

Economic perspectives

Issues on Calman tax proposals still unresolved

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Introduction

The Commission on Scottish Devolution chaired by Sir Kenneth Calman delivered its report in June 2009. Among its recommendations on transfers of powers to the Scottish Parliament were a set of income tax proposals.

Following the publication of the Calman report (2009), we identified certain technical problems relating to these tax proposals. These were the subject of an open letter we sent to the Calman Commission in July 2009. This note begins with a review of the position on these technical problems, in the light of the publication of the Westminster Government's White Paper on Calman in November, (Cm 7738). Our conclusion is that, far from addressing the technical problems we had identified, the proposals in the White Paper in fact make one of these problems significantly worse. We also consider other significant issues relating to the potential effect of the proposed tax-sharing arrangements on the relations between the Scottish and Westminster governments, and to the potential utility of the new tax raising powers of the Scottish government as an instrument of fiscal policy. We conclude that both the original Calman report, and the White Paper, have failed to address these issues adequately.

Problems identified in original Calman proposals

The basic proposal on income tax in the Calman report can be summarised as follows. All income tax rates in Scotland would be reduced by 10p. At the same time, the Block Grant coming to the Scottish government would suffer a once and for all reduction equal to the amount which the 10p tax cut had cost the Whitehall Exchequer. The Scottish government would then be able to set its own rate of income tax, which would be levied as an addition to the reduced UK rate in Scotland. Hence, if the Scottish government set its rate at 10p, it would, at least initially, return its revenues to their original value. Note that the determination of the income tax bands themselves would remain under the control of Westminster.

What was the expected result of this change? In essence, the Commission was seeking to achieve greater financial accountability for the Scottish Parliament, while at the same time trying to strengthen the UK union. Under its proposal,

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the Scottish government would have to make a definite decision about what rate of income tax it would set. For the first time, the Scottish parliament would be accountable not only for how it spent its money but also for how it raised a major part of the money it wanted to spend. In criticising the existing Block Grant method of funding the Scottish Parliament, the Commission noted that the Parliament “has no fiscal powers that can be used as policy instruments and it does not have a direct financial stake in the performance of the Scottish economy”: (Calman report, para. 3.87). We can reasonably assume that the Commission intended its proposal to remedy these defects.

In the open letter which we sent to the Calman commission in July 2009, we identified two technical problems with these proposals.¹ The first of these related to the possibility that there could be an undesirable outcome from the proposed new tax system in that, under certain circumstances, the income tax revenues coming to the Scottish government might move in a different direction to overall income tax revenues collected in Scotland. (As shorthand, we describe this situation as the possibility of an anomalous movement in tax revenues). The second technical problem related to the adverse effects which fiscal drag was likely to have, if the Calman proposals were implemented. We now consider these two problems in more detail.

Possible anomalous movements in tax revenues

Let us assume a Scottish government is operating under the income tax system as originally proposed by Calman. If the Scottish government sets the Scottish rate of income tax as x pence in the £, then it will receive a proportion $\frac{x}{x+10}$ of

the basic rate income tax revenues collected in Scotland, (with corresponding proportions $\frac{x}{x+30}$ and $\frac{x}{x+40}$ for the

intermediate and highest tax bands respectively). Since $\frac{x}{x+10}$ is an increasing function of x , the Scottish

government will receive an increasing proportion of the overall basic rate revenues raised in Scotland, as it increases its tax rate x , (and similarly for the other tax bands). It will receive a decreasing proportion as it reduces x .

A number of different outcomes are possible. Suppose that the Scottish government reduces the Scottish rate of tax, and at the same time, overall income tax revenues in Scotland decrease. Then the Scottish government will be receiving a decreasing proportion of a decreasing total – so that the revenues coming to the Scottish government will clearly be reducing. In this case, the overall income tax revenues collected in Scotland, and those coming to the Scottish government, are moving in the same direction.

Suppose, however, that when the Scottish government decreases the Scottish rate of tax, overall income tax revenues collected in Scotland increase. Then the Scottish government will be getting a decreasing fraction of an

increasing total. In these circumstances, it is not possible to say without further information whether the tax revenues coming to the Scottish government will be increasing or decreasing. But in principle, it is clear that, if the increase in overall tax revenues is not particularly large, then the amount of revenue coming to the Scottish government might decrease. That is, total income tax revenues, and the income tax revenues coming to the Scottish government, could move in different directions.

