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Balanced Budget Multipliers for Small Open Regions
within a Federal System:
Evidence from the Scottish Variable Rate of Income Tax *

by

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Abstract

This paper explores the impact on aggregate economic activity in a small, open region of an income-tax funded expansion in public consumption that has no direct supply-side effects. The conventional balanced budget multiplier produces an unambiguously positive macroeconomic stimulus, but the incorporation of negative competitiveness elements, through the operation of the local labor market, renders this positive outcome less certain. Simulation using a single-region Computable General Equilibrium (CGE) model for Scotland demonstrates that the creation of local amenity effects, and the extent to which these are incorporated into local wage bargaining, is central to the analysis.

JEL Numbers: C68, D58, H71, R13, R23

Key words: Fiscal federalism, devolution, regional labour market, Scotland, CGE analysis.
1. INTRODUCTION

Individual states or regions within a federal system typically face a very restricted range of macroeconomic policy options. The region has no monetary policy levers and often has only a limited number of fiscal options. This paper revisits the theory of a balanced budget fiscal expansion within a small open regional economy context. In particular, it investigates the ability of the regional government to stimulate the local economy through an expansion in expenditure on a public consumption good financed through an increase in the rate of local income tax. The formal analysis is extended through simulation results from an inter-temporal Computable General Equilibrium (CGE) model for Scotland: this policy option is at present available to the Scottish Government.

For pedagogic reasons, public consumption is assumed to have no direct supply-side effects. In this type of situation, the conventional Keynesian, fixed-price, balanced-budget multiplier produces an unambiguously positive macroeconomic stimulus (Haavelmo, 1945). But the incorporation of negative competitiveness elements, through the operation of the local labor market, renders the analysis more complex and the positive outcome much less certain (Knoester and van der Windt, 1987). This is particularly the case in a regional context, where there is typically a high degree of openness both in factor and product markets (Liebeg et al., 2007). Two key parameters are important in determining not only the size but also the sign of the macroeconomic effect. These are the extent to which households value public expenditures, weighed against the increase in taxes, and the degree to which this valuation is incorporated into the local wage bargain.
2. THE BASIC THEORETICAL MODEL

We adopt a long-run, small, open-economy (SOE) model of the region in the spirit of the disaggregated approach presented in Layard et al. (1991, Ch. 6) which incorporates imperfect competition in the regional labor market. Given the SOE assumption, the price of imports and the cost of capital are both set exogenously, in perfectly integrated national/international markets with regional exports determined through conventional trade functions (Armington, 1969; Engel and Rogers, 1996). That is to say, output produced under conditions of perfect competition within the region, is not a perfect substitute for the output of other regions.\footnote{This permits adjustments in the price of the regional good relative to the price of extra-regional goods and allows variation in the regional real and nominal wage in the long run.} Production occurs under well-behaved, linear homogeneous production functions, with two factors, capital and labor, and there is a single representative household. In the present context, long-run equilibrium implies that both the regional capital stock and population are optimally adjusted. Given that we assume no natural change in population, this implies that long-run equilibrium states are characterised by zero net investment and zero net migration.

An important assumption is that the balanced budget increase in public consumption, financed by the increase in the rate of income tax, has no direct supply-side effects: it has no direct impact on the productivity of capital and labor or the efficiency of production.\footnote{Much expenditure on arts and culture, the police, town planning, the environment, and care for senior citizens would likely fall in this category. However, the primary reason for adopting this assumption is heuristic. It simplifies the analysis in a way that captures the traditional}
macroeconomic view of public expenditure. In this analysis there is a positive demand-side impact on regional output through a variant of the standard balanced budget multiplier (Haavelmo, 1945). However, this is likely to be accompanied by negative supply-side effects operating via the impact of higher income tax rates on the nominal wage, generated by regional bargaining. We begin with a more detailed specification of our analytical model.

Net migration is identified as a positive function of the relative real wage and employment rates across regions (Layard et al., 1991) which is an extension of the Harris and Todaro (1970) model. Equation (1) states the zero net migration equilibrium condition. It identifies the set of changes in the post-tax real consumption wage, $dw$, the employment rate, $de$, and the proportionate rate of income tax, $d\tau$, for which the net migration is zero:

$$
(1) \quad dw = z_e de - \beta d\tau \quad z_e \leq 0, \beta \geq 0.
$$

Equation (1) indicates that there is a negative relationship between the change in the post-tax real wage and the change in the employment rate across zero net migration (long-run) equilibria. The coefficient $z_e$ is derived directly from setting the standard flow-migration equation to zero and rearranging terms. The sign of $z_e$ is negative because if the attractiveness of a region to migrants is to remain constant, when the employment rate falls, the real wage must rise and vice-versa.

However, equation (1) is rather unconventional in that it also includes the term $-\beta d\tau$ in an attempt to capture the effect on the migration decision of the locally financed public amenity (Rodriguez-Pose and Ketterer, 2012). $\beta$ is the parameter indicating the degree to which households value public, as against private, consumption. It represents the subjective net valuation by households of the beneficial use of the increased public expenditures weighed
against the accompanying increase in their income tax payment. Where households attach no value to the amenity created by the increased expenditures, $\beta = 0$ and the standard formulation of the net migration condition applies, with the post-tax real consumption wage governing migration decisions. However, if public expenditure generates a positive amenity, so that households attach any value at all to the services produced by the additional expenditure, then $\beta > 0$. For a given employment rate, the larger the value of $\beta$, the lower the post-tax real consumption wage required to preclude net outmigration. When $\beta = 1$, the potential migrant is indifferent between marginal changes in local public expenditure and private consumption so that in this case the pre-tax real consumption wage drives migration. Where $\beta > 1$, there is a positive marginal preference for local public expenditure over private consumption.

The change in pre-tax nominal wage, $dW$, is defined in equation (2)

$$dW = dcpi + dw + d\tau$$

where $dcpi$ is the change in regional consumer price index, given as

$$dcpi = cpi_w dW \quad 1 > cpi_w \geq 0.$$  

Equation (3) expresses the change in the regional consumer price index as a function solely of the change in the regional nominal wage. This parsimonious specification is permitted by the exogeneity of import prices and the cost of capital, together with the linear homogeneous nature of production.

