The 4th Canadian Conference on Nonlinear Solid Mechanics (CanCNSM 2013)
McGill University, July 23-26, 2013
Montréal, Canada

(Extended abstract) Paper ID 796

The Role of Uncertainty in Creep Mechanics

James BOYLE, Department of Mechanical & Aerospace Engineering, University of Strathclyde, Glasgow, Scotland jim.boyle@strath.ac.uk

The development of constitutive models for the analysis of the creep (and fatigue) of structures at high temperature has a long history. Nevertheless, with a few exceptions, it is common in engineering practice to use relatively simple models, typically based on time- or strain-hardening coupled with a simple power-law derived from steady-state behavior. The use of simple constitutive models is usually adopted even when advanced finite element simulation of complex structures is undertaken. An inherent feature of the use of such simple models is that the presence of scatter in the original creep data is ignored – the parameters in the material models are obtained from a 'best-fit' to the raw scattered data. In practice creep design is often carried through using 'worst-case' property values, although these can also be difficult to accurately define. This is quite problematic for design: the assessment of the stress, strain, and possibly failure, of complex structures at high temperature must then be carried out in the context of major uncertainties about the fundamental materials modeling – and usually by analysts whose experience of stress analysis could be principally based on low temperature behavior where elastic properties such as Young's Modulus and, to some extent, time independent inelastic properties such as uniaxial yield stress and post-yield hardening are fairly certain, showing little variation. This situation in high temperature design leads to the identification of both types of known uncertainties in creep modeling and analysis: *aleatory* – scatter and randomness, that is stochastic uncertainty, and *epistemic* – lack of knowledge. The former has been studied in the literature, but to no great extent and has scarcely been embraced in design with the exception of the estimation of creep lifetime. The latter, to the writer's knowledge, has hardly been studied for high temperature design. Epistemic uncertainty in the present context extends not only to lack of experience/knowledge on the part of the designer or analyst, but also to the use of simplified constitutive models based on limited, and scattered, creep tests. The latter can be reduced, for example, by using more detailed material models and conducting more tests. A significant feature of epistemic uncertainty is that much of what we do know about creep scatter has been gained from carefully conducted laboratory tests: how this relates to real components under actual service conditions is largely unknown. The aim of this paper is to review available work on aleatory uncertainty in creep mechanics related to the effect of scatter in stress analysis, to examine the consequences for modeling and design and to propose a way forward.

A Brief Overview Despite numerous studies of scatter in creep rupture data, and to a lesser extent in standard creep curves from tensile testing, studies of the possible effects of scatter in creep data on the prediction of stress and strain levels in components under creep are rare. Indeed, to the author's knowledge, only *three* determined attempts have been made to evaluate the effect of random material parameters on stress analysis of components under creep:

Cozzarelli & Huang studied the steady state creep of a three-bar truss [1] and a beam in bending [2] using a power-law creep law $\dot{\varepsilon}_c = B\sigma^n$ where σ is stress, $\dot{\varepsilon}_c$ is the strain rate and *B*, *n* material parameters. In the case of the three-bar structure it was assumed that *B* and *n* were random parameters which were decoupled, with the former due to random temperature fluctuations and the latter due to random material imperfections. Both were assumed to have lognormal distributions based on data available in the literature. In the analysis these random parameters were in fact random functions of distance along each bar so the resulting solutions for stress and strain rate were random variables. However, in the case of the beam in bending the material parameters *B* and *n* were random processes dependent upon distance through the beam section; the resulting stress and displacement rate were similarly random processes. In both the three-bar truss and beam in bending the derived stress values in the structures showed very little random fluctuation, whereas the displacement rate was highly random: in the case of the beam in bending this randomness came both from variations in the material parameter *n* but also in the spatial variation of the material parameters.

Westlund and Broberg also considered the effect of random material parameters on simple structures: in [3] a simple hyper-static two-bar structure and a pressurized thick-walled cylinder were analyzed, while in [4] a beam in bending was considered. Again steady-state creep was considered; in [3] a modified power-law creep was adopted in the form $\dot{\varepsilon}_c = B(1+\alpha H(x))\sigma^n$ where $\alpha H(x)$ was considered to be a normally distributed ergodic stochastic process of Markov type; *x* is some geometric co-ordinate (distance along a bar, or through the cylinder thickness). This assumes that scatter is originates from statistical variations in *B* only, while *n* is constant; it was further assumed that the random variation was in one principal direction only in the case of the pressurized cylinder. In [4], for the beam in bending, the statistical variation in *B* included all principal directions. While the statistical variations were considered to be quite simple in these studies, the writers established that the statistics of the derived stress and deformation rates had an obvious structural and size effect, that is the statistical variation in the derived results for each structure analyzed decreased as the component size or structural redundancy increased. This could have been a significant finding, especially for design of real components, but was never taken further.