Annex 1 examines the algebra surrounding this situation in more detail. It identifies precisely when the anomalous effect of the Scottish government’s income tax revenues moving in a different direction to overall income tax revenues collected in Scotland will occur. The relevant conditions derived in Annex 1 are summarised here:

Within the basic rate band of income tax: *if the effect of a unit decrease in the income tax rate in Scotland is to increase the overall income tax revenues from the basic rate, but by less than 5%, then the revenue coming to the Scottish government from the basic rate band will decrease.*

Within the higher rate bands: *if the effect of a unit decrease in the income tax rate in Scotland is to increase total revenues, but by less than 7.5% for the 40% band, or 8% for the 50% band, then the revenue coming to the Scottish government from the relevant band will decrease.*

Two questions therefore become very relevant:

- a) Would it matter if a Scottish government found itself operating in a position where this anomaly held?
- b) How likely is it that the conditions for this anomaly to hold will actually be met?

As regards the first question, we argue that it would matter a great deal if a Scottish government found itself in this position. Suppose that total income tax revenues could be increased by a reduction in the Scottish rate of tax, probably supported by other stimulatory measures. If, however, the tax elasticities were such that the anomaly held, then the Scottish government would have to reconcile itself to suffering a permanent reduction in its own revenues if it wanted to stimulate the Scottish economy by lowering taxes – even though the Whitehall exchequer would benefit from increased revenues. More realistically, at a time of stringent pressure on its finances, a Scottish government operating under these conditions could well find itself forced to raise its tax rates to boost its own revenues – at the expense of deflating the Scottish economy, and also at the expense of the Whitehall exchequer. So if the anomalous conditions actually held, the Scottish government could be forced into actions which were perverse – both from the point of view of the Scottish economy, and of the Whitehall exchequer. By contrast, if the tax elasticities were exactly the same, but the Scottish government’s income tax revenues always moved

in the same direction as total income tax revenues raised in Scotland, then a Scottish government could lower its tax rate - to the benefit of the Scottish economy, its own revenues, and the UK exchequer.

This then makes the second question very important. Are the conditions of the Calman anomaly so unlikely that the possibility of their occurring can be discounted: or is there a significant chance that these conditions could actually hold? We do not have a definitive answer to this question, but the following points are relevant.

- a) There is little or no experience of this particular situation. What is relevant is what happens to tax revenues in Scotland, when the income tax rate in Scotland is varied, with the tax rate in the rest of the UK remaining fixed. This is a situation which has never been met with before in the UK.
- b) In practice, a Scottish government would probably deploy a package of measures, as well as a cut in the Scottish rate of income tax, if it were trying to stimulate the economy. Such measures might include, for example, cuts in non-domestic rates, or in water prices. So the relevant question is not what happens to total income tax revenues in Scotland when tax rates are varied while everything else is held constant, (which is the conventional Laffer curve), but what the response would be to an overall stimulus package.
- c) Without attempting an overall literature review, it is relevant to note one piece of evidence. This is the paper by Gagné, Nadeau, and Vaillancourt, (2000), which suggests that in Canada the response to income tax rate changes at the provincial level could well have a negative elasticity in the higher rate tax bands.

Overall, the view we take is that a stimulus package for the Scottish economy, including a reduction in income tax rates in Scotland relative to England, might boost the Scottish economy sufficiently to produce a modest increase in total income tax revenues in Scotland. Given this, we argue that a precautionary principle should be adopted in designing any new taxation system: so that whatever income tax arrangements are eventually implemented for the devolved Scottish government, there should be no possibility of total income tax revenues, and the income tax revenues coming to the Scottish government, moving in opposite directions. The original Calman proposals fail this test. We set out later, for interest, a modification of the Calman proposals which would avoid the problem.

Effects of fiscal drag

If the Scottish government sets a tax rate of around 10p, then it will receive approximately 50%, 25%, and 20%, respectively of the total tax revenues raised in Scotland from the basic rate, intermediate, and highest rate bands. These widely differing proportions would, in practice, give rise to the following two problems:

- a) Because of the effects of fiscal drag, there is likely to be a consistent shift through time in the proportion of overall tax revenues raised from the different bands. Combined with the differing proportions of revenues coming to the Scottish government within each band, this will then lead to a consistent trend in the tax revenues going to the Scottish government, over and above any trend in overall tax revenues. Since the effect of fiscal drag is usually to increase the proportion of tax raised at the higher bands, (from which the Scottish government receives a smaller proportion of the revenues), the expected outcome is that there would be a declining trend in the overall proportion of tax revenues in Scotland actually coming to the Scottish government.
- b) Secondly, whenever the UK government, which is in charge of the UK income tax system, changed the tax thresholds or the structure of the system, there would be a shift in the relative amounts of tax collected in the different bands – and hence, a change in the amount of tax allocated to the Scottish government. At the very least, this would open the Scottish government to the danger of unpredictable and unplanned changes in its tax revenues. At its worst, this situation could be manipulated deliberately by a UK government, if it wished to trim the resources going to the Scottish government. Either way, the Scottish government would be placed in an insupportable position.

