Optomechanical self-organization in cold atomic gases

(Invited Paper)

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***Abstract*—We discuss the formation of optomechanical structures from the interaction between linear dielectric scatterers and a light field via dipole forces without the need for optical nonlinearities. The experiment uses a high density sample of Rb atoms in a single mirror feedback geometry. We observe hexagonal structures in the light field and a complementary honeycomb pattern in the atomic density. Different theoretical approaches are discussed assuming either viscous damping of the atomic velocity or not. The interplay between electronic and optomechanical nonlinearities is analyzed. A prediction for dissipative light - matter density solitons is given. The investigations demonstrate novel prospects for the manipulation of matter in a pattern forming system in which quantum effects should be accessible.**

I. Introduction

Spontaneous self-organization of coupled light-matter systems due to optomechanical effects attracted a lot of attention in recent years [1]–[10]. Typically, in these systems a pump field interacts with a cloud of cold atoms or a quantum degenerate state and an additional field arises spontaneously along the same or another axis. The interference of these two fields generates a light pattern which causes a bunching of the cold atoms due to dipole forces. The resulting density grating scatters then pump light into the instability mode, which sustains the instability. The emerging field is coherent, i.e. these instabilities can be understood as a kind of unconventional lasing where the gain arises from four-wave mixing and the 'mirrors' stem from distributed feedback from self-pumped [5], [6] or externally induced [11] density modulations. In the simple one-dimensional (1D) case where pump and emerging field are co-propagating this was coined collective atomic recoil lasing (CARL, [1], [5], [6]).

Another intriguing aspect is spontaneous symmetry breaking which plays a key role in our understanding of nature and self-organization. The spontaneously emerging field in [3], [4], [7], [10] can have two possible phases which correspond to two possible spatial phases of the density grating which the system can choose spontaneously. However, in these experiments the length scales (typically on the wavelength scale) and the symmetry of the density patterns are completely determined by the angle between the pumping field and another distinguished axis along which the spontaneously generated field is scattered. This second imposed axis might be a cavity axis [3], [4], [7], [10], the long axis of an elongated condensate [2], [9] or a seed beam [8]. In contrast, we are reporting on coupled density and optical structures arising in the plane orthogonal (transverse) to a single pump axis. In this scheme two continuous symmetries (rotations and translations in the plane) are spontaneously broken and the length scales of the resulting transverse structures (or the angle the instability sidebands enclose with the pump axis) are resulting from longitudinal interference conditions only. Corresponding structures are known from hot atoms interacting with light [12]–[15] and are just one example for spontaneous self-organization as it is ubiquitous in systems driven out of thermal equilibrium in nonlinear science, technology and nature.

In ensembles of hot atoms the dipole forces arising from the modulated light field have a negligible influence on the center-of-mass motion of the atoms and the resulting gratings giving feedback are spatial modulations in the internal degrees of freedom of the atoms like electronic states or Zeeman-substates. However, for cold atoms optomechanical effects are significant and an optomechanical nonlinearity can arise even if the atoms can be considered as linear Rayleigh scatterers. For a light field tuned to the blue (higher frequency) side of the resonance, atoms are low-field seekers and are expelled out of intensity maxima. As the refractive index seen by a blue detuned field is negative, a reduction of atomic density implies an increase of refractive index and the resulting nonlinearity is self-focusing. In a first approximation, one can think of a Kerr-like index, $n(I) = n_0 + n_2 I$ (n_0 background index of homogeneous medium without light, I intensity, $n_2 > 0$ nonlinear coefficient), but one needs to keep in mind that the nonlinearity is non-local in principle and mediated by transport processes of matter. For negative detuning, the nonlinearity is also-self-focusing, but matter is attracted to intensity maxima. The latter case is well known for synthetic optical nonlinearities in colloidal suspensions of dielectric beads [16], [17]. The case of negative polarizability corresponding to positive detuning in atoms was realized only very recently in soft matter systems [18]. It should be noted that dielectric beads in suspension will always show a strong viscous coupling to the environment whereas this coupling can be suppressed or controlled in ensembles of cold atoms.

