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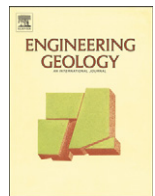
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Q21 An approach for predicting the stability of vertical cuts in cohesionless soils above the water table

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ABSTRACT

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Temporary vertical excavations in cohesionless (granular) soils pose a problem for conventional 'two-phase' soil mechanics theory since non-zero collapse height is not predicted using the classical 'dry/saturated' shear strength criterion, given that cohesionless soils above the water table are assumed to be dry. An extension of the classical shear strength equation to account for the effect of matric suction on the effective stress in partially saturated soil is presented here that is incorporated into the bound theorems of plasticity. A simple validation experiment is reported to test the concept following which, a case study is presented that explores the extent to which matric suction and its impact on shear strength can explain the large safe vertical cut height that is often observed in cohesionless pozzolan deposits in the field. Lastly, the impact of rainfall events and subsequent ponded infiltration is investigated using a very simple analytical technique based on the classical Terzaghi consolidation solution. The research presented here gives practitioners with no particular expertise in the mechanics of unsaturated soil, techniques to assess the stability of geostructures involving unsaturated cohesionless soils that are based on simple calculation techniques taught in undergraduate courses.

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1. Introduction

The stability of vertical excavations in cohesionless (granular) soils is an important problem in geotechnical engineering and engineering geology. Because excavations are typically carried out above the water table cohesionless soils are unsaturated. In routine geotechnical engineering and engineering geology calculations cohesionless soils (with no or little clay fraction) above the water table are generally assumed to be dry. Nonetheless this assumption is not accurate.

Soils above the water table, due to matric suction, are in fact partially saturated and also exhibit significantly higher shear strength than dry soils. As a result, vertical cuts up to several metres in height may remain stable (Tsidzi, 1997; Whenham et al., 2007; De Vita et al., 2008) in cohesionless soils. The beneficial effect of partial saturation is often exploited by contractors who typically cover the bank adjacent to the excavation with an impermeable membrane to divert surface runoff during heavy rainfall, thus preserving partial saturation. The beneficial effect of partial saturation on the stability of vertical and near-vertical cuts is recognised by engineering geologists when analysing and modelling bank retreat and delivery of bank sediments to river (Rinaldi and Casagli, 1999; Simon et al., 2000; Rinaldi et al.,

2004). These mechanisms should potentially be incorporated into morphodynamic models of the evolution river platforms (Langendoen et al., 2012; Nardi et al., 2012). Partial saturation plays an important role in the stability of trenches (Vanapalli and Oh, 2012), which are used in a variety of applications in assessing geologic hazards in engineering geology, and tailing dams (Zandarin et al., 2009). The effect of partial saturation is also well known to children when erecting sand castles.

Classical 'dry/saturated' soil mechanics fails to predict a non-zero safe vertical cut height in cohesionless soils above the water table, as they are assumed to be dry. In theory a dry cohesionless soil exhibits a zero collapse height, as is evidenced by the lower bound theorem of plasticity (Chen, 2007) or experimentally by observation, since it is impossible to fabricate a cylindrical sample of dry sand.

However, practitioners and academicians still find it convenient to disregard the contribution of partial saturation to shear strength as this leads to conservative design. This point of view can be questioned. Significant costs might be saved if 'new' geostructures are designed to account for the effects of partial saturation. Furthermore, geotechnical engineers and engineering geologists are often confronted with 'existing' stable yet potentially hazardous geostructures, e.g. steep slopes, for which conventional soil mechanics theory offers no explanation of the current state of equilibrium. In this case, a realistic analysis of the current state of stress is required, including characterisation of the partially saturated zone above the water table. This is essential when assessing the likelihood of future instability and hence, is a key to

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ensuring the proposal of appropriate precautionary or remedial measures.

To quantify the effects of partial saturation on the stability of geostructures, methods should be developed to analyse collapse conditions in cohesionless partially saturated soils. So far, this problem has received little attention from researchers working on the mechanical behaviour of unsaturated soils.

This paper presents an approach based on the upper and lower bound theorems of plasticity. By assuming that the shear strength of partially saturated soils is controlled by the average skeleton stress, the classical approach developed for dry and saturated soils can easily be extended to cater for problems involving partially saturated soils.

Experimental evidence of the validity of this concept is provided in the form of collapse tests performed on cylindrical samples of partially saturated sand. Because changes in suction and vertical stress along the sample height are not negligible, this unconfined compression test is regarded as a boundary value problem rather than an element test. The theoretical analysis of the column collapse load based on the upper and lower bound theorems is thus essentially the same as the analysis that will be carried out to determine the collapse height of a vertical cut.

The principal goal of the paper is to verify whether the upper and lower bound collapse loads determined theoretically, bracket closely the values observed in the model tests. This is aimed at validating an approach to predict the collapse height of unsaturated cohesionless soils based on the bound theorems of plasticity.

A case study concerning the safe unsupported vertical cut height potentially achievable in pyroclastic silty sand (Pozzolan deposits) is then presented. This illustrates the utility of the proposed extension of the bound theorems of plasticity when assessing geotechnical and engineering geology problems in the field involving unsaturated soils.

2. Extension of the bound theorems of plasticity to unsaturated soils

The upper and lower bound theorems of plastic collapse set limits to the collapse load of a structure and can be proven for the case of perfectly plastic materials with associated flow rule (Chen, 2007). In two-phase soils, the failure (yield) criterion under ultimate conditions can be defined by the following equation:

$$\tau = (\sigma - u) \tan \phi' \quad (1)$$

where τ is the shear stress, σ is the normal stress, u is the pore pressure and ϕ' is the effective angle of shearing resistance. Pore pressure equals the pore-water pressure u_w in saturated soils and the pore-air pressure u_a in ideally dry soils. Using the failure criterion given by Eq. (1), the ultimate conditions of soil structures such as retaining walls, foundations, vertical cuts, and slopes can be assessed for saturated and dry soil (Atkinson, 1981; Chen, 2007).

The application of bound theorems of plasticity to soil structures above the water table requires the definition of a suitable failure criterion for partially saturated soils. For compacted (aggregated) soils, shear strength under partially saturated states can be expressed by the following equation (Tarantino and Tombolato, 2005):

$$\tau = (\sigma - u_w S_{re}) \tan \phi'_s = (\sigma + s S_{re}) \tan \phi'_s \quad (2)$$

where u_w is the pore-water pressure, s is the suction ($s = -u_w$), and S_{re} is an effective degree of saturation (degree of saturation of the macro-pores), which is given by:

$$S_{re} = \frac{e_w - e_{wm}}{e - e_{wm}} \quad (3)$$

where e is the void ratio (volume of voids per volume of solids), e_w is the water ratio (volume of water per volume of solids), and e_{wm}

is the 'microstructural' water ratio, which separates the region of inter-aggregate porosity from the region of intra-aggregate porosity (Romero and Vaunat, 2000). The parameter e_{wm} may conveniently be determined by best fitting of shear strength data and the validity of Eq. (2) in conjunction with Eq. (3) has been proven by Tarantino (2007) and Tarantino and El Mountassir (in press) for a wide range of clayey soils, including compacted, and natural soils.

On the other hand, reconstituted and non-clayey soils are generally non-aggregated and the 'microstructural' water ratio e_{wm} may therefore be expected to be zero for these soils. Indeed, this has been demonstrated to be the case for a wide range of non-clayey soils by Tarantino and El Mountassir (in press).

For non-aggregated soils, the failure criterion can therefore be defined by the following equation (Öberg and Sällfors, 1997):

$$\tau = (\sigma - u_w S_r) \tan \phi'_s = (\sigma + s S_r) \tan \phi'_s \quad (4)$$

If Eq. (2) or (4) is used in place of Eq. (1), collapse of geostructures in partially saturated soils can be analysed in a very similar manner by introducing a few simple modifications. To derive the upper bound solution, the work done by the internal stresses W_i for the case of translational failure can be written as (assuming an effective cohesion $c'_s = 0$):

$$W_i = \delta \sin \phi'_s \int_l s S_{re} dl \quad (5)$$

where δ is the magnitude of the block displacement, ϕ'_s is the effective (saturated) angle of shearing resistance, s is the suction, S_{re} is the effective degree of saturation, and l is the length of the failure surface. It is worth mentioning that the work done by the internal stresses W_i is written here in terms of total stresses whereas the external work associated with the gravitational load is calculated by considering the (total) soil unit weight.