The change in labor demand is a function of the change in the nominal pre-tax wage and the tax rate, so that

$$dn = n_w dW + n_t d\tau \quad n_w \leq 0, n_t \geq 0.$$  

It is important to note that equation (4) represents a general equilibrium relationship,
constructed on the basis of full income endogeneity. Labor demand is negatively related to the nominal pre-tax wage through competitiveness and factor substitution effects. This means that any consumption effects from a higher nominal wage are more than offset by adverse substitution effects in production and reduced exports and import substitution. On the other hand, labor demand is a positive function of the tax rate, reflecting the operation of the conventional Keynesian balanced budget multiplier. This implies that a transfer of income from the private to the public sector generates a net demand stimulus, primarily because of the lower saving and import propensity of the public sector, together with the greater labor intensity of public sector activity.

Finally, through the bargaining function, the change in the real post-tax consumption wage is positively related to the change in the regional employment rate (Layard et al., 1991)

\[ dw = b\gamma de - \alpha \beta d \tau \quad b\gamma \geq 0. \]

The motivation for the positive sign on the employment rate term is that union power increases as the employment rate rises, resulting in a higher bargained real wage. However, in this formulation of the regional bargaining function, the local amenity generated by the expenditure is allowed to influence wage bargaining behavior directly. The parameter \( \alpha \), which takes a value between 0 and 1, reflects the extent to which the value of the amenity is taken into account in the wage bargaining process.

The possible amenity effects on the local bargained real wage appear to be neglected in the literature on fiscal federalism. Since the amenity provision is exogenous to the individual worker, in a perfectly competitive labor market, the amenity is ignored in the worker’s work/leisure choice, so that only the post-tax real consumption wage matters. This
corresponds to one situation in which the value of $\alpha$ is zero. This value will also apply in our model if bargaining is atomized so that it fails to take into account any public expenditure implications. However, in the bargaining context the public good externality will be internalised in so far as local unions cover a significant section of the labor force and act co-operatively. Under many local/regional income tax arrangements, including the Scottish Variable Rate, the scale of the amenity is tied directly to tax income and therefore to the bargained wage. For these cases the value of $\alpha$ will rise above zero. In the mid 1970s, the Social Contract between the unions and the UK Labour Government explicitly limited wage bargaining in response to government commitments to increase the social wage. Similarly, the present Scottish First Minister, Alex Salmond, has emphasised the potential importance of the social wage, which includes elements of public as well as private consumption.

3. THEORETICAL ANALYSIS OF A BALANCED BUDGET FISCAL EXPANSION

Changes in the Employment Rate, $de$, and the Nominal Pre-Tax Wage, $W$

Using equations (1)-(5), the change in the five endogenous variables $dn$, $de$, $dcpi$, $dw$ and $dW$, can be determined, given the change in the value of the exogenous tax rate, $dr$. A key variable in the analysis is the change in regional competitiveness that accompanies the fiscal expansion, which, in the present model, is identified by the change in the nominal pre-tax wage, $dW$. This, together with the change in the employment rate, is determined by the interaction of the migration and bargaining functions. Combining equations (1), (2) and (3) produces the zero net migration (ZNM) function, expressed in terms of the pre-tax nominal wage
If $d\tau$ is set to zero, equation (6) gives the initial zero net migration function in the change in nominal pre-tax wage-employment rate space. This is represented by the curve $Z_0$ in Figure 1, which has a slope equal to $z_e/(1-cpi_w) < 0$ and passes through the origin.

Similarly, combining equations (2), (3) and (5) gives the bargaining function in pre-tax nominal wage form

\[ dW = \frac{b_e}{1-cpi_w} de + \frac{1-\alpha\beta}{1-cpi_w} d\tau. \]

Again, setting $d\tau$ to zero in equation (7) produces the initial bargained real wage function (BRW) with a slope equal to $b_e/(1-cpi_w) > 0$ which also passes through the origin. This is curve $B_0$ in Figure 1.

Equations (6) and (7) are sufficient to tie down the change in the employment rate and nominal wage, $de$ and $dW$, that accompany the fiscal expansion, $d\tau$, producing

\[ de = -\frac{\beta(1-\alpha)}{b_e - z_e} d\tau \leq 0 \]

and

\[ dW = \frac{b_e(1-\beta) - z_e(1-\alpha\beta)}{(1-cpi_w)(b_e - z_e)} d\tau. \]

It is useful to analyze these results diagrammatically using Figure 1. Where a local income tax increase of $d\tau$ is levied, the ZNM function (equation 6) moves vertically by an amount equal to $((1-\beta)/(1-cpi_w))d\tau$. The tax increase also shifts the BRW function (equation 7) vertically but by $((1-\alpha\beta)/(1-cpi_w))d\tau$. Note that the parameter restrictions imply that $1-\alpha\beta \geq 1-\beta$ so that the BRW
function cannot experience a smaller upward movement than the ZNM function. We begin by considering the impact of the fiscal expansion under alternative assumptions about the labour market. These are encapsulated in the value of $\alpha$ which must lie between zero and unity.

A Unified Regional Bargain over the Social Wage: $\alpha = 1$. Where $\alpha = 1$, the amenity value of the public expenditure is fully reflected in the bargaining equation. This would apply where the whole workforce is covered by a single bargain. In essence, this would be where workers bargain over a social wage. In this case the ZNM and the BRW functions both move vertically by the same amount. There is no change in the employment rate, $\Delta e = 0$, so that the equilibrium lies on the $dW$ axis, that is on the line AJ0C in Figure 1. The equilibrium point along this line depends solely on the value of $\beta$.

Where $\beta = 0$, the additional public expenditure produces no amenity value to local residents, the ZNM and the BRW functions both shift upwards by $\Delta r/(1-cpi_w)$ to $B_1$ and $Z_1$ respectively and the equilibrium is at A. The change in the pre-tax nominal wage is $\Delta r/(1-cpi_w)$ so that the full tax increase is incorporated into higher nominal wages, including the term $(1-cpi_w)^{-1}$ to accommodate the increase in regional cpi.