Unlike previous studies, Harlow & Delph, [5,6], based on evidence in the literature, assumed that the material parameters in the power law were simple random parameters, but highly correlated requiring a joint probability distribution, with *B* log-normal and *n* normal with a joint distribution function log-normal/normal. The material parameters were also assumed to be spatially invariant with the observation that standard tensile creep curves were reasonably deterministic for a single specimen with a random variation from specimen to specimen; however it was recognized that this may not be a reasonable assumption for real large-scale components. They used a novel probabilistic finite element method [5] to analyze a cantilevered beam under constant uniform load: the resulting tip deflection rate could vary by a factor of 5 with a highly skewed probability distribution function. At the time these computations were found to be numerically intensive and not suitable for design.

Discussion In summary, our understanding of the effect of material scatter on the creep of structures is *very* limited. Yet, the studies described above have generally yielded quite significant results. None have really been followed up, nor have the implications for design been investigated. One reason for this was highlighted by Harlow & Delph [5]. Even though the Stochastic Finite Element Method has been reaching maturity, and has been developed for inelastic material behavior (see for example Sett [7] among others), there appear to be no published studies on highly nonlinear, time-dependent material behavior such as creep. Harlow & Delph argue that "… in any case, any such application would be somewhat academic, because experimental data relating to spatial variation in creep properties are almost completely lacking …" further "… *the primary difficulty in making probabilistic failure predictions … does not lie in analysis techniques, but rather in the paucity of experimental data upon which to base the calculations* …" and finally "… analytical predictions in this area are only as good as the data upon which they are based …". In other words, the problem is epistemic uncertainty rather than aleatory.

So, is there a way forward for the treatment of scatter in high temperature design of real components? Is it possible to get some indication of the likely variation in computed results from stress analysis where random material behavior is expected? In the UK an assessment approach to the structural integrity of components under creep conditions has been developed based on the so-called 'reference stress' approach. In this method some characteristic deformation rate, $\dot{q}$, of a creeping structure is expressed in the form $\dot{q} = \delta \times \dot{\varepsilon}_c(\sigma_R)$ where δ is some 'scaling factor' and σ_R is the 'reference stress' while $\dot{\varepsilon}_c(\sigma_R)$ is the result of a creep test held at the reference stress. In general both the scaling factor and reference stress are *independent* of the creep law used; of course this is an approximation, but good enough for design purposes. For a power law it has been found that the scaling factor and reference stress can be well estimated from limit analysis. However the author [8] has demonstrated a method in which the reference stress is obtained by making the scaling factor virtually independent of random variations in the material parameters by minimizing its variance: several creep analyses are required – if there are *M* independent material parameters then 2^M analyses are required. The method was demonstrated for a simple bar structure, beam in bending and a pressurized cylinder. Thus, if the results of a creep stress analysis are expressed in the reference stress form, using scaling factor minimum variance, then theoretically the likely variation in the characteristic deformation rate can be estimated from replicated tests at a single stress level, or a more focused statistical analysis of available creep data. At the time, this work was not continued but has recently been adopted by some companies involved in high temperature design. This paper will elaborate on the method and provide the results of some new finite element creep analyses on more complex structures.

Acknowledgments
The new work to be described in this paper has been carried out under commercial confidentiality. The results are being cleared for presentation at the conference. The author would like to dedicate this paper to the late Professor Jan Hult, of Chalmers University of Technology, Gothenburg, Sweden, who first prompted him to consider the important role of uncertainty in creep while working in the Solid Mechanics Group in Chalmers with Rolf Westlund and Hans Broberg.

References
[1] F. A. Cozzarelli and W. N. Huang. Effect of random material parameters on nonlinear steady creep solutions. *Int. J. Solids Structures*, 7: 1477-1494,1971.

[2] W. N. Huang and F. A. Cozzarelli. Steady creep bending in a beam with random parameters. J. Franklin Inst., 294(5):323-337,1972.

[3] H. Broberg and R. Westlund. Creep in structures with random material properties. *Int. J. Solids Structures*, 14: 365-374,1978.

[4] R. Westlund. Properties of a random creep process. *Int. J. Solids Structures*, 18(4): 275-283,1982.

[5] T. J. Delph and J. E. Yukich. A finite element method for probabilistic creep of solids. *Int. J. Num. Meth. in Eng*., 35:1171-1182,1992.

[6] D. G. Harlow and T. J. Delph. Creep deformation and failure: effects of randomness and scatter. *Trans ASME, J. Eng. Matl. Tech.*, 122:342-347,2000

[7] K. Sett. *Probabilistic Elasto–Plasticity and Its Application in Finite Element Simulations of Stochastic Elastic–Plastic Boundary Value Problems*. PhD thesis, UC Davis, 2007

[8] J. T. Boyle. Approximations in the reference stress method for creep design. *Int. J. Mech. Sci.*, 22:73-82, 1980