To derive the lower bound solution, the failure criterion must not be exceeded at any point in the soil. This occurs if none of the Mohr's circles cross the failure envelope in the $\sigma + s S_{re}$, τ plane (rather than the σ , τ plane as in the case of saturated or dry soils).

3. Laboratory validation

3.1. Material

Experimental verification of the proposed extension to the bound theorems of plasticity was performed using a fine silica sand with grain size range of 0.075–0.2 mm and specific gravity, G_s , of 2.73 (derived experimentally). This material was expected to commence desaturation under applied suctions of less than 5–10 kPa, allowing a simple negative water column technique to be used to apply suctions to the base of a sample.

3.2. Water retention behaviour

3.2.1. Apparatus

To derive the water retention characteristics of the fine sand a simple negative water column method was employed. The apparatus used is shown schematically in Fig. 1. A cylindrical cell was used to house the sand sample and allow application of suction at the base. Different magnitudes of suction were applied to the sand sample by raising and lowering a water reservoir on a frame. A ThetaProbe sensor (Delta-T Devices Ltd., 1999) was placed in the sample at the surface to measure the volumetric water content at a known location. A high air-entry filter (with an air entry value greater than the maximum suction to be applied during the experimental procedure) was specifically devised to provide an interface between the sand and the hydraulic reservoir, thus maintaining suction.

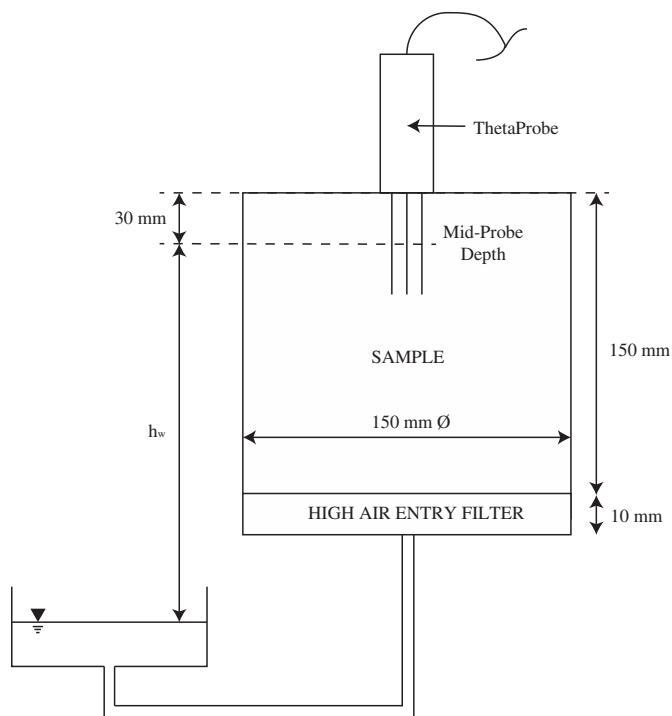


Fig. 1. Schematic of apparatus used to derive soil water retention behaviour of very fine uniform sand.

3.2.2. Sensor for water content measurement

The ThetaProbe sensor was used to measure the bulk dielectric permittivity of the soil, which is then correlated to the soil water content via calibration. The probe has a sensing length of 60 mm and the measurements taken in this investigation were assumed to be representative of the soil water content at the mid-depth of the probe (i.e. 30 mm from the surface of the sample).

The sensor outputs a voltage that is correlated to the soil bulk dielectric permittivity ε by the following relationship:

$$\sqrt{\varepsilon} = 1.07 + 6.4 \cdot V - 6.4 \cdot V^2 + 4.7 \cdot V^3 \quad (6)$$

where V is the output voltage of the probe (Gaskin and Miller, 1996; Delta-T Devices Ltd., 1999). To convert the dielectric permittivity measurement to the soil water content the following relationship is suggested by the manufacturer:

$$\theta = \frac{\sqrt{\varepsilon} - a_0}{a_1} \quad (7)$$

where a_0 and a_1 are soil specific calibration parameters.

To confirm the accuracy of the relationship in Eq. (6), the dielectric constant of some common laboratory solvents (Acetone, Acetic Acid and Ethanol) was measured and checked against values quoted by Budevsky (1979), yielding an average percentage discrepancy of approximately $\pm 1.2\%$, which was deemed acceptable.

Soil specific calibration of parameters a_0 and a_1 was then conducted on silica sands compacted into a mould of 150 mm height and 100 mm diameter. Compaction was achieved in three layers using a 250 g sliding hammer dropped from a height of 300 mm, with fifteen blows being applied per layer. Four target soil water contents in the range of 0–0.35 were tested to represent dry, damp, wet and saturated samples in both fine sand and coarse sand with grain size ranges of 0.075–0.2 mm and 0.4–0.6 mm respectively. After compaction of the sample the ThetaProbe was inserted and measurements taken for a period of 10 min, from which the time averaged root dielectric value was calculated using Eq. (6). Following this the soil water content of each sample was

derived experimentally. Fig. 2 shows a plot of the measured root dielectric permittivity of the fine and coarse silica sand with respect to the measured volumetric water content showing that soil dielectric permittivity was not significantly grain size dependent. The calibration parameters a_0 and a_1 were determined using the method of least squares, yielding values of 1.492 and 9.743 respectively. The average discrepancy of the calibration function from the measured soil water content was ± 0.03 , which is less than the capability of the device as quoted by the manufacturer and thus deemed acceptable.

3.2.3. High air entry filter preparation

A simple high air entry filter was created using uniform silt with an estimated air-entry suction of approximately 20–30 kPa. The filter allows the transmission of water relatively rapidly at low applied suctions (0–15 kPa) but not air, thus allowing hydraulic suction to be maintained at the base of the sample.

The silt filter has two critical advantages in comparison to commercial high air-entry porous ceramics. It provides adequate air-entry suction for sand whilst ensuring higher hydraulic conductivity and subsequently shorter equalisation periods than for example, a commercial 100 kPa air-entry suction ceramic. Additionally, it ensures proper contact with the sand and eliminates possible wall effects (large pores at the interface between a flat surface and a granular material) that could prevent suction being transmitted to the sample.

To construct the filter, first, a woven mesh was placed in the base of the test chamber, which was covered by a paper filter. Liquefied silt slurry was then poured on top of the filter paper and allowed to settle under gravity, generating a targeted 10 mm depth of filter. Excess water was then drained from the base, with clear water indicating a successful filter and cloudy water indicating failure. Following successful generation of the filter a test suction of 17 kPa was applied (which was greater than the target maximum suction of 15 kPa), whilst allowing complete drainage of the cylindrical cell and consolidation of the filter without filter desaturation. This exposed the filter to air, thus testing the ability of the filter to maintain hydraulic suction at the base of the sample. Before use each of the filters was tested in this manner.

3.2.4. Experimental procedure

Following creation of a successful air entry filter oven dried fine silica sand was rained into the cell, to attain a sample height of 150 mm with uniform density. The ThetaProbe was then placed into the centre of the sample, with an accompanying latex cover being used to isolate the sample from the atmosphere, thus minimising evaporation of pore water from the sample to the atmosphere. The cell was connected to the hydraulic reservoir using transparent plastic tubing, with an air trap at the point of lowest pressure in the system.

