sail lightness number	0.0482	0.0535	0.0625	0.0442	0.0440	0.0440
Required sail characteristic acceleration	0.29 mm s^{-2}	0.32 mm s^{-2}	0.37 mm s^{-2}	0.262 mm s^{-2}	0.261 mm s^{-2}	0.261 mm s^{-2}
Time to 60° solar latitude	5.0 years	5.0 years	5.0 years	5.0 years	5.0 years	5.0 years
Time to 90° solar latitude	6.8 years	7.0 years	7.3 years	5.5 years	5.7 years	6.1 years

Table 3: Comparison of 2-phase and 3-phase transfers for various mission architectures

Table 4 to Table 9 show the maximum spacecraft and payload masses which could be carried for a variety of mission architectures, considering both the 2- and 3-phase options. The range of possible values shown corresponds to the maximum values that could be carried depending on the mass of the solar sail deployment booms selected.

Table 10 demonstrates that depending on the mission architecture selected, a spacecraft using the 3-phase transfer can carry up to 33kg extra payload mass when compared with the 2-phase transfer. This is a significant increase in the payload mass available and could allow for multiple extra instruments to be flown. Architecture option A^2 offers the greatest capacity, with a maximum payload mass of 71kg available when a solar sail of 2.5μm thickness is used in conjunction with a 3-phase transfer.

Due to the consistency in the characteristic acceleration for all of the 3-phase transfer options, the maximum spacecraft and payload mass values vary only slightly for all architecture options. Conversely, it is seen that in the case of the 2-phase transfer, the payload and mass that can be carried falls away sharply as the target orbit radius increases. Hence, by making use of the 3-phase transfer, a payload of the same mass could be placed in a target orbit further from the Sun than would be achievable using the 2-phase transfer, resulting in lower thermal stresses throughout the bulk of the mission lifetime. In addition, the consistency in the allowable payload means that an initial spacecraft platform and payload design could be carried out before a final orbit is selected.

System Trades

Whilst the 3-phase transfer has been shown to offer advantages such as reduced transfer times and increased payload capacity, there are also disadvantages associated with the use of this transfer trajectory. The most critical of these are the increased thermal stresses the system would experience as a result of approaching the Sun closely during the cranking phase. To account for this it is likely that the thermal subsystem will need to be improved, resulting in an increase in the platform mass.

Previous analysis of a similar mission using a 2-phase transfer has found that for orbits smaller than 0.34AU there is a steep increase in the mass of the spacecraft platform[12]. It suggests that there is a specific solar radius for which the overall system mass will be minimised; beyond this value the necessary increase in the platform mass will outweigh the possible mass savings due to a smaller sail being required close to the Sun.

It is likely that a similar relationship exists in the case of the 3-phase transfer and a trade study would be required to determine the optimal trajectory to provide the greatest possible payload mass. It is hoped that the general perturbation solution described here would facilitate such a system trade and allow for a more straightforward analysis of the complete system.

	Architecture option A^2 with 7.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	*negative*	*negative*
75 m	< 12	< 2	< 29	< 6
100 m	21 – 71	4 – 14	51 – 102	11 – 21
125 m	75 – 150	15 – 30	121 – 198	24 – 40

Table 4: Comparison of payload masses for option A^2 with 7.5µm film

	Architecture option B^2 with 7.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	*negative*	*negative*
75 m	*negative*	*negative*	< 30	< 6
100 m	< 39	< 8	52 – 103	10 – 21
125 m	24 – 100	5 – 20	123 – 200	24 – 40

Table 5: Comparison of payload masses for option B^2 with 7.5µm film

	Architecture option C^2 with 7.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	*negative*	*negative*
75 m	*negative*	*negative*	< 30	< 6
100 m	*negative*	*negative*	52 – 103	10 – 21
125 m	< 34	< 7	123 – 200	24 – 40

Table 6: Comparison of payload masses for option C^2 with 7.5µm film

	Architecture option A^2 with 2.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	< 7	< 2
75 m	38 – 67	8 – 13	54 – 85	11 – 17
100 m	120 – 170	25 – 34	149 – 200	30 – 40
125 m	229 – 305	46 – 61	275 – 352	55 – 71

Table 7: Comparison of payload masses for option A^2 with 2.5µm film

	Architecture option B^2 with 2.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	< 7	< 2
75 m	20 – 49	4 – 10	55 – 86	11 – 17
100 m	87 – 138	17 – 28	151 – 202	30 – 41
125 m	179 – 254	36 – 51	278 – 354	55 – 71

Table 8: Comparison of payload masses for option B^2 with 2.5µm film

	Architecture option C^2 with 2.5 µm film			
	2-Phase		3-Phase	
Sail Side Length	Platform Mass (kg)	Payload Mass (kg)	Platform Mass (kg)	Payload Mass (kg)
50 m	*negative*	*negative*	< 7	< 2
75 m	< 26	< 5	55 – 86	11 – 17
100 m	45 – 96	9 – 19	151 – 202	30 – 41
125 m	113 – 188	23 – 38	278 – 354	55 – 71

Table 9: Comparison of payload masses for option C^2 with 2.5µm film

	Payload Mass Saving for 3-Phase over 2-Phase (kg)					
	A^2		B^2		C^2	
Sail Side Length	**7.5 µm**	**2.5 µm**	**7.5 µm**	**2.5 µm**	**7.5 µm**	**2.5 µm**
50 m	-	2	-	2	-	2
75 m	4	3 – 4	6	7	6	11 – 12
100 m	7	5 – 6	10 – 13	13	10 – 21	21 – 22
125 m	9 – 10	9 – 10	19 – 20	19 – 20	24 – 33	32 – 33

Table 10: Payload mass savings for various architecture options

V. CONCLUSION

A mission capable of providing long-term views of the poles of the Sun, with short revisit times is highly desirable. Using a 3-phase transfer from the Earth it is possible to achieve such an orbit within 7 years, with an inclination of 52.75° achieved within 5 years of launch. This is significantly faster than the results which can be achieved using a 2-phase transfer. It also offers the potential for a greater payload mass to be carried to the target orbit increasing the potential for useful science. The dependency of the required characteristic acceleration on the cranking orbit radius alone means that there is a consistency in the allowable payload mass regardless of the final target orbit. This could allow for parallel design of the spacecraft platform and payload, and the orbit and transfer trajectory.

The reduction in transfer time and increase in payload mass facilitated by the 3-phase transfer comes at the cost of a closer solar approach and hence greater thermal stresses on the system. A complete system trade study is necessary to determine the optimal trajectory for which the increased payload mass is not outweighed by the increased mass of the thermal subsystem. The general perturbation solution described here should allow for such a trade to be carried out. In addition, it provides a realistic description of a potential transfer trajectory and has been shown to offer time savings when compared with existing numerical solutions.

This work demonstrates how the general perturbation solution can be used to investigate the system and compare with existing options. A baseline trajectory was defined and the corresponding spacecraft platform and payload masses estimated. Further analysis of the mission as well as the spacecraft should allow for a more detailed mass budget to be prepared and an optimal trajectory selected to allow for the greatest scientific return.

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