Where $\beta = 1$, so that the value of the increased public expenditure to local residents just equals the forgone private consumption, neither the ZNM nor the BRW curve moves. There is simply a switching of part of the pre-tax wage from private to public expenditure: there is no change in the employment rate and no loss of competitiveness through higher nominal wages. The new equilibrium remains at the origin.
Where $\beta > 1$, so that the residents have a positive preference for public as against private consumption, the ZNM and BRW functions move downwards so that the nominal pre-tax wage falls to an equilibrium at a point such as C. With unified region bargaining, a balanced budget fiscal expansion has no impact on the employment rate but affects the nominal wage, and therefore competitiveness, in a manner inversely related to the value of public expenditure to the residents.

**Bargaining solely over the Private Real Wage: $\alpha = 0$.** Where the social wage is irrelevant, such as under atomised bargaining or under a perfectly competitive labour market, $\alpha = 0$. From equation (7) this means that in Figure 1, for any value of $\beta$ the BRW curve moves upwards by the amount $d\tau/(1-\text{cpi}_W)$ to $B_1$. The subsequent competitive labour market equilibrium will lie on this line, ADMLE. Where the public amenity has no value, so that $\beta = 0$, the new equilibrium is at A, with the change in the nominal pre-tax wage as $d\tau/(1-\text{cpi}_W)$. Where $\beta = 1$, the ZNM curve remains static at $Z_0$ and the new equilibrium is at D. Using equation (8) gives the result that at D, the change in the nominal pre-tax wage is positive and is given as

$$dW = -\frac{z_e d\tau}{(1-\text{cpi}_W)(b_e - z_e)} > 0.$$

Values of $\beta$ between zero and one generate equilibria along the line segment AD and values of $\beta$ greater than 1 lead to equilibria further down the BRW function $B_1$, to points such as M, L and E. In all cases the employment rate, $de$, falls.

**The General Case: $1 \geq \alpha \geq 0$.** Where $\alpha$ lies between zero and unity, for a particular value of $\beta$ the associated values of $dW$ and $de$ lie on the appropriate ZNM line. For example, with $\beta = 1$, the appropriate ZNM function is $Z_0$. In this case the equilibrium will lie on the line.
0HD, where the closer the value of $\alpha$ is to zero, the closer is the equilibrium to D. For lower values of $\beta$ ($1 > \beta \geq 0$), the ZNM function is above and parallel to 0HD. The relevant range of equilibrium values will again lie between the vertical zero employment rate change line, AJ0C, and the BRW function, $B_1$, shown as ADMLE. Again, the less workers bargain for a social wage, the closer the equilibrium will be to the ADMLE curve.

It is clear that the equilibrium must lie in the shaded area in Figure 1. Where $1 \geq \beta \geq 0$, the equilibrium is within the darker shaded triangle, AD0. With these parameter restrictions, there is only one point where inter-regional competitiveness is not negatively affected by the fiscal expansion. This is where $\alpha = \beta = 1$, the equilibrium at the origin. In every other outcome in the triangle AD0, regional competitiveness is reduced. Where $\beta > 1$, the possible equilibria are represented by the lighter shading. In these cases there are combinations of the BRW and ZNM functions where the nominal wage falls, so that regional competitiveness could increase with a local fiscal expansion.

**Changes in Total Employment, $dn$**

The results in Figure 1 give changes in the nominal wage and the employment rate, but our central concern is the change in economic activity and specifically in aggregate employment. In general the level of employment and the employment rate diverge because the population (and therefore the work force) is endogenous. Figure 1 shows that under a wide range of parameter values, a balanced fiscal expansion generates an increase in the nominal wage and therefore reduces regional competitiveness. However, where this is the case, the change in employment is the result of the trade-off between the positive demand-side stimulus, generated
by the Keynesian balanced budget multiplier, and the negative competitiveness effects, produced by the higher nominal wage.

This analysis follows that of Knoester and van der Windt (1987) who argue that, at a national level, forward tax shifting by workers produces a reduction in competitiveness and therefore a possible inverted Haavelmo effect; that is, a negative balanced budget multiplier. Substituting equation (8) into equation (4) gives the employment change as

\[
(9) \quad dn = \left[ n\gamma (1-cpi) + n_w (1-\beta - z\epsilon (1-\alpha\beta)) \right] d\tau \frac{1}{(1-cpi)(b-\epsilon)}
\]

where \( \frac{\partial (dn)}{\partial \alpha}, \frac{\partial (dn)}{\partial \beta}, \frac{\partial^2 (dn)}{\partial \alpha \partial \beta} \geq 0 \).

Clearly the change in employment is positively related to the value of the amenity generated by the government expenditure, \( \beta \), and the extent to which this is reflected in the regional bargained wage, \( \alpha \). However, our central concern is the sign of the employment change that accompanies the balanced fiscal expansion. Again we approach this both diagrammatically and algebraically.

First, setting \( dn = 0 \) in equation (4) and rearranging gives the value of \( dW \) for which the fiscal expansion has a zero employment impact

\[
0 = -\frac{n}{n_w} d\tau \geq 0.
\]

This line is plotted in Figure 1 as GMHJK, where the intercept J on the \( dW \) axis is \( -(n/ n_w) d\tau \). All combinations of the change in pre-tax nominal wage and employment rate below GHJK produce an increase in employment. All points in Figure 1 above this line result
in a reduction in employment.

Equilibria involving no increase in the pre-tax nominal wage are unambiguously associated with an expansion in employment. This includes the origin, which is the equilibrium where $\alpha = \beta = 1$. Here no price changes accompany the fiscal expansion so that the regional economy operates as under the standard Keynesian balanced budget multiplier with $dn = n_d \Delta r$. But there is also a range of equilibria where the change in pre-tax nominal wage is positive, so that regional competitiveness falls but employment still rises. The corollary is that as long as there is a positive demand side stimulus from the balanced fiscal expansion, so that $n_r > 0$, there is always some set of values for $\alpha$ and $\beta$ in the range $1 \geq \alpha, \beta \geq 0$ where employment change will be positive. In Figure 1 the equilibria falling in the triangle 0HJ are in this category.