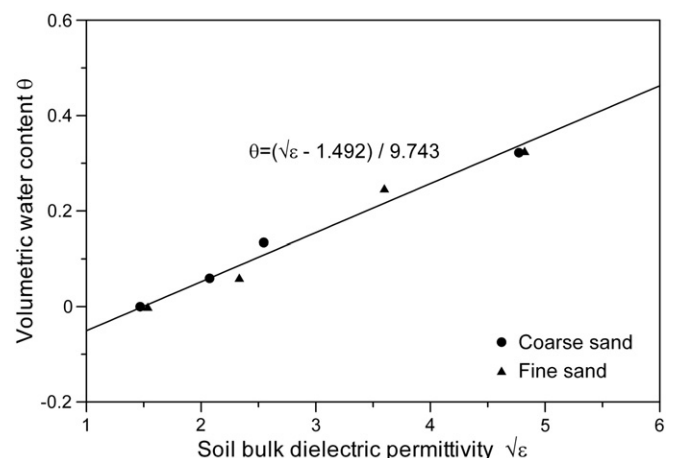


Fig. 2. Calibration of parameters a_0 and a_1 for fine and coarse grained silica sands.

278 Upon assembly of the apparatus, the water reservoir was raised above the
 279 surface of the sand in the cylindrical cell to commence saturation,
 280 which was achieved when the water table was observed to be above
 281 the surface of the sand and the ThetaProbe was indicating a constant
 282 measurement.

283 After saturation of the sample, the drying phase was initiated by
 284 lowering the water reservoir in increments followed by the wetting
 285 phase by raising the water reservoir. This allowed the investigation
 286 of the hysteretic hydro-mechanical properties of the soil.

287 3.2.5. Experimental results

288 Fig. 3 presents the change in soil volumetric water content (θ)
 289 with respect to time in hours, with the final measurement points used
 290 to define the soil water retention drying and wetting curves indicated.
 291 A change in sample porosity was evident between the start and the
 292 end of the experiment. Fleuerau et al. (1993) observed in silty non-
 293 plastic soils that changes in void ratio were apparent during the drying
 294 phase but only significantly before the air entry suction was reached. If
 295 the same behaviour is assumed to be apparent here it is reasonable to
 296 assume all volumetric changes occurred before the air entry suction
 297 of the sand was reached. The porosity at all suctions exceeding the
 298 air-entry value could thus be assumed to be equal to the final porosity,
 299 which was measured at the end of the experiment.

300 After correction of the initial soil water content measurement
 301 based upon this assumption, the van Genuchten (1980) relationship
 302 describing soil water retention characteristics was fitted using the
 303 least squares method for both the drying and wetting water retention
 304 curves as illustrated in Fig. 4. The following equations were used
 305 to model the main drying curve and the scanning wetting curve
 306 respectively:

$$S_r = \left(\frac{1}{1 + (\alpha_d s)^{n_d}} \right)^{m_d} \quad \text{(Main drying)} \quad (8)$$

308
$$S_r = S_{r0} + (1 - S_{r0}) \left(\frac{1}{1 + (\alpha_w s)^{n_w}} \right)^{m_w} \quad \text{(Scanning wetting)}. \quad (9)$$

309 The scanning wetting curve was modelled by setting a 'residual'
 310 degree of saturation S_{r0} greater than 0. Table 1 summarises the
 311 Van-Genuchten parameters fitted for both the drying and wetting
 312 paths.

313 The difference in parameters used to fit the Van-Genuchten rela-
 314 tionship is evidence of the hysteretic behaviour of the silica sand
 315 investigated. Thus for a given suction, two distinct degrees of satura-
 316 tion could exist dependent upon whether the soil is in a drying or
 317 wetting path.

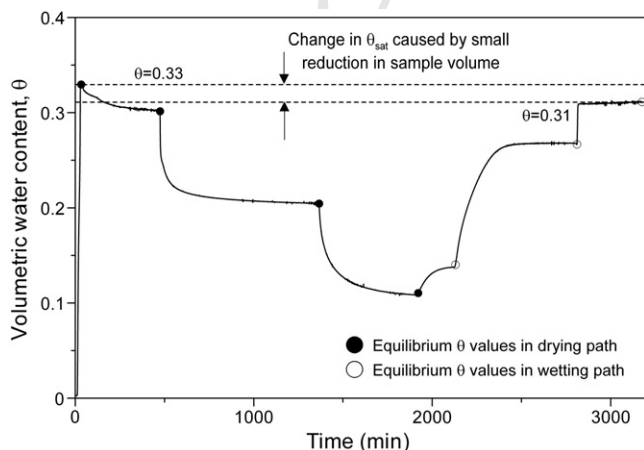


Fig. 3. Equalisation of hydraulic suction during water retention characteristic curve derivation experiment, performed on uniform fine sand.

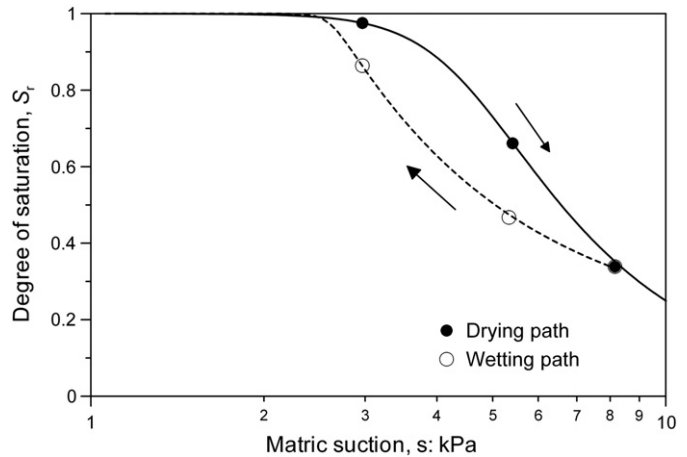


Fig. 4. Water retention curves for drying and wetting paths fitted using the Van-Genuchten relationship.

319 wetting cycle. Thus in relation to shear strength, if the shear strength
 320 of the sand is assumed to be a function of suction multiplied by degree
 321 of saturation, then the soil can exhibit two distinct shear strengths at the
 322 same suction, depending on whether the degree of saturation lies on the
 323 drying or wetting curve. This hypothesis was investigated by performing
 324 simple column collapse tests on samples on both the drying and wetting
 325 paths.

326 3.3. Column collapse tests

327 3.3.1. Apparatus

328 A 100 mm diameter triaxial base and split-form was used in place
 329 of the cylindrical cell to form the samples for the column collapse tests.
 330 A high air entry filter was created in the base of the split-form following
 331 the same method as previously described for the soil water retention
 332 experiment (Figure 5).

333 3.3.2. Experimental procedure

334 Following testing of the air entry filter, oven-dried fine sand was
 335 rained into the split-form to create a sample of uniform density, with
 336 depth of 180 mm and diameter of 100 mm. Next the sample was satu-
 337 rated by raising the water reservoir to provide a small positive head
 338 potential at the surface of the sample. This was followed by either dry-
 339 ing to a target applied suction or drying to an applied suction of 8.2 kPa,
 340 followed by wetting to a target applied suction at the base of the sam-
 341 ple. Due to a ThetaProbe not being placed in-situ in the sample, to
 342 facilitate collapse testing on a virgin sample, it was not possible to
 343 observe constant soil-water content using the ThetaProbe in these ex-
 344 periments. As a result, a period of 24 h was allowed for equalisation of
 345 matric suction following the application of a suction increment either
 346 in drying or wetting. This was seen as a conservative estimate of the
 347 time required for equalisation of suctions within the sample according
 348 to the response observed in the water retention test.

349 After the allowed equalisation period the split-form was removed,
 350 revealing a cylindrical column of sand that could be loaded in com-
 351 pression to failure. Loading was facilitated using a triaxial loading

Table 1

Van-Genuchten parameters fitted for drying and wetting hydro-mechanical behaviours.