A modification to the Calman proposals which would avoid the above problems

In our open letter, we derived conditions which a tax sharing system of the Calman type would have to satisfy, in order to ensure that both of the difficulties identified in the preceding section were avoided. The argument is as follows.

Annex 2 derives the conditions under which, for a general class of tax sharing systems, total tax revenues, and the revenue going to the Scottish government, will always move in the same direction. In words, the key requirement is that, in any tax band, if overall tax revenues drop as the Scottish tax rate increases, then the percentage increase in the share of tax revenues going to the Scottish government should be smaller than the percentage decrease in overall tax revenues. But since the potential decrease in tax revenues (if any) resulting from an increase in the Scottish tax rate is unknown, the only way that it can be guaranteed that the condition will always be met is if the percentage increase in the share of tax revenues going to the Scottish government is zero: that is, within each tax band, if the tax sharing system gives the Scottish government a fixed share of the tax revenues raised in Scotland in that band.

The circumstances under which the fiscal drag problem will be avoided are that the percentage share of the tax revenues from each tax band going to the Scottish government must be the same.

The implication is that, if both the problem of anomalous revenue movements, and the problems associated with fiscal drag, are to be avoided, then the Scottish government should receive a fixed percentage share of the overall income tax revenues raised in Scotland.

Hence a tax sharing system which would always avoid both problems would involve the Scottish government setting its own rate of tax, (as proposed by Calman), but the Scottish government then receiving the same, fixed, percentage share of overall income tax revenues collected in Scotland.

Note that, while this modification of the Calman proposals would solve the identified technical problems, we are not ourselves advocating that this modification should be implemented, for the following reason. Under the modified Calman system, a decision by the Scottish government to change the Scottish rate of income tax would clearly have a direct impact on the revenues received by the UK government: and conversely, any change in tax rate by the UK government would have a direct impact on the revenues of the Scottish government. Successful operation of this system would thus require that the UK and devolved governments were willing to operate in a collegiate manner – being appreciative of, and respecting, the impact that their own actions will have on the revenues of the other parties. The implication is that a successful tax sharing system would have to involve a more federal way of working than is the current practice in the UK. It would be a mistake to introduce such a system unless all the potential ramifications of this had been clearly thought through in advance. (Note that the Calman proposals themselves would have an effect on UK tax revenue, but this would be a second order, rather than a first order effect.)

The White Paper on Calman of 25th November

On 25th November 2009, the UK government published its White Paper with its proposals for taking forward the Calman recommendations. As regards income tax, the White Paper proposed that the Calman recommendations should be implemented virtually in their original form. There is, however, one important exception, which we discuss in this section. This relates to the transitional arrangements proposed in the White Paper. These would in fact have a very significant effect on the operation of the tax sharing system – and in a way which makes the potential for anomalous revenue movement much worse. It is fair to say that the implications of the White Paper's transitional arrangements seem to have escaped general comment.

The transitional arrangements are described in paragraphs 4.18 and 4.19 of the White Paper. It is stated there that the Calman proposals on income tax will need to be phased in carefully “at a time of major fiscal adjustment and economic uncertainty”. During the phasing in period, the adjustment to the Block Grant will not be done on a one off basis, but will be calculated afresh for each new Public Expenditure

spending review – that is, every three years. The UK government would forecast tax receipts from Scottish taxpayers for the three years of each spending review, and a sum equivalent to a 10p reduction in the rate of income tax would then be subtracted from the Scottish Block Grant for each of these years.

The White Paper puts no specific time limit on the operation of these transitional arrangements, but says that there would be a move towards the full model proposed by Calman as soon as economic and fiscal circumstances permitted.

Given that current economic uncertainties look set to persist for the foreseeable future, it seems likely that the transitional arrangements would operate for a significant length of time.