Theoretically, optomechanically induced density gratings in a 2D plane orthogonal to a single pump axis were considered in different geometries: single-pass propagation [19], counter-propagating beams [20] and ring cavities [21]. First indications for transverse structure in a low aspect ratio situation

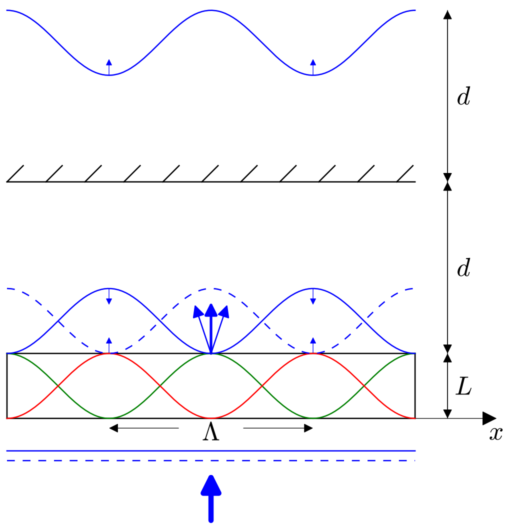

Fig. 1. Scheme of experimental setup and mechanism of instability. Red: density profile, green: refractive index profile, blue: intensity profile, dashed-blue phase-profile, blue arrows: propagation direction of light. At a distance $2d$ after the medium, the feedback beam reentering the medium is reproduced and can be detected with a camera.

were observed in counterpropagating beams in an elongated cloud [8]. In our experiment we employ a simple feedback scheme [22]–[24] based on the retro-reflection of the laser beam passing through the atoms by a high-reflectivity plane mirror at a distance d after the center of the medium (Fig. 1). In its simplest case, the medium is assumed that be thin enough that diffraction in the medium can be neglected compared to the diffraction taking place in the feedback loop. Imagine now a periodic perturbation of the atomic density leading to a corresponding (but spatially anti-phase for positive detuning) fluctuation of the refractive index. As the input beam passes through the medium, the fluctuation in the index of refraction induces phase fluctuations. After reflection by the mirror and propagation back to the sample, these phase fluctuations have turned to intensity fluctuations due to diffraction. This in turn will change the density of the medium via dipole forces, closing the feedback loop and leading to the growth of a macroscopic modulation or pattern out of infinitesimally small fluctuations. The length scale of the instability is given by the Talbot effect [24], [25]: A spatial sideband propagating at an angle Θ to the pump axis has a transverse wavenumber $q = \Theta k$ where $k = 2\pi/\lambda$ is the wavenumber of the light. It acquires a phase shift of $\exp\left(-iq^2/(2k)\delta z\right)$ with respect to the on-axis wave over a propagation distance δz. At the distance where the phasor has the value of i or $-i$ an initial phase perturbation (with a phase of i between on-axis waves and sidebands) will have been converted to an amplitude modulation either spatially in phase or in anti-phase with the original phase modulation. Positive feedback gives then a length scale $\Lambda = 2\pi/q$ of the emerging pattern of

$$\Lambda \approx \sqrt{4d\lambda}, \tag{1}$$

for a self-focusing medium.

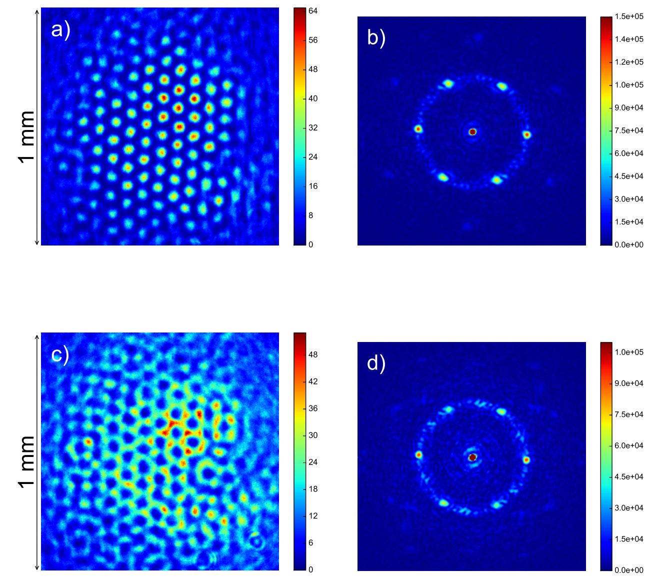

Fig. 2. Typical hexagonal patterns observed in the transmitted beam: Re-imaged (10 mm after cloud) near field intensity distribution of transmitted pump (a) and probe (b) beam; b, d) numerically calculated Fourier transform of a), d). The DC peak is capped to a value slightly above the amplitude of the sidebands. Parameters for the pump beam: $I = 129\,\mathrm{mW/cm^2}$, $\delta = +7\,\Gamma$, and $d = 5$ mm.