Main drying		Scanning wetting	
α_d	0.219	α_w	0.384
n_d	5.81	n_w	29.10
m_d	0.32	m_w	0.046
		S_{r0}	0.15

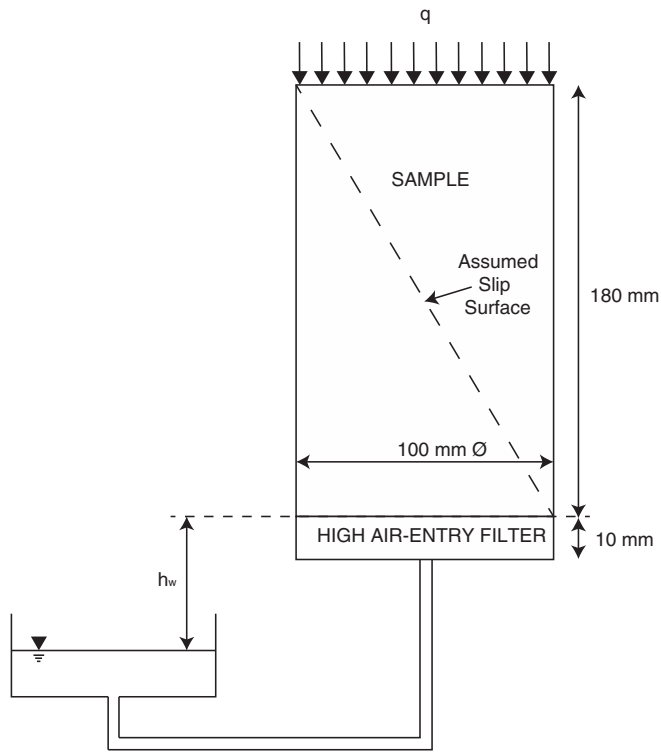


Fig. 5. Schematic of apparatus used to generate collapse in the sand sample.

cap and a plastic hopper, into which small ball bearings were placed until failure. The mass of the loading cap and ball bearings at failure allowed calculation of the failure boundary pressure for the sample.

3.3.3. Experimental results

Collapse boundary pressures are plotted against the suction applied at the base of the sample in Fig. 6. The results clearly show that there is a hysteretic effect and it will later be demonstrated that this is associated with the hysteresis of the water retention curve.

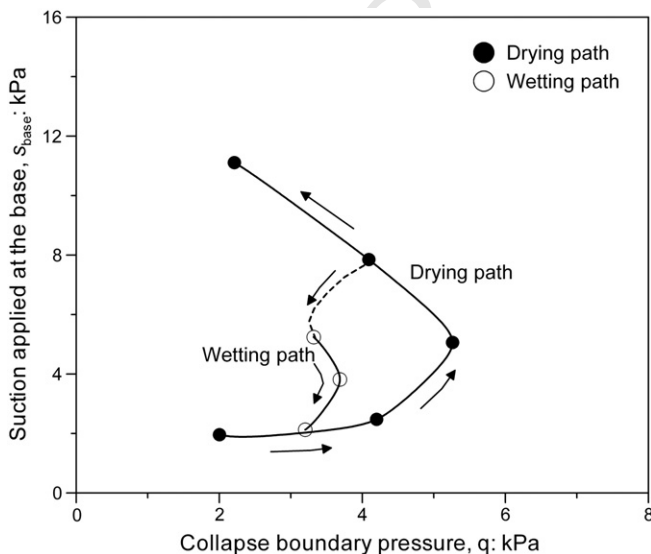


Fig. 6. Collapse boundary stresses measured on samples subjected to drying and wetting hydraulic paths.

3.4. Prediction of the upper and lower bounds of collapse pressure 360

3.4.1. Failure criterion 361

The following failure criterion was adopted for the sand according to Öberg and Sällfors (1997): 362 363

$$\tau = (\sigma + sS_r) \tan \phi' \tag{10}$$

364

The internal angle of friction ϕ' of the fine sand was estimated in a very simple manner using a tilting slide mechanism. Sand was placed in a Perspex slide formed from three pieces of Perspex to make a slide 1 m long and 0.15 m wide with sides to contain the sand and a rough surface along the base. The sand was placed in a uniform thickness of approximately 30 mm depth. The slide was then tilted until movement of the sand was observed; indicating the angle of the slope of the slide had exceeded the angle of friction of the sand material. The slide was then tilted back toward horizontal until the movement of the sand subsided. At which point the angle of the slide was calculated using simple trigonometry; thus giving an estimate of $\phi' = 32^\circ$ for the critical angle of friction for the sand material. This simple method was preferred to the more conventional direct shear or triaxial tests as the low stresses were more representative of those apparent in the experiments presented in the previous section. 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380

3.4.2. Estimating degree of saturation 381

To model shear strength by Eq. (10), the degree of saturation S_r needs to be estimated as a function of suction. For the case where tests were performed along the draining path, the main drying curve given by Eq. (9) was used because points at any elevation in the sample all desaturated from a saturated state (State 0 in Fig. 7): 382 383 384 385 386

For the case where the sample was wetted after being partially dried by lowering the reservoir to H_w^* (see Figure 7), the scanning curve given by Eq. (10) was used. As points at different elevations in the sample had previously been dried to different degrees of saturation, they followed different scanning paths as illustrated in Fig. 7 (hydraulic paths 1–2). The scanning wetting curve was modelled by scaling the wetting curve using the parameter S_{r0} . It can be demonstrated that this parameter can be derived as follows: 387 388 389 390 391 392 393 394

$$S_{r0}(z) = \frac{\left(\frac{1}{1+[\alpha_d \cdot s^*(z)]^{n_d}}\right)^{m_d} - \left(\frac{1}{1+[\alpha_w \cdot s^*(z)]^{n_w}}\right)^{m_w}}{1 - \left(\frac{1}{1+[\alpha_w \cdot s^*(z)]^{n_w}}\right)^{m_w}} \tag{11}$$

where $s^*(z)$ is the suction at the end of the drying process generated by the water level H_w^* as shown in Fig. 7. 395 397

3.4.3. Lower bound solution of collapse boundary pressure 398

To derive the lower bound solution, we assume the axial and radial directions to be principal stress directions. The axial and radial stress, σ_a and σ_r respectively, are therefore given by 399 400 401

$$\begin{aligned} \sigma_r &= 0 \\ \sigma_a &= q + \left[\int_0^z \gamma(z) dz \right] \cdot z \end{aligned} \tag{12}$$

where q is the applied pressure at the top of the sample, z is the depth from the sample top surface, and q is the unit weight. The latter is in turn a function of the degree of saturation: 402 404 405

$$\gamma = \gamma_s(1-n) + \gamma_w S_n \tag{13}$$

where γ_s and γ_w are the specific weights of the solids and water respectively $\gamma_s = 26.7 \text{ kN/m}^3$ and $\gamma_w = 9.81 \text{ kN/m}^3$) and n is the porosity ($n = 0.31$). 406 408 409