Suppose that the transitional arrangements are in operation, and that a Scottish government reduces the Scottish rate of income tax below 10p to, say, 8p. When the Westminster government next comes to adjust the Scottish Block grant, it will subtract from the original Scottish Block grant an amount equal to the estimated yield of a 10p income tax rate in Scotland. Assuming the Westminster government is accurate in its estimate, this will be a larger amount than what the Scottish government gets back from the lower Scottish tax rate it has set. In other words, under the arrangements set out in the White Paper, if the Scottish government reduces its tax rate below 10p, it will always suffer financially. This would happen even if the reduction in the Scottish tax rate was part of a successful package to stimulate the Scottish economy, as a result of which overall income tax revenues in Scotland actually went up.

Conversely, if the Scottish government raises the Scottish income tax rate above 10p, then the Scottish government will always benefit financially – even if the effect of the tax rise had been to damage the economy, leading to a reduction in overall income tax revenues collected in Scotland.

In fact, a stronger result can be proved about the effect of the transitional arrangements. It is not just the case that a Scottish government is worse off by cutting its tax rate below 10p relative to what it would receive if it set a 10p rate: and vice versa if it set a rate above 10p. The stronger result also holds that, for all tax rates which are likely to be feasible in practice, the slope of the curve relating the Scottish government's revenues to the Scottish rate of income tax is positive. This result is proved in Annex 3. It can be seen from the Annex that the conditions under which this result holds are so general as to justify our assertion that the result holds for all feasible tax rates.

What this result means is that, under the transitional arrangements, the Scottish government will always face what is in effect an upward sloping Laffer curve, for any feasible tax rate it might consider setting. The first technical problem we identified with the original Calman proposals

has now actually become much worse. Previously, if the elasticity of overall income tax revenues was sufficiently negative there was still a chance that the elasticity of Scottish government revenues might be negative too. Under the Calman transitional arrangements, there is no such chance.

It is unclear what effect the transitional arrangements have on the second technical problem we identified, relating to fiscal drag. Under one scenario, however, the effect can be predicted. Suppose that both the unadjusted Block Grant, and total income tax revenues collected in Scotland, were rising in line with inflation. Suppose also that the yield of a 10p tax rate in Scotland represents a decreasing proportion of total tax revenues, because of fiscal drag. Then a Scottish government which set a 10p tax rate in the absence of the transitional arrangements would see its revenues rising more slowly than inflation: whereas with the transitional arrangements in place, its revenues would rise in line with inflation. Under this scenario, therefore, the transitional arrangements do indeed overcome one of the major problems associated with fiscal drag. However, this particular scenario is relatively unlikely. Under other, more complicated scenarios, it is not clear what the effect of the interaction of fiscal drag and the transitional arrangements would be.

Important issues not addressed in either the Calman Report or the White Paper

In the section above where we derived a modification to the Calman proposals which would avoid the identified technical problems, we expressed caution about implementing this change, because of the implications it would have for the relations between the Scottish and Westminster governments. This issue, however, is not just confined to this particular modification of Calman. Any system of tax sharing, including the original Calman proposals themselves, is going to have an effect on the relationship, and mode of working, between the different layers of government involved. The Calman report did indeed recommend that the liaison arrangements between the Westminster and Scottish governments would have to be strengthened, in the light of their proposals on tax sharing. But they did not analyse in detail the specific tensions and pressures which the new system would be subject to: hence it is not clear whether Calman's specific proposals on improving liaison would be able to cope.

Among important questions which need to be addressed are the following. Would implementation of the original Calman proposals, (leaving aside their technical flaws), result in fundamental shifts in the relations between the different parties which would institute a dynamic for further change? Is it possible to implement a tax sharing system which is technically acceptable, without moving to something akin to a federal system? These are fundamental questions, which the Calman report, and the White Paper, signally failed to address.

Another important set of questions which were similarly neglected in both the Report and the White Paper relates to how effective the proposed tax varying powers would be as a tool which the Scottish government could use in economic management, and fiscal policy. As we have already noted, the lack of such tools was one of Calman's criticisms of the existing block grant arrangements: and it is reasonable to assume that the Calman Commission hoped that their proposals would remedy this lack. Despite this, neither the report nor the White Paper pay any attention to examining the question of whether their proposed tax varying powers would constitute an effective economic management tool for the Scottish government. In fact, our analysis above suggests that there is a grave risk that the proposed powers would have perverse effects which could gravely damage the Scottish economy – this being particularly the case for the White Paper transitional arrangements. The implication is that the proposed powers would be worse than useless as tools of economic management. We find it very surprising that Calman, having identified the need for fiscal powers that could be used as policy instruments, and for the Scottish government to have a direct financial stake in the performance of the Scottish economy, paid no attention to the question of whether the proposed changes actually fulfilled these requirements. If the Calman Commission had considered this topic, it seems likely that they would have discovered for themselves the anomalous effects, and likely adverse implications for the Scottish economy, which are implicit in their proposals.