An alternative approach is to set $dn$ equal to zero in equation (9), and rearrange to generate the combinations of the parameters $\alpha$ and $\beta$ that produce zero employment change. For a positive employment change the $\beta$ parameter must be greater than a minimum value, $\beta$, given by

$$
\beta = \left[ \frac{b_e - z_e}{b_e - z_e \alpha} \right] \left[ 1 + \frac{n_e (1 - cp_i)}{n_w} \right]
$$

(10)

where $\frac{\partial \beta}{\partial n_e} < 0$ and $\frac{\partial \beta}{\partial n_w} > 0$, recalling that $n_w$ takes a negative sign.

Without imposing additional parameter restrictions, equations (9) and (10) are difficult to interpret intuitively. Figure 1 helps conceptually because a number of the parameters enter only one of the three functions identified there, so that the effect of varying individual parameters is clarified. However, the sign of the employment change is determined by the relative size of the different effects, which proves difficult to capture analytically. This implies
that key general equilibrium elasticities, specifically $\pi_{w}, n_{w}$ and $n_{x}$, determine not just the quantitative but also the qualitative nature of the balanced fiscal expansion. These elasticities depend upon the full operation of the economic system. We therefore pursue the analysis through numerical simulation using a regional Computable General Equilibrium model, employing extensive sensitivity analysis.\textsuperscript{10}

4. COMPUTABLE GENERAL EQUILIBRIUM MODELLING: AN APPLICATION TO SCOTTISH DEVOLUTION

Regional Computable General Equilibrium Modelling

The use of Computable General Equilibrium (CGE) simulation to identify the likely impacts of fiscal innovations is well established both at the national (Shoven and Whalley, 1992) and regional levels (Hirte, 1998; Partridge and Rickman, 2010). We adopt such an approach here in order to operationalize the analytical model outlined in Sections 2 and 3. However, the CGE simulations also generate additional information. The analytical model focuses on the aggregate labour market. The CGE simulations widen the scope of the study to incorporate the changes in the values of a range of additional aggregate variables, such as GDP, household consumption and investment, and allow for sectoral disaggregation. Similarly, the analytical model gives only long-run equilibrium values while the CGE dynamic simulations identify the evolution of the economy to the new long-run equilibrium.

One problem in tackling this issue through simulation is specifying the appropriate values for the parameters $\alpha$ and $\beta$. The UK has no experience of a local income tax and there is no
consensus, even at the national level, on the nature of long-run tax effects on the bargained real wage (Church et al., 1993). Further, whilst there is evidence from other countries on values of $\alpha$ and $\beta$, the results are extremely mixed and depend on the composition of public expenditures (Bartik, 1992; Cebula, 2002; Dahlberg and Fredriksson, 2001; Dalenberg and Partridge, 1995; Day, 1992; Feld and Kirchgassner, 2002; Fisher, 1997; Gabe and Bell, 2004; Helms, 1985; Mofidi and Stone, 1990; Wallace, 1993). The available empirical evidence therefore does not allow us to tie down the values of $\alpha$ and $\beta$ precisely. However, our reading of the literature is that the tendency of conventional macroeconomic analysis to ignore the potential direct benefits of regional public expenditures is rejected by those studies that provide a balanced treatment of tax and expenditure effects (e.g. Gabe and Bell, 2004). Furthermore, the suggestion that the composition of expenditures influences key parameter values implies that these are sensitive to policy choices. Against this background, there is a strong case for progressing the analysis via numerical simulation as long as the sensitivity of the results to the values taken for $\alpha$ and $\beta$ is a central feature.

**AMOS: A Macro-Micro Model of Scotland**

AMOS is a CGE modelling framework calibrated to a base year Social Accounting Matrix for Scotland built around the 2004 Scottish Input-Output Table (Office of the Chief Economic Adviser, Scottish Government, 2007). AMOS is an acronym for A Micro-Macro Model Of the Scottish Economy, so named because it combines micro and macro-economic elements. Essentially, it is a fully specified, empirical implementation of the skeletal theoretical model developed in Sections 2 and 3. It has three domestic transactor groups. These are the personal sector, corporations and government; and four major components of final demand:
consumption, investment, government expenditure and exports. There are eleven commodities/activities but in the simulation results reported in Table 2, these are aggregated into three broad industrial groups: manufacturing, non-manufacturing traded and a sheltered sector.\textsuperscript{12}

In this version of the model, consumption and investment decisions reflect inter-temporal optimization with perfect foresight. The core equations for this element of the model are available in Lecca et al. (2010; 2013). Real government expenditure is equal to the base year level plus an additional amount that just exhausts the incremental revenues raised by the local income tax. This implies that government expenditure becomes dependent on the entire general equilibrium of the system, which is exactly what would happen if the Scottish Variable Rate were to be implemented. The demand for Scottish, Rest of the UK (RUK) and Rest of the World (ROW) exports is determined via conventional export demand functions where the default price elasticity is set at 2.0. Imports are obtained through an Armington link (Armington, 1969) and therefore relative-price sensitive with trade substitution elasticities of 2.0 (Gibson, 1990).

In all the simulations in this paper there is one representative household and a single Scottish labour market characterised by perfect sectoral mobility. All industrial sectors are taken to be perfectly competitive and produce using multi-level CES production functions. The elasticity of substitution in the production of gross output and value added is 0.3 and is Leontief for intermediate demands (Harris, 1989). We do not explicitly model financial flows, our assumption being that Scotland is a price-taker in competitive UK financial markets.
As regards demographic developments, we assume no natural population change but the size of the labour force adjusts over time through migration. This adjustment is determined using the econometrically parameterised regional net migration function reported in Layard et al (1991), augmented to accommodate the amenity effects discussed in Sections 2 and 3. The model starts in long-run equilibrium with a zero net migration flow. Subsequently, between periods migration updates the labour force. The level of net migration is positively related to the gap between regional and national real tax-adjusted wages, and negatively related to the gap between regional and national unemployment rates

\[ m = \zeta - 0.08 \ln(u^S) - \ln(u^R) + 0.06 \left[ \ln \left( \frac{w^S}{cpi^S} \right) - \beta \ln(1-\tau) - \ln \left( \frac{w^R}{cpi^R} \right) \right] \]

where \( m \) is net in-migration as a proportion of the regional population; \( u \) is the unemployment rate; \( \beta \) is the relative valuation of the public expenditure; the \( S \) and \( R \) superscripts stand for Scotland and the Rest of the World, respectively; and \( \zeta \) is a parameter calibrated to specify zero net migration in the base period. In the long run, there is an implied zero-net-migration condition that corresponds to the ZNM function given as equation (1). This is

\[ \ln \left( \frac{w^S}{cpi^S} \right) = b + 1.33 \ln(u^S) + \beta \ln(1-\tau) \]

where \( b \) is again a calibrated parameter.