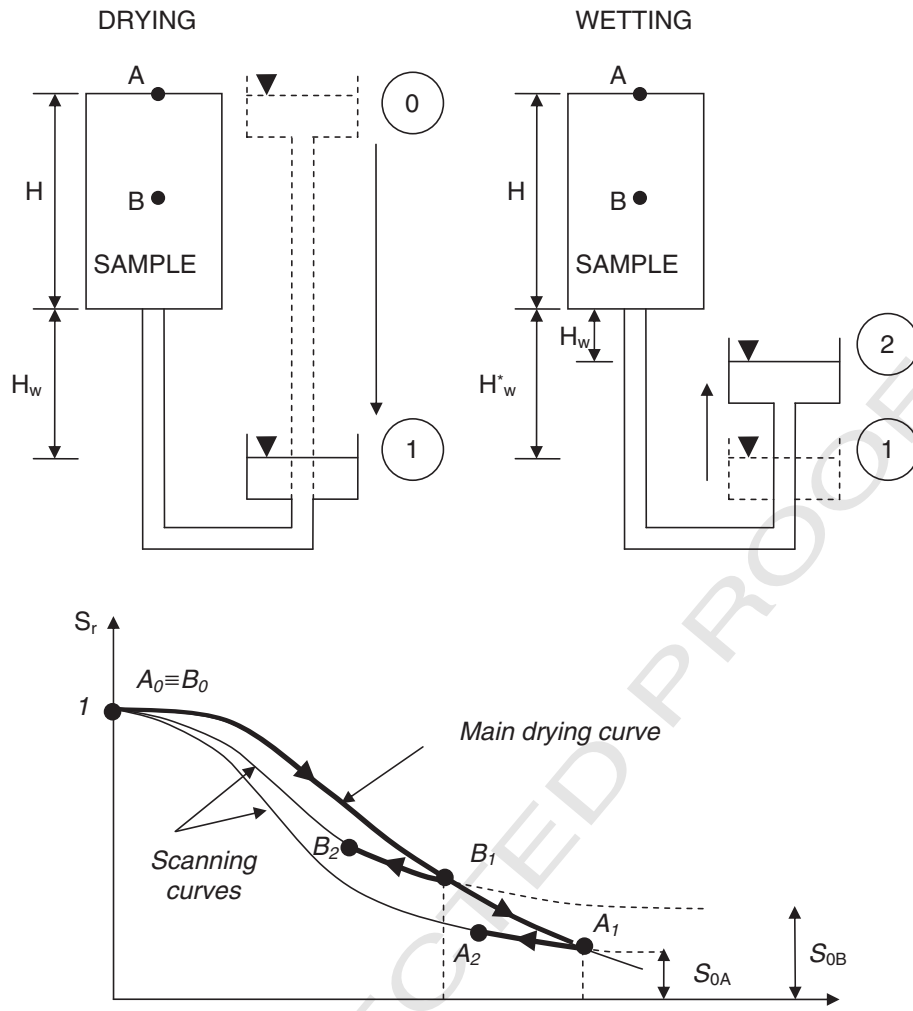


Fig. 7. Hydraulics paths followed at different elevations (e.g. A and B) in the samples during the drying path (0-1) and wetting path (1-2).

410 The lower bound solution for the collapse pressure q is obtained
 411 by imposing that the Mohr circle at the base of the sample is a tangent
 412 to the failure envelope in $\sigma'-\tau$ space as illustrated in Fig. 8:

$$q_l = (k_p - 1) [s(H)S_r(H)] - \int_0^H \gamma(z) dz \quad (14)$$

413 where k_p is the passive earth coefficient.

3.4.4. Upper bound solution of collapse boundary pressure

415 The upper bound solution was derived by considering a single
 416 block mechanism with a planar failure surface formed at an angle β
 417 to the vertical as illustrated in Fig. 9. It was found that the minimum
 418 upper bound value of the collapse pressure is obtained for the angle
 419 generating a failure surface that cuts the cylinder in two halves as
 420 shown in Fig. 9.

421 The upper bound collapse load is obtained by equating the external
 422 work associated with the pressure q and the self-weight W with the internal
 423 work done by shear and normal stresses along the failure surface:
 424

$$\left(W + q \frac{\pi d^2}{4} \right) \delta \cos(\beta + \varphi') = \delta \sin \varphi' \int_L s S_r dL \quad (15)$$

where δ is the displacement of the block, d is the sample diameter, q is
 425 the pressure applied at the boundary, and W is the self-weight of the
 426 sliding block. By rearranging this equation we obtain:
 427
 428

$$q_u = \frac{4}{\pi d^2} \left[\frac{\sin \varphi'}{\cos(\beta + \varphi')} \int_L s S_r dL - W \right] \quad (16)$$

429 with the failure stress q_u , the self-weight W and the integral $\int s S_r dL$ cal-
 430 culated numerically by subdividing the problem vertically into 100 dis-
 431 crete parts. Therefore when calculating the degree of saturation along a
 432 scanning wetting path, 100 different scanning curves were used which
 433 were described by the scaling parameter defined by Eq. (11).
 434

3.5. Discussion

435
 436 The lower and upper bound envelopes for the drying and wetting
 437 paths are shown in Fig. 10 together with the experimental results. The
 438 lower and upper bound solutions appear to bracket the experimental
 439 data showing that the theorems of bound plasticity can adequately
 440 capture the collapse behaviour even for partially saturated soils. Al-
 441 though simplistic, this, to the authors' knowledge, represents the first
 442 validation of limit analysis for partially saturated soils.

443 The lower and upper bound solutions were derived under two as-
 444 sumptions that might seem to be questionable at first glance: (i) an

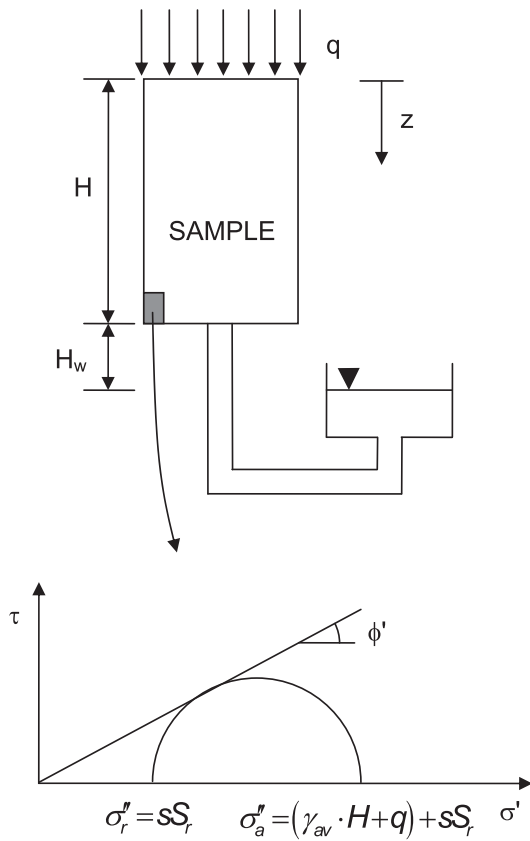


Fig. 8. State of stress to determine a lower bound collapse pressure using the lower bound theorem of plasticity.

445 associative flow, i.e. a dilatancy angle, $\psi = 32^\circ$; and (ii) a friction angle
 446 equal to the critical (ultimate) friction angle, ϕ'_{crit} . The first assump-
 447 tion, although unrealistic, leads to an upper bound solution that
 448 coincides with the solution obtained by the limit equilibrium method,
 449 which is the simplest approach to understand and apply in geotechnical
 450 design. On the other hand, the adoption of the critical friction angle
 451 allows for a conservative estimate of the collapse load that tends to
 452 compensate for the overestimation associated with the associative flow.
 453

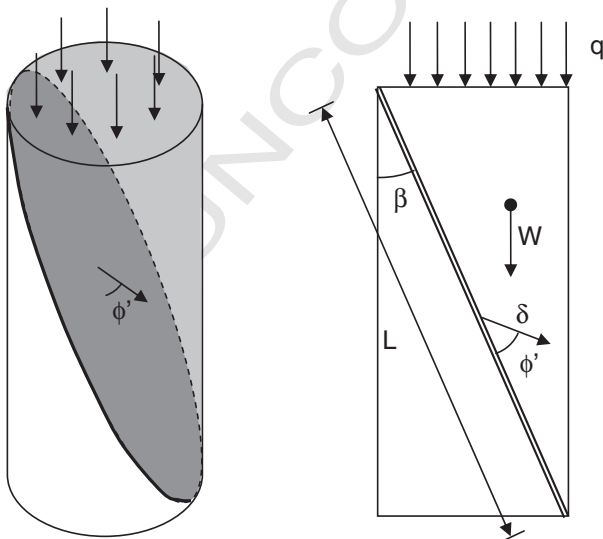


Fig. 9. Kinematic mechanism to determine an upper bound collapse pressure using the upper bound theorem of plasticity.

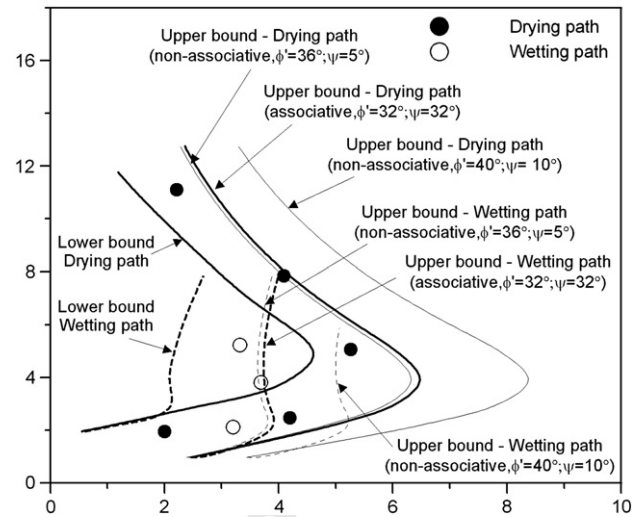


Fig. 10. Collapse boundary stresses predicted using the bound theorems of plasticity and comparison with experimental results.