If the questions identified in this section are not addressed, and likewise, if the identified technical problems are not rectified, then implementation of the Calman tax proposals could rapidly prove very unsatisfactory for all the parties concerned. This could, paradoxically, weaken the existing union – quite counter to Calman's stated aim of introducing change which would actually strengthen the union.

Conclusions

We draw three main conclusions from the above.

1. The transitional arrangements introduced in the White Paper make the first of the technical problems identified in our open letter significantly worse. The effect is that, with the transitional arrangements in operation, the curve relating the revenues a Scottish government will receive to the Scottish rate of income tax will always be upward sloping. Even if a Scottish government were able to stimulate the Scottish economy by a combined stimulus package involving income tax cuts, and even if this had the effect of increasing overall income tax revenues in Scotland, its own revenues would reduce. Conversely, under the transitional arrangements, a Scottish government could always increase its revenues by raising the Scottish rate of tax, even if this meant severe deflation of the Scottish economy, and a reduction in overall income tax revenues in Scotland. These effects are so perverse, that we argue it would be disastrous if

the White Paper on the Calman proposals on income tax were implemented as they stand.

2. The complete absence from the White Paper of any discussion of technical issues relating to tax, and the introduction in the White Paper of transitional arrangements, without any apparent appreciation of the significant effects these would have, both indicate an altogether inadequate level of technical treatment in the drafting of the White Paper. We conclude that any serious attempt to reform the taxation arrangements for the Scottish government should proceed on the basis of a much more informed level of technical discussion. A cynic might say that the absence of consideration of technical issues in the White Paper suggests that the White Paper is more of a political document, rather than a serious attempt to reform the taxation arrangements for devolution.
3. The Calman report, and the subsequent White Paper, are alike deficient in the limited attention they pay to analysing the effect which the introduction of their proposals on taxation would have on the way in which the different levels of government interact. There are fundamental questions here which need to be addressed in much more detail. In particular, it is not clear whether it would be feasible to devise a workable system of tax sharing without moving towards a more federal system of government. Similarly, both documents neglect the important question of how effective the proposed tax varying powers would be as a tool which the Scottish government could use in economic management, and fiscal policy. We conclude that any attempt to reform the tax arrangements for devolution should involve a much more detailed assessment of such issues.

References

Calman Report, (2009): "Serving Scotland Better: Scotland in the UK in the 21st Century", Final Report, Commission on Scottish Devolution, June 2009.

Gagné, R., Nadeau, J.-F., Vaillancourt, F., (2000): "Taxpayers' Response to Tax Rate Changes: A Canadian Panel Study", CIRANO Working Papers no. 2000S-59, University of Montreal.

White Paper, (2009): "Scotland's Future in the United Kingdom, Building on ten years of Scottish devolution", Cm 7738, November 2009.

Endnotes

¹(The text of our open letter, together with subsequent correspondence with the Scotland Office, can be found at www.cuthbert1.pwp.blueyonder.co.uk).

Annex 1: The conditions under which anomalous movements in tax revenues occur

Suppose that the original Calman proposals on income tax are in operation.

The basic rate is considered first.

Let $T(y)$ = total basic rate tax revenues in Scotland, when tax levied at rate y :

let $f(x)$ = basic rate revenue going to the Scottish government, when it sets its tax rate at x .

Now when the Scottish government sets its tax rate at x , it will, to a good approximation, receive a proportion $\frac{x}{10+x}$ of the basic rate revenues raised in Scotland when the aggregate rate is $(10+x)$. So the fundamental

relationship between f and T is that

$$f(x) = \frac{x}{(10+x)} T(10+x)$$

The anomalous condition that we are interested in occurs, by definition, when the slopes, (that is, the first derivatives), of $f(x)$ and $T(10+x)$ have opposite signs.