Wage setting is determined by a regional bargained real wage function that embodies the econometrically derived specification given in Layard et al (1991), again augmented by amenity effects

\[ \ln \left( \frac{w^S}{cpi^S} \right) = c - 0.113 \ln(u^S) + \alpha \beta \ln(1-\tau) \]

where \( \alpha \) represents the extent to which the amenity effect is reflected in the wage bargain and \( c \)
is a calibrated parameter.\textsuperscript{13} This formulation explicitly acknowledges imperfect competition and unemployment in the labour market.

\textbf{5. SIMULATION RESULTS}

Under the Scotland Act (1998) the Scottish Parliament has the authority to make a balanced-budget adjustment in public expenditure, over and above the expenditure funded by the present block grant. This fiscal adjustment has to be accompanied by a corresponding local modification to the basic rate of income tax of up to three pence in the pound. This tax facility is known as the Scottish Variable Rate (SVR).\textsuperscript{14} In the present paper we use AMOS to simulate the long-run effects of activating the full Scottish Variable Rate in order to produce a balanced budget increase in public consumption. Given that the model is parameterised on the 2004 Social Accounting Matrix for Scotland we use the 2004 HM Treasury Budget estimate that these fiscal powers would raise £810 million at 2004 prices, which represents a 1.52 percentage point rise in average personal income tax in AMOS.

Table 1 reports the long-run proportionate changes in total Scottish employment after the introduction of such a tax for combinations of $\alpha$ and $\beta$, where $\alpha \in [0,1]$, $\beta \in [0,2]$. That is to say, this table reports results for a wide variation in the value that residents (and potential migrants) place on additional public expenditure and the incorporation of public expenditure decisions into the wage bargain. Figure 2 illustrates these results graphically.

These outcomes are consistent with the analytical results generated in Section 3.2. When the parameters $\alpha = \beta = 1$, replicating the standard Keynesian balanced budget multiplier, the
employment change is positive at 0.60 percent. Further, the proportionate change in employment is positively related to the values of the parameters $\alpha$ and $\beta$, so that positive figures are reported for parameter value combinations towards the upper right-hand corner of Table 1. However, it is clear that relatively modest reductions in the value of either parameter $\alpha$ or $\beta$ below unity leads to employment falling with a balanced budget fiscal expansion. For example, if $\beta < 0.6$, there is no value of $\alpha$ which will generate a positive employment change. Similarly, if $\alpha < 0.24$ even a value of $\beta$ equal to 2 - so that public expenditure is valued at a level twice that of private expenditure - generate a negative employment change. This implies that where there is no social bargaining, so that $\alpha = 0$, employment falls for any $\beta$ value within the range adopted here.\textsuperscript{15}

In Table 2 we give the proportionate changes in a more comprehensive set of economic variables for four particular combinations of $\alpha$ and $\beta$. This allows a fuller investigation of the economic forces at work in each of these cases. The first column shows results from the simulation where $\alpha = \beta = 1$, which emulates the conventional Keynesian balanced budget multiplier case, albeit with endogenous investment and population. This is represented by the outcome at the origin in Figure 1.

In this simulation there is no change in either the pre-tax nominal wage or the unemployment rate, so that there are no adjustments in a number of key endogenous variables. These include long-run value-added prices, the cost-minimising choice of production technique in each industry and the level of exports. Essentially the economy operates as an extended input-output system where, in each sector, output, employment and capital stock vary by the same proportionate amount.\textsuperscript{16} The level and composition of exports remains the same and the
demand disturbance comes solely through the switching of private consumption expenditure to
public expenditure.

As argued in Sections 2 and 3, this expenditure switching has a general expansionary impact
on the regional economy. The 3.57 percent increase in government expenditure produces an
increase in Scottish GDP of 0.40 percent and in employment and population of 0.60 percent.
However, the adjustment in consumption and government demand has an uneven effect across
sectors. Value added in the sheltered sector, which is most strongly represented in government
expenditure, increases by 2.11 percent. In the other two sectors, which are more strongly
related to household consumption, value added falls, but by a relatively small amount, -0.03
percent, in manufacturing and by -0.31 percent in the non-manufacturing traded sector.17

The second column gives the simulation results with the parameter values $\alpha = 0$ and $\beta = 1$.
This simulation is for an expansion in public expenditure that is valued equally to an increase
in private consumption but where the labour market fails to incorporate this in wage bargains.
This equilibrium is represented by point D in Figure 1. This combination of parameter values
generates increases in the nominal pre-tax wage (2.39 percent) and the unemployment rate
(1.35 percent), resulting in negative competitive effects as value-added prices increase in all
sectors. The fall in aggregate exports swamps any expansionary impacts generated by the
other shifts in final demand. Scottish GDP, total employment and population decline by 0.84
percent, 0.78 percent and 0.68 percent respectively. Activity still increases in the sheltered
sector. However, this is more than offset by corresponding falls in the manufacturing and non-
manufacturing traded sectors. The decline in Scottish real income is associated with a smaller
rise in the endogenous public expenditure, which increases by 3.27 percent with these
parameter values.