This is demonstrated by a simple calculation of the upper bound
 collapse load using a non-associated flow rule. Bolton (1986) has
 demonstrated that sands tend to dilate even at relatively low relative
 densities and relatively high mean stresses. A non-zero dilatancy
 leads, in turn, to a peak friction angle that is greater than the
 critical state angle. Bolton proposed a widely used relationship between the
 peak friction angle, ϕ'_{peak} , the critical friction angle ϕ'_{crit} , and the
 dilatancy, ψ :

$$\phi'_{peak} - \phi'_{crit} = 0.8\psi. \tag{17}$$

As an example, for the sand tested in this programme ($\phi'_{crit} = 32^\circ$),
 $\psi = 5^\circ$ generates peak friction angle $\phi'_{peak} = 36^\circ$ and $\psi = 10^\circ$ gener-
 ates peak friction angle $\phi'_{peak} = 40^\circ$.

To estimate the upper bound load for soils with non-associated
 flow rules, Drescher and Detournay (1993) suggested using rigid
 block mechanisms with reduced discontinuity strength. This reduced
 strength is a function of the friction and dilatancy angle, ϕ'^* and fol-
 lows the formula derived by Davis (1968):

$$\tan\phi'^* = \frac{\cos\psi \cdot \sin\phi'}{1 - \sin\psi \cdot \sin\phi'} \tag{18}$$

where ϕ'^* and ψ are the friction and dilation angles respectively.

To appreciate the role of dilatancy, the upper bound collapse load
 was calculated using Eq. (18) for two values of dilatancy angles, $\psi = 5^\circ$
 and $\psi = 10^\circ$, and corresponding values of peak friction angle $\phi'_{peak} =$
 36° and $\phi'_{peak} = 40^\circ$ respectively. The results from this analysis are
 shown in Fig. 10 where it can be seen that the non-associated solution
 for a small value of the dilatancy angle ($\psi = 5^\circ$) is very similar to the
 one obtained by assuming associative flow using the critical friction
 angle ($\phi'_{crit} = 32^\circ$ and $\psi = 32^\circ$) and the solution obtained for higher
 dilatancy angle ($\phi'_{peak} = 40^\circ$ and $\psi = 10^\circ$ in Figure 10) leads to a sig-
 nificant overestimation of the collapse load.

This demonstrates that the classical upper bound solution based
 on associative flow and friction angle equal to the critical one, widely
 used in geotechnical design even if disguised in the form of the limit
 equilibrium method, is acceptable for engineering purposes.

4. Case study: Pozzolan Quarry

A demonstration of the application of this approach is to consider
 the maximum unsupported vertical cut height in a cohesionless soil
 in the field. De Vita et al. (2008) described vertical cuts up to 15 m

high in a pyroclastic Pozzolan deposit in a quarry in the Campi Flegrei area near Naples in Italy with a water table depth of a few tens of metres. The authors were successful in capturing the correct order of magnitude of the critical height but they used a rather simplistic approach that would be problematic to use in engineering practice. They estimated the contribution of suction to shear strength using a linear relationship (Fredlund et al., 1978), which is conceptually and experimentally incorrect since the failure envelope with respect to suction has been demonstrated to be markedly non-linear (Escario and Sáez, 1986). They also assumed a constant suction throughout the excavation, which is inadmissible since the suction varies with depth in a profile that depends on the groundwater table level and the hydraulic boundary condition at the ground surface.

By using the approach proposed and validated in the previous section, a more accurate estimate can be attained, which accounts for the depth of the water table and incorporates a more realistic shear strength criterion.

4.1. Mechanical and hydraulic characteristics of pozzolan deposit

The Pozzolan soil relevant to this case study was investigated by De Vita et al. (2008) by means of 7 samples labelled C1 to C7. The material is characterised by a field porosity n in the range 0.54–0.68 and a specific unit weight of the solids γ_s in the range of 23.6–25.2 kN/m³ (average values of n of 0.63 and γ_s of 24.4 kN/m³ are used in these calculations). The grain size distribution is characterised by a silt fraction in the range 0.32–0.50, a sand fraction in the range 0.45–0.52 and the absence of any clay fraction.

Water retention characteristics of the soil were investigated using a conventional tensiometer by De Vita et al. (2008). Water retention data is shown in Fig. 11a together with the van Genuchten function (Eq. (8)) which was optimised to fit the experimental data.

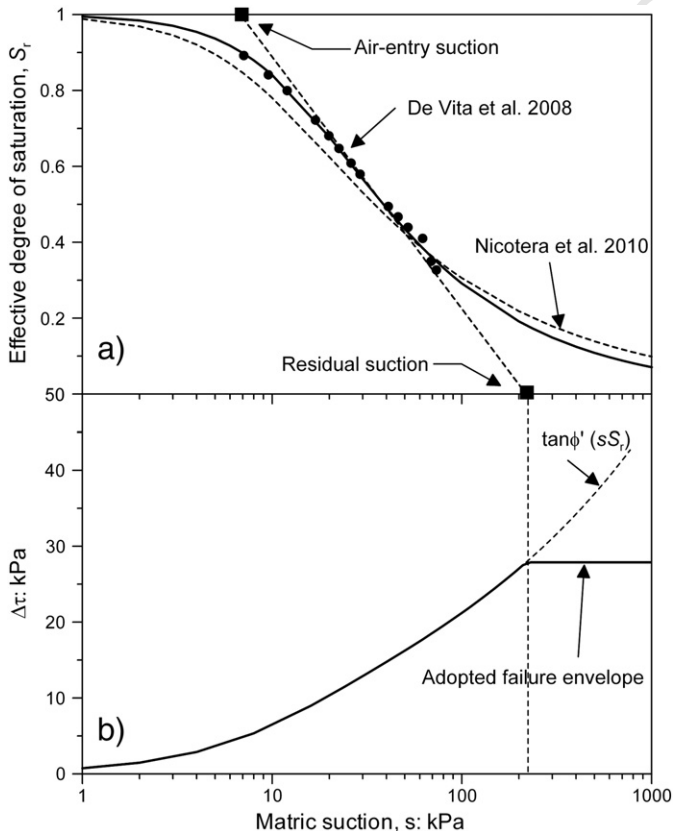


Fig. 11. Water retention curve (a) and shear strength criterion (b) for the pozzolan soil in Campi Flegrei near Naples.

De Vita et al. (2008) did not carry out suction-controlled or suction-monitored tests on the pozzolan pyroclastic soils. However, shear strength of a very similar pyroclastic soil was investigated by Papa et al. (2008), whose water retention curve (Nicotera et al., 2010) is compared with the one obtained by De Vita et al. (2008) in Fig. 11a. Papa et al. (2008) observed that Eq. (4) models the experimental data in the range of suction 0–20 kPa very well with a friction angle $\phi' = 36.9^\circ$.

When the shear strength criterion given by Eq. (4) is extrapolated at high suctions, it is found that the contribution of suction to shear strength, $\Delta\tau = s \cdot S_r \cdot \tan\phi'$, indefinitely increases with suction (Figure 11b), which is not intuitively acceptable. Eq. (4) is physically based on the effects of bulk water on the soil skeleton and can be anticipated to fail when pore-water is predominantly present in the form of menisci or adsorbed water as occurs at high suctions. As a first approximation, the residual suction shown in Fig. 11a may be assumed to delimit the range of menisci/bulk water and, hence, to limit the validity of Eq. (4). This assumption seems to be corroborated by Cattoni et al. (2007), demonstrating that Eq. (4) holds in the range of suctions bounded by the residual suction.