II. Experiment

In the experiment, a laser-cooled cloud of cold ^{87}Rb with a temperatures of about 290 μK is used. Doppler broadening is negligible compared to the natural width $\Gamma/(2\pi) = 6.06$ MHz of the atomic transition ($D2$ line at λ = 780.2 nm). The cloud has a roughly Gaussian density profile with dimensions (full width at half maximum, FWHM) of $10 \times 10 \times 5$ mm and contains about 5×10^{10} atoms. The optical density (OD) in line center is about 150. The experimental sequence alternates a preparation stage where the atoms are trapped and cooled in a magneto-optical trap (MOT), and a measurement stage where the MOT (trapping lasers and magnetic field) is shut down and the pump beam is turned on for a duration t_{pump}. This pump beam is spatially filtered by a single-mode fiber and collimated to a spot size of 1.9 mm (FWHM). The experiment is performed in the vicinity of the $F = 2 \rightarrow F' = 3$ hyperfine transition, which is closed. A repumper tuned to the $F = 1 \rightarrow F' = 2$ transition counteracts hyperfine pumping due to the residual excitation of other states. Typically, we use linear input polarization, but the experiment works equally well with circular input polarization. This, and an insensitivity to the presence of magnetic fields of different configurations, indicates that Zeeman pumping is not at the origin of the observations.

Pattern formation is observed for a wide range of positive detunings to the $F = 2 \rightarrow F' = 3$ transition. Fig. 2a shows a typical single-shot image. We observe high-contrast positive hexagonal patterns. This hexagonal symmetry is also evident in the numerically calculated Fourier transform of the intensity distribution (Fig. 2b). The pump beam in the center is suppressed for clarity and the six off-axis beam are the spontaneously generated sidebands. They arise on top of a faint ring in Fourier space which is the critical wavenumber selected by the phase-amplitude conversion (in the simplest case the Talbot effect in vacuum, Eq. 1, see [26] for the experimental situation with a diffractively 'thick' cloud). The sidebands break the rotational symmetry spontaneously (within limitations imposed by unavoidable imperfections of any real setup) as evidenced by shot-to-shot fluctuations.

Ten microseconds after the pump beam is switched off, a weak probe beam, which does not experience feedback and is detuned a few linewidth to the red side of the resonance, is injected the medium. It shows a honeycomb pattern. This is consistent with the expectation for dispersive imaging and complementary patterns in the light field and the atomic density. Atoms are expelled from the pump filament and gather along the ridges of the honeycomb pattern, where they attract the light of the negatively detuned probe beam due to the enhancement of refractive index there.

Even more importantly, the presence of a structure a few microseconds after the pump beam is switched off excludes electronic excitation, i.e. a population of the excited state, as the source of the atomic grating, as it should decay in a few times the natural lifetime of $\Gamma^{-1} \approx 26$ ns. The time scale of the probe pattern decay of about 80 μs and corresponding measurements of the turn-on dynamics [26] are compatible though with optomechanical dynamics and hence we conclude on the presence of a significant density grating.

III. Theory: Patterns

In the theoretical description, the state of the cloud is described by its phase-space distribution function $f = f(\mathbf{x}, \mathbf{v}, t)$ (with $\mathbf{x}$ and $\mathbf{v}$ position and velocity vectors in the plane transverse to the field propagation, respectively). Its dynamics is described by a collision-less Boltzmann equation with a driving term given by the dipole force:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}_{\text{dip}}}{M} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0\,. \tag{2}$$

Here M is the atomic mass and $\mathbf{F}_{\text{dip}} = -\partial_{\mathbf{x}} U_{\text{dip}}$ the dipole force with $U_{\text{dip}} = (\hbar\delta/2)\log(1 + s(\mathbf{x}, t))$, and s is the saturation parameter, i.e. the intensity divided by the saturation intensity. The spatial density $\rho(\mathbf{x}, t)$ is obtained by integrating f over the entire velocity space, with the normalization chosen so that the spatially homogeneous solution corresponds to $\rho = 1$. The saturation parameter s is given by the sum of the suitably normalized intensities of the forward field g_F and the backward field g_B, i.e. $s = |g_F|^2 + |g_B|^2$.