The results in the third column are derived where $\beta = 0$. This corresponds to point A in Figure 1, where the amenity has no value to existing or potential Scottish residents. In this simulation there is no change in the post-tax real consumption wage or the unemployment rate. The nominal pre-tax wage increases by 2.60 percent, the full extent of the tax plus the rise in the consumer price index. Scottish GDP and employment fall by 0.95 percent and 0.90 percent respectively. The impact on individual sectors is qualitatively similar to the case where $\alpha = 0$ and $\beta = 1$, but with the activity in all sectors slightly lower than in the earlier simulation. This is the worst-case scenario for the impact of the Scottish Variable Rate on economic activity.

The final simulation, reported in column four, adopts the parameter values $\alpha = 0.80$, $\beta = 1.20$ and represents an equilibrium lying in the area 0HML in Figure 1 where $\beta > 1$ and both employment and the nominal pre-tax wage increase. The 0.05 percent rise in the pre-tax nominal wage following the introduction of the Scottish Variable Rate reduces exports in all sectors. However, the other expansionary fiscal demand impacts produce a more than offsetting effect on overall Scottish aggregate activity. Therefore, although there are small employment falls in the manufacturing and non-manufacturing traded sectors of 0.07 percent and 0.37 percent, employment in the sheltered sector rises by 2.14 percent, producing an aggregate increase in GDP and employment of 0.37 percent and 0.57 percent respectively.

6. THE ADJUSTMENT PROCESS AND SENSITIVITY

Up to this point, the analysis and the simulations have involved changes in long-run
equilibrium values. In this section we consider the adjustment path and undertake some sensitivity analysis. We look in particular at the sensitivity of the aggregate results to the openness of the regional economy.

One criticism of CGE models is that they are not fully econometrically estimated and the results might be very sensitive to imposed parameter values. In this sensitivity exercise, we follow the method used by Harrison and Vinod (1992) in which the values of key parameters are varied randomly around their default levels. Specifically, 1000 simulations are run in which the value of the constant elasticity of marginal utility is selected from a uniform distribution with the range \((0.2 - 1.6)\), the value-added production function substitution elasticities from the range \((0.1 - 0.5)\) and the trade elasticities from the range \((0.1 - 4.0)\). We divide each distribution into 4 equal intervals and adopt a complete randomized factorial design. Each of the 1000 simulations is run for 50 periods.

Figures 3 and 4 report the results of systematic sensitivity analysis on the period-by-period simulations for two of the \((\alpha, \beta)\) combinations reported in Table 2: \((1,1)\) and \((0.8, 1.2)\). In each period the graphs show the mean solution value of the percentage increase in total employment of the 1000 simulations together with the plus-or-minus-one-standard-deviation range of results. Two broad conclusions can be drawn.

The first is that the economy adjusts very rapidly to the fiscal expansion. In both sets of simulations the mean employment change in period 1 is around 60 percent of the long-run change and the mean employment reaches its long-run level in period 3. There is some overshooting at this point but the adjustment is generally very rapid. Second, the one standard
deviation confidence limits are small and fall over time. This is because in these two cases, migration and investment reduce the price deviations upon which the production and demand elasticities bite. This is particularly apparent in the simulation results reported in Figure 3, where $\alpha$ and $\beta$ are both unity. In this simulation in the long run there are no relative price changes, so that extended Input-Output results hold. Therefore variation in price elasticities plays no role and the confidence range ultimately collapses to a single point (McGregor et al., 1996). In Figure 4, where $(\alpha, \beta)$ values are $(0.8, 1.2)$, price changes are still present in the long run so that employment remains sensitive to these parameter values.

As is apparent from equation (4), the overall employment change from a balanced budget fiscal expansion depends on the trade-off between the stimulus coming through the conventional Keynesian demand shock and the potential negative competitiveness effect generated in the labour market. The absolute size of any adverse competitiveness effect will be positively related to the openness of the economy. This line of enquiry is particularly relevant for regional economies, which are typically much more open than national economies for two reasons. First, production for extra-regional exports generally makes up a much greater share of the local economy. Second, the price elasticity of demand for regionally traded products will be higher than corresponding national trade because regional commodities are closer substitutes and transactions costs are lower.\textsuperscript{18} We can gain a limited indication of the strength of these competitiveness effects by showing the sensitivity of our results to changes in the export demand elasticities.

Figure 5 gives the percentage changes in Scottish long-run aggregate employment for different values of the price elasticity of demand for all (rest of the UK and rest of the world) exports.
Two illustrative cases are considered which involve simulations that differ in their $\alpha$ and $\beta$ values. These simulations, which we label A and B, have the corresponding $(\alpha, \beta)$ values: (0.8, 0.8) and (0.4, 1.2).

The default value for the export demand elasticity is 2. The results in Table 1, which were simulated using the default parameter values, indicate that simulation A registers a positive, 0.08 percent, and simulation B a negative, -0.10 percent, change in employment. These results are replicated in Figure 5. However, note that the total employment change is negatively related to the export demand elasticity and this can affect not simply the magnitude, but also the sign, of the employment change. In particular, at elasticity values above 2.74, simulation A now generates a fall in aggregate employment, whilst at elasticity values below 1.295 simulation B produces an employment increase. Clearly, the more open the economy, the more likely that inverted Haavelmo effects will occur. The corollary is that the more closed the economy, the more likely is the conventional Keynesian outcome.\textsuperscript{19}

7. CONCLUSIONS

This paper analyzes the net impact of a regional tax increase made in order to finance a rise in public consumption that is assumed to have, of itself, no direct supply-side implications. The key finding is that the effect on the overall level of activity in the region can be positive or negative, depending upon the relative size of positive demand and potentially negative competitiveness effects. Further, the sign of this net impact depends crucially on the value that existing and potential residents place on the resulting public amenity and the extent to which this value is reflected in a moderation of local pay claims. These are important results, given
the very large number of regions in the EU and North America that are able to engage in balanced budget fiscal expansions. Further, because regions are typically more open than nations, the potential negative competitive effects of local tax changes are more important for fiscal decisions taken at this level.