Accordingly, the contribution of suction to shear strength was assumed to become constant as the residual suction is exceeded as illustrated in Fig. 11b.

4.2. Stability of a vertical cut in pozzolan deposit

To derive an upper bound of the critical height H , the simplest kinematic mechanism was considered, which consisted of a single block with a planar slip surface inclined by the angle α as shown in shown in Fig. 12a. Considering that the unit weight γ of the soil is given by:

$$\gamma = (1-n)\gamma_s + n \cdot S_r \gamma_w \tag{19}$$

where n is the porosity, γ_s and γ_w are the specific unit weights of the solid particles and water respectively and S_r is the degree of saturation. The weight W of the block can be calculated as follows:

$$W = \gamma_s(1-n) \tan\alpha \frac{H^2}{2} + n\gamma_w \tan\alpha \int_H^0 S_r(z)(H-z)dz \tag{20}$$

hence, the work done by the external forces W_e is equal to:

$$W_e = \left[\gamma_s(1-n) \tan\alpha \frac{H^2}{2} + n\gamma_w \tan\alpha \int_H^0 S_r(z)(H-z)dz \right] \delta \cos(\alpha + \phi') \tag{21}$$

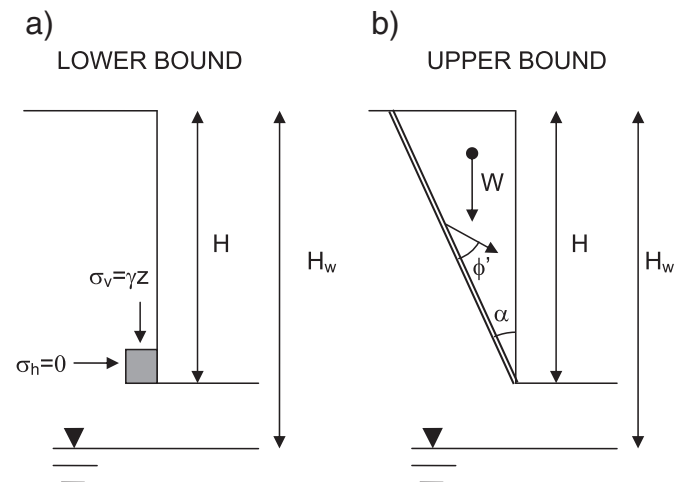


Fig. 12. (a) State of stress adopted to derive a lower bound solution and (b) kinematic mechanism to derive an upper bound solution for the critical height of a cut slope.

557 On the other hand, the internal energy dissipation W_i is given by:

$$W_i = \delta \frac{\sin \phi'}{\cos \alpha} \int_H^s s(z) S_r(z) dz. \quad (22)$$

558 An upper bound solution for the critical height can then be
 560 obtained by equating W_e with W_i . It can then be demonstrated (see
 561 for example Stanier and Tarantino, 2010) that the minimum 'upper
 562 bound' value is obtained for:
 563

$$\alpha = \frac{\pi}{4} - \frac{\phi'}{2}. \quad (23)$$

564 To derive a lower bound value for the critical height, we assume
 566 the vertical and horizontal directions to be principal directions of
 567 stress. Accordingly, the equilibrium stress state is given by:
 568

$$\begin{cases} \sigma_z = \gamma z \\ \sigma_x = 0 \end{cases} \quad (24)$$

569 where σ_z and σ_x are the vertical and horizontal stresses. Based on the
 571 shear strength criterion given by Eq. (4) a lower bound can be obtained
 572 by imposing that the Mohr stress circle in the " $\sigma + sS_r, \tau$ " plane relative
 573 to a point at the base of the excavation (Figure 12b) is a tangent to the
 574 failure envelope:

$$s(H) \cdot S_r(H) = k_a \{ [(1-n)\gamma_s + n \cdot S_r(H) \cdot \gamma_w] H + s(H) S_r(H) \} \quad (25)$$

576 where k_a is the active earth coefficient, s is the suction that is a function
 577 of H , and S_r is the degree of saturation that is in turn a function of suction
 578 s .

579 The lower and upper bound solutions obtained by assuming a
 580 hydrostatic suction profile are plotted in Fig. 13 as a function of the
 581 depth H_w of the water table. As expected, the critical height increases
 582 with the depth of the groundwater table, although the effect becomes
 583 less and less important at large values of the groundwater table
 584 depth. Fig. 13 shows that the calculated critical height is significant
 585 in this cohesionless material and this is in general agreement with
 586 field observations (see De Vita et al., 2008).

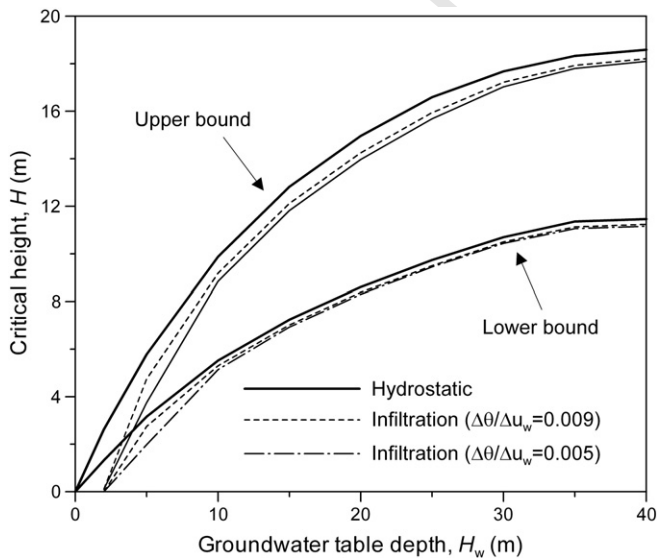


Fig. 13. Upper and lower bound solutions of critical height in pozzolan pyroclastic soil as a function of groundwater table depth assuming an hydrostatic suction profile and suction profiles generated by ponded infiltration.

4.3. Effect of rainfall on suction profile

587

588 Stability of vertical cuts in cohesionless soils has been demonstrated
 589 to rely on matric suction. However, its effect could partially or totally
 590 vanish when rainwater infiltrates at the ground surface. For engineering
 591 purposes, it becomes crucial to assess the potential impact of rainfall
 592 on matric suction and, hence, on vertical cut stability. A very simple
 593 method that allows preliminary assessment of this risk of vertical cut
 594 collapse is proposed here that is based upon solutions available in clas-
 595 sical geotechnical textbooks focusing on saturated soils. The principal
 596 advantage of the method is that it can be used by practitioners lacking
 597 specific expertise in modelling water flow in unsaturated soils above
 598 the water table.

599 As a simplification the water flow equation may be linearised by
 600 assuming that hydraulic conductivity is constant and the water reten-
 601 tion curve is linear. For conservatism the hydraulic conductivity is
 602 assumed to be equal to the saturated value ensuring a maximal infil-
 603 tration rate and, hence, the highest reduction in suction and shear
 604 strength. Under these circumstances the water flow equation becom-
 605 es (Tarantino et al., 2010):

$$\left(\frac{k_{sat}}{\gamma_w \frac{\Delta \theta}{\Delta u_w}} \right) \frac{\partial^2 u_w}{\partial x^2} = \frac{\partial u_w}{\partial t} \quad (26)$$

606 where u_w is the pore-water pressure, t is the time, z is the vertical
 607 coordinate, k_{sat} is the saturated hydraulic conductivity, γ_w is the unit
 608 weight of water, and $\Delta \theta / \Delta u_w$ is the slope of the linearised water reten-
 609 tion curve. The water retention curve is highly non-linear and we sug-
 610 gest two possible linearisations in Fig. 14. It will be demonstrated later
 611 that these linearisations are essentially equivalent for the purpose of
 612 estimating suction profiles following rainfall.