Neglecting diffraction effects inside the cloud (thin medium approximation), the interaction between a forward pump field of amplitude g_F and the cloud of laser-cooled two-level atoms is described by the following equation

$$\frac{\partial g_F}{\partial z} = -\frac{\text{OD}\,(1 - 2i\delta/\Gamma)}{2L\,[1 + 4(\delta/\Gamma)^2]}\,\frac{\rho}{(1+s)}\,g_F\,, \tag{3}$$

where L is the medium thickness. The $1/(1+s)$ term describes electronic saturation and takes into account that in the experiment optomechanical and electronic nonlinearities are potentially simultaneously significant, at least in some parameter regimes. To obtain g_B, we first integrate Eq. (3) under the assumption of a longitudinally homogeneous ρ and s and obtain the transmitted field at the exit face of the medium. The free-space propagation to the mirror (distance d) and back can be solved exactly in Fourier space, see [27] for details.

Typical results obtained at high saturation parameter (far above threshold) for a 1D geometry are shown in Fig. 3. For short pump durations, a pattern develops in the light field. It is accompanied by a pattern in the excited state (not shown). The velocity distribution is not changed from the initial Maxwellian one and the density remains flat. The interpretation is that, at the high value of the saturation parameter used, the instability can be triggered by electronic effects alone. If the pump is acting for longer, the atoms are moving in the dipole potential created by the modulated light field, a density modulation evolves and the velocity distribution is modified. The contrast of the light pattern is enhanced due to additional scattering from the density grating. In this nonlinear growth regime, harmonics of the fundamental wave number arise. Their strength increases further for increasing pumping duration, as well as the modulation depth of the density and the width of the velocity distribution. The long term saturation dynamics are currently under investigation.

In contrast, for lower input saturation value (close to threshold), light and density patterns develop together and on time scales in the tens to hundreds of microsecond range. In this regime, the optomechanical instability alone is sufficient to trigger the instability [26]. Corresponding regimes are identified in the experiment [26]. It should be stressed that the optomechanical nonlinearity alone is sufficient to induce the instability [27]. The threshold for the mixed electronic-optomechanical case is much closer to the purely optomechanical threshold than to the purely electronic one, supporting that the regime close to threshold is indeed dominated by the optoelectronic nonlinearity.

IV. Theoretical prediction of localized states

Close to threshold hexagonal patterns typically possess the intriguing feature that individual constituents of the pattern can serve as 'dissipative solitons' [28]–[30]. In a region of bistability between the homogeneous state and the hexagonal pattern below the threshold for pattern formation, self-sustained localized excitations can exist on the homogeneous background, which can be written and erased again 'at will' by external perturbations, in optics conveniently via focused control pulses [31]. We study this phenomenon in a model with velocity damping following [20]. In an experiment, velocity damping can be provided by a 3D optical molasses [20], [32], [33]. Though a corresponding model can and was developed for the single-mirror case, results are currently available only for a cavity scheme, but are expected to carry over to the single-mirror case. Details of the model can be found in [21], [34], we reproduce here only the essential results on structures.

Figs. 4a), b) show a developed hexagonal pattern 5% beyond threshold, obtained for a positive detuning of the pump light to the atomic resonance. The patterns in intensity and density are complementary as in the single-mirror system. Hexagonal patterns persist down to 10% below threshold, if the pump parameter is reduced again after the instability took place. This means that the bifurcation is subcritical and