Differentiating with respect to x , we can see that

$$\begin{aligned} f'(x) &= \frac{x}{(10+x)} T'(10+x) + \frac{10}{(10+x)^2} T(10+x) \\ &= \frac{1}{(10+x)} \left(xT'(10+x) + \frac{10}{(10+x)} T(10+x) \right) \end{aligned} \quad (1)$$

It follows from expression (1) that $f'(x) > 0$ whenever $T'(10+x) > 0$: so the anomaly can never occur when $T'(10+x) > 0$.

It also follows from (1) that

$$f'(x) > 0 \quad \text{if and only if} \quad \left(xT'(10+x) + \frac{10}{(10+x)} T(10+x) \right) > 0,$$

that is,

$$\text{if and only if} \quad T'(10+x) > \frac{-10}{x(10+x)} T(10+x).$$

This implies that the anomaly will occur if, and only if,

$$0 > T'(10+x) > \frac{-10}{x(10+x)} T(10+x)$$

Looking at the key "starting position" of $x=10$ in more detail, it is possible to identify more precisely the conditions under which the anomaly will occur: namely,

$$\text{if and only if} \quad 0 > T'(10+10) > \frac{-10}{10(10+10)} T(10+10)$$

$$\text{that is if and only if} \quad 0 > \frac{T'(20) \cdot 100}{T(20)} > \frac{-100}{20} = -5.$$

The term in the middle of this last expression is approximately the percentage change in overall basic rate tax revenues in Scotland, resulting from a 1p increase in the tax rate: (this can be seen on taking the first two terms in a Taylor expansion). So the anomaly will occur if a 1p increase in the tax rate in Scotland leads to a reduction in total basic rate revenues collected, but a reduction which is less than 5%. An alternative way of expressing the same condition is that a 1p reduction in the tax rate leads to an increase in revenues collected at the basic rate, but an increase which is less than 5%. This is the form of the condition quoted in the text of the paper.

The algebra for revenues coming from the higher rate tax bands is similar, but the numbers are different. Let $H(y)$ represent total tax revenues in Scotland from the middle tax band, (currently the 40% tax band), when tax is levied at rate y : then what the Scottish government gets from this tax band, when it sets its tax rate at x , is

$$\frac{x}{(30+x)} H(30+x) .$$

A similar argument to the above shows that the critical threshold in this case is 7.5%, (as

compared to 5% for the basic rate tax band). Similarly, for the highest rate tax band, (currently 50%), the critical threshold is 8%.

Annex 2: The conditions under which a general tax sharing system will avoid the anomaly

Consider the basic rate.

As in Annex 1, let $T(y)$ = total basic rate tax revenues in Scotland, when tax levied at rate y :
let $f(x)$ = basic rate revenue going to the Scottish government, when it sets its tax rate at x .
Consider a general tax sharing system, defined by the relationship

$$f(x) = a(x) T(10+x) + c,$$

where $a(x)$ is the proportion of overall basic rate tax revenues going to the Scottish government when it sets its tax rate at x , and c is a constant, (which could be zero).

It is reasonable to assume $a'(x) \geq 0$.

Again, we are interested in the slope or first derivative of $f(x)$. Now,

$$f'(x) = a(x) T'(10+x) + a'(x) T(10+x) ,$$

Since $f'(x) = a(x) T'(10+x) + a'(x) T(10+x)$,

it follows that $f'(x)$ and $T'(10+x)$ will have the same signs if either

- (i) $T'(10+x) \geq 0$, or
 - (ii) $T'(10+x) < 0$, and $a(x) T'(10+x) + a'(x) T(10+x) < 0$,
- that is, if $T'(10+x) < 0$, and $\frac{a'(x)}{a(x)} < \frac{-T'(10+x)}{T(10+x)}$.

In words, these conditions mean that the anomaly will not occur if the effect of an increase in the rate of tax is to increase total revenues collected in Scotland: or if the effect of an increase in the rate of tax is to reduce revenues collected, while at the same time the percentage increase in the proportion of income tax revenues coming to the Scottish government is smaller than the percentage decrease in overall revenues.

Annex 3: Under the White Paper transitional arrangements, the slope of the curve relating the Scottish Government's revenues to the Scottish rate of income tax is positive, for all feasible tax rates.

For simplicity, we ignore for the present the higher rates of tax. As in the previous annexes, let $T(y)$ = total basic rate tax revenues in Scotland, when tax is levied at rate y .