613 Let us assume that the initial condition for pore-water pressure is
 614 hydrostatic and controlled by the groundwater table located at the
 615 depth H_w from the ground surface. This is a conservative assumption
 616 as evapotranspiration at the ground surface would generate suctions
 617 higher than those associated with hydrostatic conditions. To simulate
 618 infiltrating rainwater, the hydraulic boundary condition at the ground
 619 surface should be represented by an inward flux. For conservatism it
 620 is assumed that a pond immediately forms at the ground surface and
 621 that the hydraulic boundary condition is represented by zero pore-water
 622 pressure at the ground surface (i.e. ponded infiltration). This is the most
 623 conservative assumption as it returns the maximum possible infiltration
 624 and, hence, the highest reduction in suction. Therefore, the groundwater
 625 table and the ponded surface infiltration represent the hydraulic bound-
 626 ary conditions at the bottom and top of the flow domain.
 627

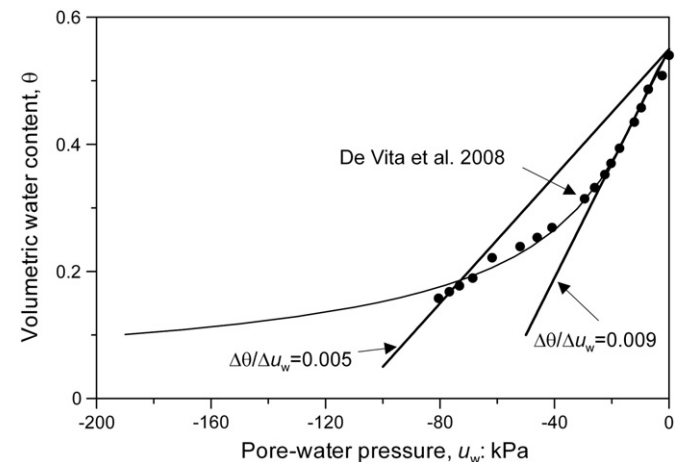


Fig. 14. Linearisation of the water retention curve for the pozzolan pyroclastic soil.

628 With these initial and boundary conditions, the problem reduces
629 to the classical Terzaghi consolidation problem with triangular excess
630 pore-water pressure and double-drainage. The solution of this problem
631 is widely found in classical geotechnical textbooks, often in graphical
632 form (e.g. [Lambe and Whitman, 1969](#)), and can therefore be exploited
633 by engineers with no specific background in unsaturated soil mechanics.
634 The solution is given by:

$$u(z, t) = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} \left\{ 1 - \left(\frac{2H}{n\pi} \right) \sin(n\pi) \right\} \sin \frac{n\pi z}{2H} \exp \left(-\frac{n^2 \pi^2 T}{4} \right) \quad (27)$$

636 where u_0 is the initial excess pore-water pressure at the ground surface
637 and T is the time factor given by:

$$T = \left(\frac{k_{sat}}{\gamma_w \frac{\Delta\theta}{\Delta u_w}} \right) \frac{1}{H^2} \cdot t \quad (28)$$

638 where H is the drainage length. By assuming $k_{sat} = 7 \cdot 10^{-7}$ m/s
639 ([Nicotera et al., 2010](#)), $\Delta\theta/\Delta u_w$ is equal to 0.005 or 0.009 (see [Figure 14](#)),
640 and a rainfall duration of 2 days, we can derive the pore-water pressure
641 profiles as shown in [Fig. 15](#) for different water table depths H_w . It appears
642 that rainfall only affects a shallow portion of the ground and its effects
643 become less and less important as the depth of the groundwater table
644 increases. If the lower and upper bound solutions are calculated by con-
645 sidering the pore-water pressure profiles after two days (ponded) infil-
646 tration, the values shown in [Fig. 13](#) are obtained.

648 It appears that there is not a significant difference between the
649 values derived under the assumption of hydrostatic pore-water pressure
650 profile and either of the ponded surface infiltration solutions. Hence, the
651 choice regarding the slope of the 'linearised' water retention curve is not
652 overly critical. In conclusion therefore, rainfall does not seem to jeopardise
653 slope stability, which compliments field observations ([De Vita et al., 2008](#)).
654 It should be stressed again that the analysis of the effect of rain-
655 water is definitively conservative since saturated hydraulic conductivity
656 and ponded infiltration was considered.

657 5. Conclusions

658 An extension to the classical limit analysis has been proposed to
659 allow assessment of the stability of excavations above the water table
660 in cohesionless (granular) soils, which accounts for the beneficial effect

of suction and partial saturation on shear strength. A modified shear
strength criterion has been incorporated into the traditional bound
theorems of plasticity approach using a relationship relating shear
strength to the product of suction, s , and saturation ratio, S_r . This has
facilitated analysis of the stability of vertical cuts in cohesionless soils
above the water table.

To assess the validity of this extension, simple small-scale column
collapse tests were performed using fine silica sand for which the
water retention characteristics were derived using a negative-water
column approach. The column collapse tests allowed assessment of the
failure boundary pressure of the column for a given boundary suction
applied to the base of the sample. Upper and lower bound solutions
were derived for this boundary value problem, generating failure
bounds that bracketed the experimental results reasonably well. To
the authors' knowledge, this represents the first experimentally vali-
dated appraisal of the application of the bound theorems of plasticity
to problems involving cohesionless soil above the water table.

The impact on practice of the findings of the laboratory validation
tests was then explored using a case study, focussing on the vertical
cut height observed in pyroclastic Pozzolan deposits near Naples,
Italy. This problem has previously been addressed by [De Vita et al. \(2008\)](#)
by introducing, however, several oversimplifications (constant
matric suction within the excavation and linear 'unsaturated' failure
envelope) that were removed in this paper. The upper and lower bounds
for the safe vertical cut height were calculated accounting for varying
suction, s , and saturation ratio, S_r , within the deposit and a non-linear
failure envelope. These were solved using numerical integration and
the calculated failure heights indicated good agreement with field
observations of stable vertical cuts in pyroclastic Pozzolan deposits.

The impact of rainfall on infiltration and vertical cut stability was
then explored. Simplifying the scenario of rainfall to a case with ponded
infiltration and maximum (saturated) hydraulic conductivity, a conserva-
tive appraisal of vertical cut stability was generated using the classical
Terzaghi consolidation solution for double drainage and a triangular
excess pore pressure distribution. The impact of 2 days of constant
rainfall causing ponded infiltration has been demonstrated to minimally
impact upon the vertical cut stability in Pozzolan soil. This would ex-
plain the long-term stability of the large vertical cuts (tens of metres)
observed in Pozzolan deposits in the field ([De Vita et al., 2008](#)).

The findings of this paper present and validate an approach to
assessing the stability of vertical cuts in cohesionless soils that are
based principally upon methods taught in most undergraduate Civil
Engineering courses and that require little specialist knowledge. Hence,
it is envisaged that these techniques may be used in the future by prac-
tising engineers, to rationalise the often unexplained non-zero vertical
cut height observed in cohesionless soils above the water table, for
which classical soil mechanics theory offers no rational explanation.

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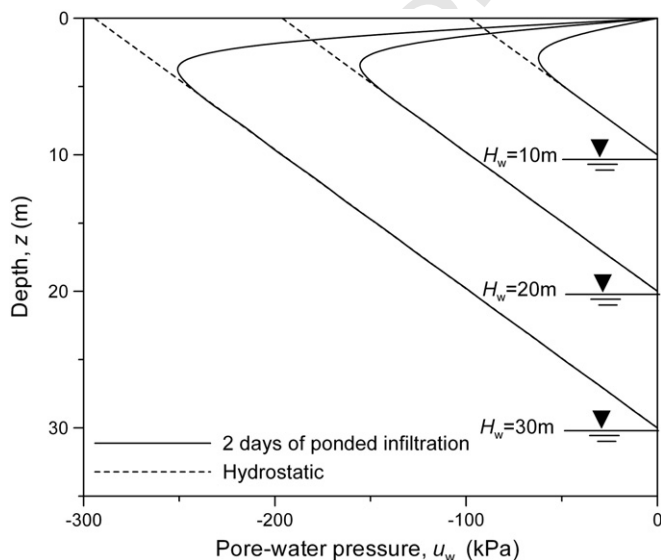


Fig. 15. Suction profiles after two days of ponded infiltration for water table depths of 10, 20, and 30 m.

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