Moderation of wage claims is not available if bargaining takes an atomistic form, where there is no mechanism to internalise the benefit of the publicly provided amenity. Our simulation results suggest that the balanced budget employment multipliers would be negative in such circumstances. However, in an imperfectly competitive labour market where unions are concerned with the general welfare of their members, such inverted Haavelmo effects do not necessarily occur. Under these circumstances an expansion in public expenditure has smaller adverse supply-side effects and balanced budget multipliers can become positive. Where supply side effects are significantly moderated by households’ valuation of the public amenity and their bargaining behavior, there can be expansionary impacts on the host economy. In the limit, these would generate outcomes of the kind associated with the traditional Keynesian vision of regional economies (here in the context of both endogenous investment and inter-regional migration). In these circumstances a key role for politicians may be to reinforce the communal benefits to wage restraint as a trade-off for increased public expenditure. Alex Salmond, Scotland’s First Minister, has made explicit reference to the importance of the social wage. The social wage is the wage made up of both public and private consumption. This may be a much more important theme in the future.

The case for balanced budget fiscal expansion would, of course, be enhanced by the presence of any degree of nominal wage inflexibility at the regional level which would inhibit adverse
competitiveness effects. Similarly, the presence of any positive supply-side impact of the public expenditure changes introduces a direct countervailing supply-side stimulus to competitiveness. For example, the provision of public infrastructure which improves the general efficiency of private firms would have such an effect. In this case, the impact on competitiveness would depend on the extent to which the increase in productivity directly offsets any rise in the nominal wage resulting from the tax increase. Future research should explore the importance of these effects, and of the potential for the various elements of government expenditure to be valued differently. Finally, the introduction of balanced budget fiscal adjustments in an inter-regional context which incorporates endogenous policy responses would be a valuable avenue of research.
REFERENCES


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Silvestre, Joaquim. 1990. “There May be Unemployment when the Labour Market is Competitive and the Output Market is Not,” The Economic Journal, 100, 899-913.


TABLE 1: Long-run % change in employment following the introduction of the Scottish Variable Rate, for combinations of parameters $\alpha$ and $\beta^*$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
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<tr>
<td></td>
<td>0.00</td>
</tr>
<tr>
<td>2.00</td>
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</tr>
<tr>
<td>1.80</td>
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<tr>
<td>1.60</td>
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<td>1.40</td>
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<td>1.20</td>
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<tr>
<td>1.00</td>
<td>-0.78</td>
</tr>
<tr>
<td>0.80</td>
<td>-0.80</td>
</tr>
<tr>
<td>0.60</td>
<td>-0.83</td>
</tr>
<tr>
<td>0.40</td>
<td>-0.85</td>
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<td>0.20</td>
<td>-0.87</td>
</tr>
<tr>
<td>0.00</td>
<td>-0.90</td>
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*The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
TABLE 2: Long-run % change in key economic variables following the introduction of the Scottish Variable Rate, for combinations of parameters $\alpha$ and $\beta^*$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regional Bargaining</th>
<th>$\alpha=\beta=1$</th>
<th>$\alpha=0\beta=1$</th>
<th>$\beta=0$</th>
<th>$\alpha=0.8\beta=1.2$</th>
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<tr>
<td>GDP</td>
<td></td>
<td>0.40</td>
<td>-0.84</td>
<td>-0.95</td>
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<tr>
<td>Consumption</td>
<td></td>
<td>-1.30</td>
<td>-1.36</td>
<td>-1.36</td>
<td>-1.30</td>
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<tr>
<td>Govt expend.</td>
<td></td>
<td>3.57</td>
<td>3.27</td>
<td>3.25</td>
<td>3.56</td>
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<tr>
<td>Investment</td>
<td></td>
<td>0.05</td>
<td>-1.03</td>
<td>-1.12</td>
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</tr>
<tr>
<td>Nominal pre-tax wage</td>
<td></td>
<td>0.00</td>
<td>2.39</td>
<td>2.60</td>
<td>0.05</td>
</tr>
<tr>
<td>Real post-tax wage</td>
<td></td>
<td>-1.93</td>
<td>-0.15</td>
<td>0.00</td>
<td>-1.89</td>
</tr>
<tr>
<td>Total employment</td>
<td></td>
<td>0.60</td>
<td>-0.78</td>
<td>-0.90</td>
<td>0.57</td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
<td>-0.03</td>
<td>-1.53</td>
<td>-1.66</td>
<td>-0.07</td>
</tr>
<tr>
<td>Non-Manufacturing</td>
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<td>-0.33</td>
<td>-2.02</td>
<td>-2.17</td>
<td>-0.37</td>
</tr>
<tr>
<td>Traded</td>
<td>Sheltered</td>
<td>2.16</td>
<td>1.27</td>
<td>1.19</td>
<td>2.14</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td></td>
<td>0.00</td>
<td>1.35</td>
<td>0.00</td>
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<tr>
<td>Total population</td>
<td></td>
<td>0.60</td>
<td>-0.68</td>
<td>-0.90</td>
<td>0.59</td>
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<tr>
<td>Price of value added</td>
<td>Manufacturing</td>
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<td>1.76</td>
<td>1.92</td>
<td>0.04</td>
</tr>
<tr>
<td>Non-Manufacturing</td>
<td>Traded</td>
<td>0.00</td>
<td>1.56</td>
<td>1.69</td>
<td>0.04</td>
</tr>
<tr>
<td>Shadow price of capital</td>
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<td>1.94</td>
<td>2.11</td>
<td>0.04</td>
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<tr>
<td>Consumer Price Index</td>
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<td>Value added</td>
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<td>-1.35</td>
<td>-1.46</td>
<td>-0.06</td>
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<tr>
<td>Non-Manufacturing</td>
<td>Traded</td>
<td>-0.31</td>
<td>-1.76</td>
<td>-1.88</td>
<td>-0.34</td>
</tr>
<tr>
<td>Sheltered</td>
<td></td>
<td>2.11</td>
<td>1.31</td>
<td>1.25</td>
<td>2.09</td>
</tr>
<tr>
<td>Capital stock</td>
<td>Manufacturing</td>
<td>-0.03</td>
<td>-0.97</td>
<td>-1.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>Non-Manufacturing</td>
<td>Traded</td>
<td>-0.29</td>
<td>-1.45</td>
<td>-1.55</td>
<td>-0.32</td>
</tr>
<tr>
<td>Sheltered</td>
<td></td>
<td>1.87</td>
<td>1.50</td>
<td>1.47</td>
<td>1.87</td>
</tr>
<tr>
<td>Exports</td>
<td>Manufacturing</td>
<td>0.00</td>
<td>-0.64</td>
<td>-0.69</td>
<td>-0.01</td>
</tr>
<tr>
<td>Non-Manufacturing</td>
<td>Traded</td>
<td>0.00</td>
<td>-0.89</td>
<td>-0.96</td>
<td>-0.02</td>
</tr>
<tr>
<td>Sheltered</td>
<td></td>
<td>0.00</td>
<td>-1.42</td>
<td>-1.54</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