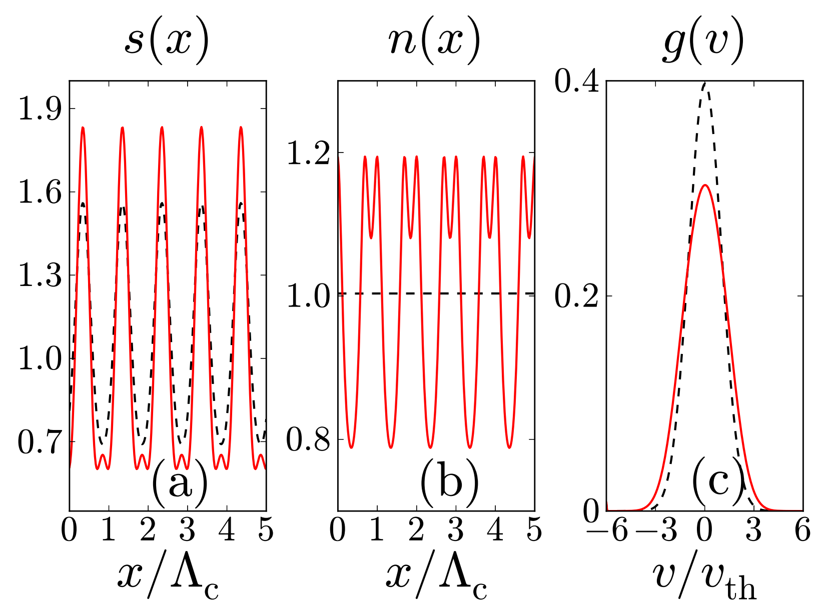

Fig. 3. Saturation profile (a), density profile (b) and velocity profile (c), i.e. the integral of the distribution function over space, 5 μs after pump switch-on (dashed black line) and 50 μs after pump switch-on (solid red line). Parameters: $|g_F|^2 = 0.6$ at input facet (purely electronic threshold at 0.2), $\delta = +10\ \Gamma$, $d = 5$ mm, $T = 300\,\mu$K, OD $= 200$.

within the hysteresis loop the homogeneous state, patterns and localized states can coexist. An example of a state with three localized states is shown in Figs. 4c), d). Their positioning and number is completely arbitrary and controllable by initial conditions as long as they don't come too close to interact [35]. In an experiment, a localized perturbation can be provided by a writing beam coherent to the homogeneous pump using constructive and destructive interference [36] or a writing beam incoherent to the pump [31], [35]. In the optomechanical case, incoherent writing can be implemented via a pulse slightly detuned to the pump perturbing the density distribution via its dipole potential. Note that depending on the pump detuning holes or peaks can be imposed on the density and are sustained after ignition by homogeneous driving only.

V. Conclusion

We demonstrated the spontaneous formation of coupled light-density structures in a 2D plane transverse to a single pump axis, breaking spontaneously two continuous symmetries in that plane. Though some form of material transport via convection (hydrodynamics), diffusion (chemistry and biology), or charge drift (gas discharges) is often present in pattern-forming systems, typically modulation of the overall matter density is neither the decisive driver nor the manifestation of self-organization. Hence, the self-structuring and manipulation of the density of matter itself demonstrated here augments the variety of pattern forming systems. Possibly even more intriguing is the prospect of localized structures in the density of matter, sustained by homogenous driving only.

Similar mechanism are expected to apply to instabilities in the electron density in plasmas due to ponderomotive forces [27] and structures in colloidal suspensions of dielectric beads, for which up to no only 1D geometries with feedback were explored [37].

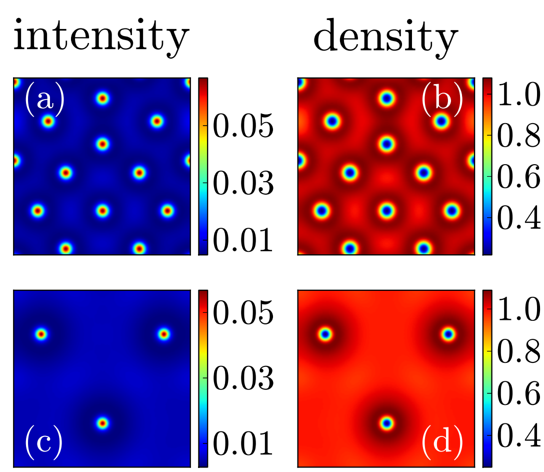

Fig. 4. Intensity (left column) and density (right column) obtained by a 2D numerical integration of a model including velocity damping. Upper row: hexagonal structures, lower row: examples for localized states. The pump is 5% above threshold for the hexagons, 5% below threshold for the solitons, other parameters as in [34].

From a quantum optical point of view it is interesting to look at classical and non-classical correlations between the fluctuations in the different sidebands and between the light and matter waves. The coupled light-matter structure can be interpreted as a self-induced and self-loaded optical lattice, but in contrast to externally imposed optical lattices it is not rigid but dynamic. This is likely to enable interesting studies on the propagation of localized and periodic perturbations (generalized 'sound') across the structure. Significant interest exists in studying self-organization involving multiple spatial modes in quantum degenerate matter, e.g. [38], [39]. As the instability described here relies only on coherent processes (diffraction and dipole forces) it should be extendable to quantum degenerate gases. The interaction between optomechanics, matter wave coherence and potentially the intrinsic matter wave nonlinearity is expected to lead to new interesting phenomena.

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