Let B = block grant which would have come to Scottish government, if the tax sharing arrangement in Calman had not been implemented. (Both B and T will also be functions of time: but for the purposes of the present argument, we lose nothing by omitting the time variable.)

Let $f(x)$ now represent the total revenues coming to the Scottish government, from both the block grant, and via its share of income tax, assuming the White Paper transitional arrangements are operational, when the Scottish rate of tax is x . Then, if the Westminster government is accurate in its estimate of how much 10p accounts for out of the total tax take in Scotland, what the Scottish government will receive will be B , reduced by the transitional adjustment of $10/(x+10)$ of $T(10+x)$, plus the amount raised by the Scottish rate of tax, which is $x/(x+10)$ of $T(10+x)$. Thus,

$$f(x) = B + \frac{(x-10)}{(x+10)}T(10+x) \quad (2)$$

$$\text{Hence } f'(x) = \frac{(x-10)}{(x+10)}T'(10+x) + \frac{((x+10)-(x-10))}{(x+10)^2}T(10+x)$$

$$= \frac{1}{(x+10)}[(x-10)T'(10+x) + \frac{20}{(x+10)}T(10+x)]$$

Looking at the expression within the square brackets, there are then four different cases to consider, as follows:-

(a) If $T'(10+x) > 0$, and if $x > 10$: in this case, $f'(x) > 0$ always.

(b) If $T'(10+x) > 0$, and if $x < 10$: in this case, $f'(x) > 0$

$$\text{if and only if } [(x-10)T'(10+x) + \frac{20}{(x+10)}T(10+x)] > 0,$$

$$\text{if and only if } \frac{T'(10+x)}{T(10+x)} < \frac{-20}{(x-10)(x+10)} = \frac{20}{(100-x^2)} \quad (3)$$

But the function on the right hand side of this expression is an increasing function of x on the range $(0, 10)$. So

condition (3) will always be satisfied if $\frac{T'(10+x)}{T(10+x)}$ is less than the value of the expression on the right evaluated at

$x=0$. But the value of the expression at $x=0$ is 0.2. So if the percentage increase in total basic rate tax revenues raised in Scotland stemming from a unit increase in the tax rate is less than 20%, (which under any feasible scenario will always be the case), then the implication is that $f'(x)$ will indeed be positive in case (b).

(c) If $T'(10+x) < 0$, and if $x < 10$: in this case, $f'(x) > 0$ always.

(d) If $T'(10+x) < 0$, and if $x > 10$: in this case, $f'(x) > 0$

$$\text{if and only if } [(x-10)T'(10+x) + \frac{20}{(x+10)}T(10+x)] > 0,$$

$$\text{if and only if } \frac{T'(10+x)}{T(10+x)} > \frac{-20}{(x-10)(x+10)},$$

$$\text{if and only if } \frac{-T'(10+x)}{T(10+x)} < \frac{20}{(x^2-100)} \quad (4)$$

But the function on the right hand side of this expression is a decreasing function of x for $x > 10$. So condition (4) will

always be satisfied for x in the range $10 < x < 15$, say, if $\frac{-T'(10+x)}{T(10+x)}$ is less than the value of the expression on

the right evaluated at $x=15$. But the value of the expression at $x=15$ is 0.16. So if the percentage decrease in basic rate tax revenues stemming from a unit increase in the tax rate is less than 16%, (which under any feasible scenario is always likely to be the case), then the implication is that $f'(x)$ will indeed be positive in case (d), at least for all x

in the range $0 < x < 15$. It would appear very unlikely that a Scottish government would ever set a Scottish rate of tax which is more than 5p higher than the rate in the rest of the UK.

In each of the four possible cases, therefore, $f'(x) > 0$ either without any further conditions, or under conditions which are always likely to be met in practice. This justifies our assertion that $f'(x) > 0$ for all feasible tax rates.

The above proof has been given for the basic rate case. Essentially the same argument goes through, and the same conclusions apply, for the intermediate and higher bands, with the following changes:-

The relevant terms in formula (2) become $\frac{(x - 10)}{(x + 30)} H(30 + x)$ and $\frac{(x - 10)}{(x + 40)} G(40 + x)$ for the intermediate and

highest rate bands respectively, where the functions H and G represent total revenues at these bands.

In case (b), the figure of 20% for the basic rate becomes 13.3% for the intermediate rate, and 12.5% for the highest rate.

In case (d), the figure of 16% for the basic rate becomes 17.7% for the intermediate rate, and 18.2% for the highest rate.