*The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
FIGURE 1: The real wage bargaining and zero net migration functions represented in nominal pre-tax wage (dW) and employment rate (de) space.
FIGURE 2: Long-run % change in employment following the introduction of the Scottish Variable Rate, for combinations of parameters $\alpha$ and $\beta^*$. The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
FIGURE 3: The sensitivity of the percentage change in total employment to parameter variability (for $\alpha = \beta = 1$)*.

*The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
FIGURE 4: The sensitivity of the percentage change in total employment to parameter variability (for $\alpha = 0.8$ and $\beta = 1$)\textsuperscript{*}.

\textsuperscript{*}The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
FIGURE 5: Sensitivity of employment change to variations in the price elasticity of demand for exports for two simulations with different values for the parameters $\alpha$ and $\beta$.

Simulation A has both the $\alpha$ and $\beta$ values set to 0.8. Simulation B has $\alpha$ and $\beta$ values set to 0.4 and 1.2 respectively. The parameter $\alpha$ measures the extent to which the local amenity created by the public expenditure is reflected in wage bargaining (equation 5) and the parameter $\beta$ measures the marginal valuation of the amenity as against private consumption (equations 1 and 5).
FOOTNOTES

1 Layard et al. (1991, Ch. 6) also assume competitive commodity markets in their analysis of disaggregated labour markets.

2 For studies that consider the impact of local taxation used to finance productive investment, such as public infrastructure and human capital improvement, see Deepak et al. (2001) and Seung and Kraybill (2001).

3 This flow equilibrium formulation is extensively used in the migration literature. See Lecca et al. (2010) for further references.

4 Equations (1) to (5) are expressed as total differentials given the following normalisation: w, W, N, cpi =1, $\tau = 0$ and n = e. The corresponding equations in level form are given in Lecca et al. (2010).

5 In the present paper the migration decision is positively related to the amenity, as in Tiebout (1956). However, a positive $\beta$ simply requires that the amenity is valued to some degree by households and is not dependent on the strict assumptions of the Tiebout model.

6 That is to say, the representative worker is indifferent between $1 marginal private consumption and the public good implications of a local fiscal arrangement where he or she pays $1 more in tax.
Silvestre (1990) describes this general equilibrium labour demand curve as the "full" demand curve for labour. No nominal inertia is implied by this formulation, which is a re-parameterised version of a specification in which labour demand is a declining function of the real product wage. In principle, this labour demand curve could be upward sloping as a result of income effects produced through changes in the terms of trade. However, in a very open regional economy we always expect the negative competitiveness impacts to dominate and this is reflected in the simulation results reported in Sections 5 and 6.

We discount the possibility that the labor supply decision could be directly influenced by the amenity, through the amenity’s being complementary to leisure, for example.

The origin is a stable equilibrium with the initial ZNM₀ and BRW₀ functions. For example, an increase in the employment rate will lead to an initial increase in the real wage through the bargaining function. However, this generates net in migration, a corresponding rise in the labor force and a subsequent reduction in the employment rate leading to a fall in the real wage. In migration will continue until the initial equilibrium wage and employment rate levels are reinstated.

Lecca et al. (2010) augments this analysis by discussing models which impose national wage bargaining or hold regional population fixed. It also reports the corresponding simulation results generated using a Computable General Equilibrium framework.

For a full specification of an early version of the model, see Harrigan et al. (1991). For the present multi-period, forward-looking version see Lecca et al. (2013).
The key characteristic of the sheltered sector is that it has very low, but not zero, exposure to trade. For a more detailed breakdown of these aggregate sectors see Lecca et al. (2010).

Equations (11), (12) and (13) are expressed, and estimated, in terms of the local unemployment rate but this is simply equal to one minus the employment rate used in Sections 2 and 3. A wage curve elasticity of around -0.1 has been found over a large number of empirical studies across different countries and time periods (Blanchflower and Oswald, 2005).

The main source of funding for the Scottish Government is a population-based block grant from the UK Government. The SVR has never been used to augment this funding. However, under the Scotland Act (2012) the Scottish Government will have to set a portion of the income tax rate for Scotland, even if this only matches the rate in the rest of the UK. But the possible variation has been increased to up to ten pence in the pound on all income tax bands. The Scottish Government has very limited alternative sources of additional revenue and has presently virtually no borrowing powers.

In Table 1 a zero employment change identifies the value of $\beta$, defined in equation (10), for a given value of $\alpha$. The relationship between $\beta$ and $\alpha$ is downward sloping and convex. This result requires a positive value for the second term on the RHS of equation (10), which occurs with a value of $n_y(1-cpi_w)$ less than the absolute value for $n_w$.

The figures reported in Table 2 do not show this precisely because the results given for the
three sectors are for aggregations of the 11 sectors in the model. For more sectoral detail see Lecca et al. (2010).

17 The per capita changes for all variables can be calculated by comparing the percentage change in the variable as against the percentage change in population. For example, GDP per head falls in all the simulations reported in Table 2. This primarily reflects the long-run changes in sectoral composition of GDP brought about by the introduction of the tax.

18 Regions are also generally more open than nations because of the greater ease of labor migration. In our model the presence of migration typically amplifies the absolute size of the employment change but only exceptionally alters the sign of that change. For a more detailed discussion see Lecca et al. (2010).

19 We also have performed sensitivity analysis on individual Armington elasticities. Economic activity is much more sensitive to varying the ROW, as against the RUK, values. Sensitivity was also performed for variations in elasticity in the bargaining function, but these had a very small impact on the change in economic activity.